Contests for Perception *

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Abstract

We observe competitive behavior in different domains of society, even without explicit monetary prizes. In this paper, I model a mechanism that may drive such behavior: I study a contest model where prizes are given by an inactive observer's posterior belief about a player's ability. In other words, prizes are determined endogenously in the model. I define the equilibrium in this game and show how expected effort changes with two exogenous parameters: the probability of an agent being high ability and the difference in productive ability between types. I show that expected effort is maximized when uncertainty about players' abilities is the highest. I identify a novel *encouragement* effect of ability asymmetry. Total expected effort can increase in ability asymmetry: when the prizes are determined in equilibrium, the *discouragement* effect from ability heterogeneity can be reversed when heterogeneity is sufficiently low. I also analyze win probabilities when allowing for the observer's prior to depend on players' identities: I identify the "underdog effect" where initially decreasing the prior belief about the ability of the player with lower expected ability can nonetheless increase her win probability.

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Nature, when she formed man for society, [...] taught him to feel pleasure in their favourable, and pain in their unfavourable regard.

—Adam Smith, *The Theory of Moral Sentiments* (1759)

1 Introduction

Social image concerns exert a strong influence on human behavior in many contexts (Goffman 1959; Bursztyn and Jensen 2017): voters turn out to vote in person to signal their civic types (Gerber, Green, and Larimer 2008; Funk 2010; DellaVigna et al. 2016), credit cards associated with high status are more likely to be used when this is visible to observers (Bursztyn et al. 2018) and merely the visibility of workplace achievement can motivate additional effort (Kosfeld and Neckermann 2011). Social image concerns can also help explain the pervasive competitive behavior we observe in society, even in the absence of explicit monetary incentives or other immediate instrumental benefits to agents. The desire to outrank peers is a strong motivating force in and of itself (Anderson, Hildreth, and Howland 2015), leading individuals to expend substantial effort and take significant risks even in high-stakes environments (Ager et al. 2022).

This paper examines a key mechanism that motivates effort in competitions without monetary rewards: information revelation about competing agents' abilities. When performance in a competitive task depends on talent, the outcome of the competition conveys information about competitors' abilities. Observers can use this information to update their beliefs about agents. When these agents care about how others perceive their ability,[1] this concern translates observers' posterior beliefs into implicit prizes. Having these implicit prizes leads to agents exerting costly effort while competing to obtain the high-ability signal of winning the contest. This paper studies how such perception-based motivation shapes competitive behavior.

I develop a contest model where prizes are determined endogenously by an observer's posterior belief about players' abilities conditional on the primitives of the model: Two players whose abilities are drawn independently compete in a task, in which the identity of the winner is determined by whose output is higher. The competing players know each other's realized ability type, but the observer only knows the distribution from which these types are drawn. The observer is only informed about the identity of the winner (and the loser) in the competition but not absolute individual output. She updates her beliefs about the competing players' abilities from this relative performance information.

1. The motive to be perceived as competent may be ingrained by culture (Markus 2016) or could be rooted in the drive to increase fitness, as argued by the literature on evolutionary biology (Schauer 2022; Lane and Briffa 2022). Self-determination theory in psychology (Deci and Ryan 2000) argues that the need for competence is a basic psychological need.

I assume that players derive utility from the observer's posterior belief (similar to Bénabou and Tirole (2006) and Bursztyn and Jensen (2017)), hence I can derive the prizes in the contest associated with this psychological payoff in equilibrium, rather than fixing prizes in the contest exogenously. I establish a direct connection between the literature on contest theory and social image concerns and show how the mechanism described above can motivate agents to exert costly effort even in the absence of explicit prizes. Deriving the equilibrium in the game described above is the first, and most conceptual contribution to the literature on how agents are motivated to exert costly effort when they have social image concerns (Frey 2007; Ellingsen and Johannesson 2007; Moldovanu, Sela, and Shi 2007; Besley and Ghatak 2008; Auriol and Renault 2008).

While several contest theory papers incorporate social image in the form of status concerns (Moldovanu, Sela, and Shi 2007; Ederer and Patacconi 2010), status utility is purely driven by relative positional concerns in these models. In contrast, utility from perception (which can be interpreted as status in this framework) depends on the likely ability of players and ability asymmetry between types, not simply realized relative position or relative wage. This model is also fundamentally different from signaling models often used in studying the behavior of agents with social image concerns (Dufwenberg and Lundholm 2001; Austen-Smith and Fryer 2005; Hupkau and Maniquet 2018; Bursztyn, Egorov, and Jensen 2019), where an agent's ability to produce a given signal depends only on her type. In contrast, the "good" signal in this model—i.e., winning the contest—is perfectly rivalrous: the *ability* to signal also depends on other agents' actions rather than exclusively on the agent's cost of doing so, as is the case in standard signaling models (Spence 1973).

Analyzing the model leads to several novel insights about how agents behave when competing for the perception of being talented. First, competition is most intense when a player is equally likely to have high or low ability. This is because winning is only informative about ability asymmetry when the competing players indeed differ along this dimension: the probability of this event occurring is highest when the two ability types are drawn with equal probabilities—in other words, when the observer has flat priors assuming she has correct beliefs. This result relates to predictions from social psychology: more similar agents have greater comparison concerns, which in turn motivate more intense competitive behavior (Festinger 1954; Kilduff, Elfenbein, and Staw 2010; Garcia, Tor, and Schiff 2013).

While it remains true in this model as well that homogeneous contests[2] yield the highest expected effort holding the probability of a player being high ability (μ) and ability asymmetry (α) constant, I provide further testable predictions related to agent similarity. Increasing ability asymmetry—thereby decreasing similarity— makes the difference between posterior beliefs conditional on winning and losing (i.e., the psychological prize

2. Homogeneity in this context refers to the case whereby competing players have the same ability in the contest.

spread) larger. This can increase effort provision in any homogeneous contest, regardless of whether ability asymmetry is increased by decreasing the productive ability of the low-ability player or by increasing that of the high-ability player. Furthermore, increasing ability asymmetry α may also increase expected effort in heterogeneous contests[3] if α is not too large. While growing ability asymmetry typically discourages effort in contests through what the literature coins as the "discouragement effect" (Dechenaux, Kovenock, and Sheremeta 2015; Fu and Wu 2019; Drugov and Ryvkin 2022), I derive conditions under which larger ability gaps—even when achieved by decreasing the productive ability of lower ability type—can increase motivation when psychological prizes adjust endogenously: I identify an "encouragement effect" of ability heterogeneity. This mechanism is present in the model because a larger ability asymmetry makes winning more informative about being high ability, potentially offsetting the usual discouragement effect from competing against a now stronger opponent.[4] While it remains true that the direct effect of increasing ability asymmetry on effort provision is weakly negative, I show that when prizes are (at least partially) given by agents' perceived ability, the increasing psychological prize may induce players to work harder as ability asymmetry grows up until some threshold value.

These results relate to the behavior of agents once abilities are realized. However, when can we expect effort to be the highest when we do not know the abilities of the competing agents? We know that with fixed prizes, competition is most intense between two high-ability agents, the probability of which occurring is highest when μ approaches 1.[5] However, when prizes are endogenously determined by some increasing function of the observer's posterior belief, agents have very little motivation to exert effort as the difference in posteriors conditional on observing a win or a loss goes to zero as the probability of a player being high ability approaches one. I then show that μ^*—the prior that maximizes unconditional expected effort—must be strictly greater than 1/2 and less than 1. When I allow an exogenous monetary prize to be awarded to the winner, this pushes the effort-maximizing prior μ^* away from 1/2 towards 1: As the motivating power of a fixed monetary prize does not depend on the observer's prior, the relative importance of psychological payoffs decreases, and we approach the classical framework of exogenously given prizes.

I extend the basic model to allow for the observer to have different priors about players' abilities. I provide sufficient conditions for an equilibrium to exist, and confirm the central

3. As opposed to homogeneous contests, competing players have different abilities in heterogeneous contests.

4. This result adds to recent work providing conditions under which the discouragement effect does not hold in contests where an interior pure strategy equilibrium exists (Drugov and Ryvkin 2022)—this does not include the all-pay contest I study in this paper.

5. In the model below I assume that both ability types can occur with strictly positive probability, hence $\mu \neq 1$, and the probability of two high-ability player competing is maximized when $\mu \in (0, 1)$ is infinitesimally smaller than 1.

intuition obtained in the analysis assuming homogeneous priors: the incentive to provide effort disappears as uncertainty about the player's ability vanishes from the observer's perspective. However, due to the power imbalance between the players introduced by the observer having different prior beliefs about their ability, maximal uncertainty does not maximize expected effort provision. I then analyze how win probabilities change when the observer's prior changes about a player's ability. Importantly, I identify the "underdog effect" discussed in the management science literature (Nurmohamed 2020; Nurmohamed, Kundro, and Myers 2021): when an observer has a low prior belief about a player's ability, decreasing this belief further can initially increase the player's win probability (the "underdog effect"), but this advantage reverses as the prior approaches zero and uncertainty about a player's ability vanishes. Prior work cites the motivation from proving others wrong as the mechanism driving this effect (see for example Nurmohamed (2020)). I provide an alternative mechanism that can generate predictions consistent with empirical findings in the literature: both the effort-inducing "underdog effect" when the prior belief associated with a player's ability further decreases until a threshold, and a "complacency effect" that starts to dominate beyond the threshold and induces the player to reduce effort leading to decreasing win probability as uncertainty about her ability gradually declines.

The paper proceeds as follows: In Section 2, I introduce the model and discuss how the assumptions relate to prior work, including sociology and social psychology. Section 3 defines and characterizes the equilibrium in the game. Section 4 analyzes expected effort provision with fixed ability match-ups, while Section 5 studies expected effort provision from the perspective of the observer who does not know the realized abilities of players. I extend the baseline model to allow for heterogeneous prior beliefs about players' abilities in Section 6 and analyze expected effort provision and changes in win probabilities as a response to changing prior beliefs. Section 7 concludes. The proofs are relegated to the Appendix.

2 Model

There are two risk neutral agents (or "players"), indexed $i \in \{1, 2\}$. Each agent can be of either high- or low-ability type. Agents' ability types are denoted $a_i \in \{h, l\}$ (high vs. low, respectively) with $h > l > 0$. I define $\alpha := h/l$ as the degree of ability heterogeneity. Ability is distributed independently and identically across agents, such that for agent i, the probability that $a_i = h$ is $\mu \in (0, 1)$. After abilities are realized, each agent can observe her own as well as the other agent's abilities. Additionally, there is an outside observer who knows the distribution from which abilities are drawn, but cannot observe the realizations.[6]

The two agents compete over a monetary prize of value $M \geq 0$ by simultaneously exerting effort $x_i \geq 0$ at cost $c_i(x_i) \geq 0$. I assume that $c_i(\cdot)$ is a linear function such that $c_i(x_i) = x_i/a_i$. Because, by assumption, $h > l$, the linear cost function implies that at any given positive effort level, both the cost and the marginal cost of effort are strictly lower for a high-ability player than for a low-ability one.

The win probability of Player i, $w_i(x_i, x_{-i})$, depends on the effort exerted by the players

$$w_i(x_i, x_{-i}) = \begin{cases} 1 & \text{if } x_i > x_{-i} \\ \frac{1}{2} & \text{if } x_i = x_{-i} \\ 0 & \text{if } x_i < x_{-i} \end{cases}. \tag{1}$$

In other words, the agent exerting strictly higher effort wins the competition. In case of a draw, a winner is chosen randomly. In this economic environment, luck plays no role as long as $x_i \neq x_{-i}$, only merit (as defined in Cappelen et al. (2023)) matters in determining the winner: the game is an all-pay contest. I assume that both players' outside option is zero if they do not compete, this assumption guarantees that both will participate with probability one.

The observer observes the identity of the winner (but not the absolute level of effort submissions) and uses this information to update her beliefs about both agents' abilities using Bayes' rule. The agents derive utility from how the observer perceives them. Specifically, both agents attach value $\kappa > 0$ to the observer's posterior belief $\tilde{a}$ that they have high ability.[7] The model, therefore, describes a psychological game (Battigalli and Dufwenberg 2022).

Agent i receives non-monetary payoffs $\overline{v} := \kappa \cdot \Pr[a_i = h \mid i \text{ wins}]$ and $\underline{v} := \kappa \cdot \Pr[a_i = h \mid i \text{ loses}]$ from winning and losing the contest, respectively. Given risk-neutrality, Player i's expected payoff from choosing effort level x_i is given by

$$EU_i = w_i(x_i, x_{-i})\,(\overline{v} + M) + (1 - w_i(x_i, x_{-i}))\,\underline{v} - x_i/a_i, \tag{2}$$

where $m_i \in \{0, M\}$ is her prize money.[8]

6. One can easily generalize the model to multiple observers, assuming that the players place the same weight on each observer's perception.

7. Restricting κ to be positive rules out the case where agents want to avoid looking as high ability, as is the case in Bursztyn, Egorov, and Jensen (2019). With $\kappa < 0$ and no monetary prizes, the model is uninteresting, with players exerting no effort and the observer's posterior equaling her prior given the uninformativeness of the contest outcome about ability. Setting $\kappa = 0$, i.e., assuming that players do not care about what the observer thinks about their ability collapses the model back to the standard heterogeneous-player all-pay contest (Hillman and Riley 1989; Kawamura and Moreno de Barreda 2014).

8. Modeling status in this way is not without controversy. While Postlewaite (1998) convincingly argues that status concerns should not be directly included in agents' utility function when status provides instrumental benefit to the agent, I follow Deci and Ryan (2000) and Anderson, Hildreth, and Howland (2015) to argue

Taken together, we have a two-player contest model whereby players compete under an endogenous prize vector $\mathbf{v} = (\bar{v} + M, \underline{v})$, where the first and second entries of the vector are the prize for the contest winner and loser, respectively—the prize vector is "partially endogenous" with strictly positive monetary prize $M > 0$.

The timing of the game is as follows:

Stage 0: Abilities a_i are realized and observed by the two players

Stage 1: Players simultaneously submit effort bid x_i and the winner is identified.

Stage 2: Observer updates her belief about each player's ability type upon observing the identity of the winner.

Discussion of the Model. I draw a close connection between perceived ability and status, an equivalence that may not be directly intuitive to the reader. I build on the literature in social psychology to connect these two concepts: status afforded by agents to another depends on the agent's "perceived instrumental social value" (Leary, Jongman-Sereno, and Diebels 2014), that is whether the agent is believed to have the ability to contribute to achieving the goals of the collective who bestows status on the individual. Productive ability is clearly a relevant characteristic in achieving a wide range of collective goals, hence the perception of having high productive ability can lead to attaining high status. Given that the possession of productive ability must also be perceived as such by those who give status to the individual, projecting high ability is as important as acquiring ability itself (Brennan and Pettit 2000; Anderson, Hildreth, and Howland 2015): agents can do so by outperforming their opponent in this model.

The key assumption I make is the inclusion of the observer's posterior belief about competing player's abilities in their utility function. We can interpret this in two ways that need not be mutually exclusive: first, the observer's perception has no instrumental benefit to the agent, yet they have preferences over how they are perceived nonetheless due to some intrinsic drive to look good in the eyes of others. This concern is then simply captured by agents' utility increasing in the posterior belief of the observer. This interpretation is supported by a large literature on the "status hypothesis" in social psychology (Anderson, Hildreth, and Howland 2015), proposing that gaining status is a fundamental motive: attaining status is an end goal in and of itself (Deci and Ryan 2000). Alternatively, we could also interpret the posterior belief in the utility function as capturing some instrumental benefit to the player proportional to the posterior belief of the observer. For example, being perceived as high ability may make it more likely that the agent attain a promotion in the future, or that she gains other benefits from being

that gaining status is—partially, at the very least—a fundamental motive. In other words, gaining status is a deep preference the same way as consumption is. When status is a deep preference, including status (perceived ability in this model) directly in the utility function leads to analysis that is not subject to the Postlewaite (1998) critique.

considered high ability by society. It is also important to highlight that the existence of an outside observer is not necessary in the model, what is required is that players believe that there is an observer forming beliefs about their abilities.

Last, I comment on how this paper's approach to modeling status as perceived ability differs from that employed by related theoretical work. Status in the theoretical literature is either directly or indirectly assigned by a principal. In Moldovanu, Sela, and Shi (2007), the contest designer assigns ranks to status classes, therefore the number and size of these status classes are fixed exogenously by a principal who wishes to motivate agents to exert effort. In Auriol and Renault (2008) and Besley and Ghatak (2008), obtaining status is linked to a specified performance threshold in the optimal contract the principal offers to agents. The principal has a more indirect influence status in the contest model of Ederer and Patacconi (2010): status is given by the earnings of a player relative to the average earning of a reference group. The principal can manipulate status incentives by setting appropriate prizes. In this current model, the "principal" (the observer) is inactive and status arises without the need for a principal to designate what performance leads to obtaining status, or to manually assign status to agents. This allows us to think about implicit incentives agents may face. It is all the more important to better understand these hidden motives given that an active principal may not be able to easily influence agents' behavior through these implicit incentives even if she wishes to do so.

It is also important to highlight that agents have no intrinsic relative positional concerns in this model in contrast to prior work in contest theory mentioned above: agents derive utility from being perceived as high ability, not high*er* ability than others. Any concern for relative position is driven by how agents can signal their ability: given that it is only the identity of the winner (and the loser) that is observable, outranking the opponent is desired in order to signal ability not to win the contest per se (at least in the case with no monetary prizes, $M = 0$). Linking status to perceived ability provides a clear advantage compared to modeling status by imposing explicit positional concerns on agents' preferences: under the assumptions of Moldovanu, Sela, and Shi (2007), achieving a some fixed ranked in the contest would yield the same status utility to the agent when most competitors are high ability compared to the status utility when most competitors are low ability. As I show below, the value of status—i.e., the posterior belief of the observer—, changes with the probabilistic ability of competitors, as well as the underlying ability asymmetry between types: status payoffs are larger when winning is more informative about having high ability. This is arguably a more realistic way to think about how agents value status when it is linked to agents' instrumental social value.

3 Equilibrium in the Game

To introduce the equilibrium concept, it is helpful to proceed in three steps. First, consider the case when players form conjectures about the psychological prizes they expect to receive based on the observer's posterior beliefs. I focus on symmetric equilibria whereby both players form the same conjecture and play the same type-dependent strategy in equilibrium. Denote the conjectured prizes by $\bar{v}^c$ for winning and $\underline{v}^c$ for losing, assuming $\bar{v}^c > \underline{v}^c$ and define the marginal value of winning as $\Delta^c := M + \bar{v}^c - \underline{v}^c$. Later these conjectures will be endogenized, but for now assume them to be exogenously fixed.

Given these conjectured prizes, players simultaneously select their effort levels, x_1 and x_2. As established by contest theory literature (e.g. Barut and Kovenock (1998)), a symmetric equilibrium in pure strategies cannot exist in this game. To illustrate, suppose a pure strategy equilibrium exists with fixed effort levels x_1 and x_2. If $x_1 = x_2$, either player can increase their effort slightly to secure the win and the higher prize. Conversely, if $x_1 \neq x_2$, the player with higher effort has an incentive to marginally decrease effort, thus maintaining the win but lowering effort costs. In both scenarios, a profitable deviation arises, contradicting the assumption of a pure strategy equilibrium.

I distinguish between two types of contests in the following: A contest is said to be "homogeneous" when the two competing players have identical ability types. In "heterogeneous contests" a high-ability player competes against a low-ability opponent. The unique mixed-strategy equilibrium for each case is characterized in Lemma 1.

Lemma 1 (Equilibrium Strategies with Exogenously Fixed Marginal Value of Winning)**.**

1. *When the marginal prize Δ^c is zero, players exert no effort and each has a win probability of* $1/2$.
2. *When $\Delta^c > 0$, players in homogeneous contests randomize effort according to the equilibrium cumulative distribution function*

$$F_i^*(x_i) = \frac{x_i}{a_i \Delta^c} \text{ for } x \in [0, a_i \Delta^c]. \tag{3}$$

When $\Delta^c > 0$, players in heterogeneous contests randomize effort according to the equilibrium cumulative distribution functions

$$F_{-i}^*(x_{-i}) = 1 + \frac{x_{-i}}{h\Delta^c} - \frac{l}{h} \text{ for } x \in [0, l\Delta^c], \tag{4}$$

$$F_i^*(x_i) = \frac{x_i}{l\Delta^c} \text{ for } x \in [0, l\Delta^c], \tag{5}$$

assuming that $a_i = h$ and $a_{-i} = l$.

Let $w_1^*(a_1, a_2)$ be Player 1's win probability given the strategies described in (4) and (5) *and abilities a_1 and a_2. Win probabilities $w_1^*(a_1, a_2)$ are given by*

$$w_1^*(h,h) = w_1(l,l) = \frac{1}{2}, \quad w_1^*(h,l) = 1 - \frac{1}{2\alpha}, \quad w_1^*(l,h) = \frac{1}{2\alpha}. \tag{6}$$

Keeping in mind that the equilibrium cumulative distribution functions do not put mass points on strictly positive effort levels, we can verify that the above equilibrium mixed strategies are indeed mutual best responses. Take first the case when the players have the same ability. Then Player i's expected payoff by exerting any fixed effort level x_i in the equilibrium support is

$$F_{-i}^*(x_i)\Delta^c - \frac{x_i}{a_i} = \frac{x_i}{a_i\Delta^c}\Delta^c - \frac{x_i}{a_i} = 0. \tag{7}$$

It is also easily confirmed that exerting more effort than the upper bound of the equilibrium mixed strategy $a_i\Delta^c$ yields strictly negative payoff even though it guarantees the prize for Player i. Since any pure strategy x_i in the equilibrium support yields zero expected payoff, any mixed strategy on the same support yields zero expected payoff as well when the opponent plays the equilibrium strategy described in Lemma 1.

Similarly in a heterogeneous contest, Player i's (the player with high ability) expected payoff is

$$F_{-i}^*(x_i)\Delta^c - \frac{x_i}{h} = \left(1 - \frac{l}{h}\right)\Delta^c \ \text{ for } x \in [0, l\Delta^c]. \tag{8}$$

This equals her expected payoff if she were to exert marginally higher effort than the upper bound of the equilibrium mixed strategy, guaranteeing her win. Further increasing effort leads to a strictly lower expected payoff as the prize value does not change but effort costs increase.

We can also confirm that when the high-ability player randomizes according to the equilibrium strategy, the low-ability player earns zero in expectation for any effort level in the equilibrium support, with

$$F_i^*(x_{-i})\Delta^c - \frac{x_{-i}}{l} = \frac{x_{-i}}{l\Delta^c}\Delta^c - \frac{x_{-i}}{l} = 0 \ \text{ for } x \in [0, l\Delta^c]. \tag{9}$$

Again, bidding above the upper bound of the equilibrium support yields negative expected payoff.

Given the win probabilities in Lemma 1, one can calculate the posterior beliefs using Bayes rule. These posterior beliefs are the same for both players given that they are ex ante identical in the eyes of the observer. We can then, without loss of generality focus on the posterior belief about Player 1's ability: when $\Delta^c > 0$, the observer's posterior belief about her ability when she wins is given by

$$\Pr\Big[a_1 = h \mid i \text{ wins}\Big] = \frac{\mu\left[\frac{1}{2}\mu + w_1^*(h,l)\,(1-\mu)\right]}{\mu\left[\frac{1}{2}\mu + w_1^*(h,l)\,(1-\mu)\right] + (1-\mu)\left[\frac{1}{2}(1-\mu) + (1-w_1^*(h,l))\,\mu\right]}. \tag{10}$$

Similarly, the posterior belief conditional on Player 1 losing the contest is given by

$$\Pr\Big[a_1 = h \mid i \text{ loses}\Big] = \frac{\mu\left[\frac{1}{2}\mu + (1-w_1^*(h,l))\,(1-\mu)\right]}{\mu\left[\frac{1}{2}\mu + (1-w_1^*(h,l))\,(1-\mu)\right] + (1-\mu)\left[\frac{1}{2}(1-\mu) + w_1^*(h,l)\,\mu\right]}. \tag{11}$$

Note that the above posterior beliefs in (10) and (11) are functions of the conjectured posterior beliefs through win probability $w_1^*(h,l)$. When $\Delta^c > 0$, this win probability turns out to be independent of the conjectured posterior beliefs following from (6) in Lemma 1.

As a last component of the equilibrium, I require the conjectured posterior beliefs and the posterior beliefs implied by these conjectures to be equal. In other words, conjectured prizes must be correct in equilibrium. This consistency requirement is the non-standard part that is required in this psychological game theory set-up.[9]

To summarize, players make symmetric conjectures about prizes and then choose effort that are mutual best responses taking the conjectured prizes as given. Last, equilibrium requires the conjectured prizes to be correct in the sense that applying Bayes' rule with the conjectured prizes must generate the same prizes. The equilibrium concept is formally stated in Definition 1.

Definition 1 (Equilibrium in the Game)**.** *Assume that players' conjectures are symmetric, such that* $\underline{v}_i^c = \underline{v}_{-i}^c$ *and* $\bar{v}_i^c = \bar{v}_{-i}^c$*. Let* σ_i^* *be probability distribution over effort* $x_i \geq 0$*. An equilibrium in the game is a tuple* $(\sigma_1^*, \sigma_2^*, \underline{v}^*, \overline{v}^*)$ *such that*

1. *given* $(\underline{v}^c, \bar{v}^c)$, $\quad EU(\sigma_i^*, \sigma_{-i}^*) \geq EU(\sigma_i, \sigma_{-i}^*) \quad \forall i$
2. *and* $(\underline{v}^c, \bar{v}^c) = (\underline{v}^*, \bar{v}^*)$ *such that* $(\underline{v}^*, \bar{v}^*)$ *are the posterior beliefs derived via Bayes' rule using conjectures* $(\underline{v}^c, \bar{v}^c)$.

Given the results in Lemma 1 and the equilibrium definition above, we are now ready to characterize the equilibrium in the game.

9. The equilibrium concept in this paper is similar to that in Dufwenberg and Lundholm (2001), but we do not need to make assumptions about off-equilibrium path beliefs given that the game described in this paper is not a dynamic game of incomplete information from the players' perspective.

Theorem 1 (Equilibrium)**.**

1. *When $M > 0$, there is a unique equilibrium in the game. The equilibrium prize vector is given by* $\mathbf{v}^* = (\bar{v} + M, \underline{v})$ *with*

$$\bar{v} = \kappa \frac{\mu\left[\frac{1}{2}\mu + \left(1 - \frac{1}{2\alpha}\right)(1-\mu)\right]}{\mu\left[\frac{1}{2}\mu + \left(1 - \frac{1}{2\alpha}\right)(1-\mu)\right] + (1-\mu)\left[\frac{1}{2}(1-\mu) + \frac{1}{2\alpha}\mu\right]}, \tag{12}$$

$$\underline{v} = \kappa \frac{\mu\left[\frac{1}{2}\mu + \frac{1}{2\alpha}(1-\mu)\right]}{\mu\left[\frac{1}{2}\mu + \frac{1}{2\alpha}(1-\mu)\right] + (1-\mu)\left[\frac{1}{2}(1-\mu) + \left(1 - \frac{1}{2\alpha}\right)\mu\right]}. \tag{13}$$

The marginal value of winning the contest is given by

$$\Delta = M + \bar{v} - \underline{v} = \kappa \frac{2(\alpha - 1)(1-\mu)\mu}{\alpha} + M. \tag{14}$$

Equilibrium expected effort in homogeneous contests are given by

$$\mathbb{E}[X_i^*] = \frac{a_i \Delta}{2}. \tag{15}$$

In a heterogeneous contests expected effort is given by

$$\mathbb{E}[X_i^* \mid a_i = h] = \frac{l\Delta}{2} \qquad \mathbb{E}[X_{-i}^* \mid a_{-i} = l] = \frac{l^2 \Delta}{2h}. \tag{16}$$

2. *When $M = 0$, there is a further equilibrium in addition to the one described above: the posterior beliefs equal the prior belief irrespective of the contest outcome and $\Delta = 0$.*

The additional equilibrium when there is no monetary prize ($M = 0$) warrants further discussion. This equilibrium arises when players conjecture that the marginal prize is zero given that the observer believes the contest signal is uninformative of players' abilities. This conjecture can be consistent with Bayes' rule when assuming that both player will remain inactive, and therefore, the winner is picked randomly irrespective of ability. On the other hand, this conjecture cannot be consistent for an arbitrarily small monetary prize since the pure strategy of exerting no effort is no longer a Nash equilibrium.

Nonetheless, even with $M = 0$, conjecturing that $\Delta = \kappa \frac{2(\alpha-1)(1-\mu)\mu}{\alpha}$ remains consistent, and therefore it is an equilibrium of the game. Given that contest outcomes are informative of competitors' abilities for any conjectured nonzero marginal prize, I focus on this equilibrium in the following analysis.

4 Motivation in Contests for Perception

In this section I analyze how motivation defined as the expected effort exerted by a player in a fixed ability pairing is affected by (1) the probability of a player having high ability μ, and (2) the magnitude of ability asymmetry between the two types, α. To build intuition, I first consider how changing these parameters affect the winner and loser (psychological) prize in the competition, and the psychological prize spread $\bar{v} - \underline{v}$.

The posterior belief conditional on a win in the contest—the psychological prize for winning the contest—, is strictly increasing in both the probability of a player having high ability and ability asymmetry. Both results are quite intuitive. When μ is high, it is less likely that both players are low ability, in which case winning the contest is also less informative about a player having high ability. A higher μ, therefore, decreases "false positives", in the sense that a winner is less likely to be a low-ability player. Increasing ability asymmetry also decreases false positives, but in heterogeneous contests: a low-ability player is less likely to win against a high-ability player when ability asymmetry is more severe. Holding μ constant, this implies that winning the contest is a stronger signal of a player having high ability.

Lemma 2 (Comparative Statics: $\bar{v}$)**.** *The endogenous winner prize increases*

(I.) in μ, the probability of a player having high ability, and

(II.) in α, the degree of ability asymmetry between the two types.

Let us look at the same comparative statics for the loser prize $\underline{v}$. As for the win prize $\bar{v}$, increasing μ increases the value of the loser prize $\underline{v}$ since with higher μ it becomes more likely that the player is high ability regardless of the contest outcome. The intuition is again straightforward: the chance of a high-ability matchup increases with μ, and losing it is guaranteed in such contests that high-ability player loses which in turn decreases the informativeness of losing about the player's ability. However, increasing ability asymmetry has the opposite effect; higher ability asymmetry α decreases loser prize $\underline{v}$. When ability asymmetry increases, the probability that a high-ability player wins a heterogeneous matchup increases: a loss becomes more indicative of a player being low ability.

Lemma 3 (Comparative Statics: $\underline{v}$)**.** *The endogenous loser prize*

(I.) increases in μ, the probability of a player having high ability, and

(II.) decreases in α, the degree of ability asymmetry between the two types.

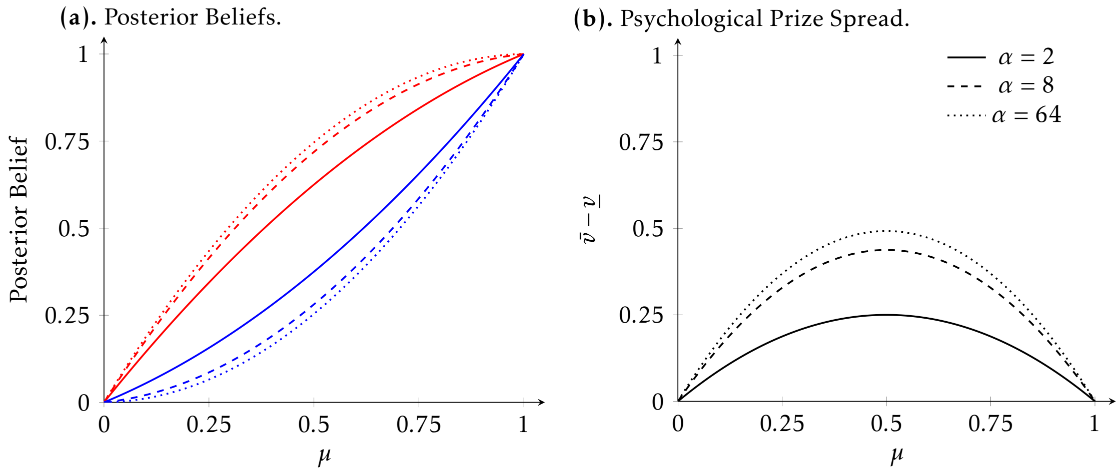

Figure 1: Posteriors beliefs and the psychological prize spread assuming $\kappa = 1$. The red function is the posterior conditional on observing a win ($\bar{v}$), while the blue function is the posterior conditional on the observing a loss from the respective player ($\underline{v}$). I plot these and their difference for three levels of ability heterogeneity: $\alpha = 2$ (solid line), $\alpha = 8$ (dashed line) and $\alpha = 64$ (dotted line).

What affects players' motivation to exert costly effort directly is the extent to which winning increases the observer's posterior compared to losing: $\bar{v} - \underline{v}$ (see panel (b) of Figure 1 for how this psychological prize spread changes with μ). When μ is high, it is more likely that the player is high-ability regardless of the contest outcome: the chance of two high-ability players competing increases. In the limit, all players are high ability, hence if their motivation to exert costly effort in the contest comes from only the psychological payoff of being perceived as high ability, they will not do so as they will be thought of as high ability regardless of them winning or losing in the contest. Players need the observer to be sufficiently unsure about their abilities in order for the posterior beliefs conditional on the contest outcome to differ. The first order condition of $\bar{v} - \underline{v}$ with respect to μ characterizes the observer's prior that maximizes the psychological prize spread in the contest, which equals $\mu = 1/2$. This result implies that the psychological prize spread is the largest when the observer is most uncertain—or equivalently, when the chance of a heterogeneous contest is the highest. Given that high ability share μ only enters expected effort through the marginal prize Δ, we have directly that expected effort is maximized when $\mu = 1/2$.

Proposition 1 (Motivation by Potential Heterogeneity)**.**

(I.) The psychological prize spread $\bar{v} - \underline{v}$ is maximized at $\mu = 1/2$.

(II.) With the two players' realized ability types given, individual expected effort is maximized when $\mu = 1/2$.

Proposition 1 shows that when players exert highest effort when they believe that the observer has flat priors about their ability types. This result adds to the line of research on social comparison in psychology arguing that opponents who are more similar display increased concern about social comparison, which in turn increases competitive behavior (Garcia, Tor, and Schiff 2013). The related literature focuses on the motivation arising from competing players comparing themselves to opponents. Motivation, defined as expected effort provision, is higher in homogeneous contests in my model as well, but this result is driven by a strategic effect rather than increased psychological payoffs from potentially winning against someone who is more similar in ability.

In my model, it is the observer who implicitly compares the two players, hence motivation arises from how the observer forms beliefs given the limited information she has of the players and their performance. It is then not only the realized abilities of the competing players that effect their motivation in the contest, but also the potential heterogeneity induced by the distribution players' abilities are drawn from as described in Proposition 1. This way of thinking about social competition introduces a novel source of motivation through the possibility to signal high ability through contest outcomes to an observer.

In this model, I assume that the observer holds correct beliefs about the distribution of ability. In other words, I assume that the probability that a given player is high ability μ equals what the observer believes this probability to be: $\mu^o = \mu$ where μ^o denotes the observer's belief. The model can, however, easily accommodate incorrect beliefs from the observer, assuming this is known to players: given that μ affects behavior only through the observer's posterior belief and it does not affect win probabilities, we can simply replace μ with $\mu^o \neq \mu$ in the Bayes' formula to get the prize vector players will compete under. If the observer believes (incorrectly) that players are more talented on average relative to μ, this can motivate players to increase effort ifμ^o is closer to 1/2 than μ, and demotivate them if μ^o is further away from 1/2 than μ. Similarly, the observer expecting players to be less talented on average can both motivate and demotivate effort provision depending on whether μ^o is closer to, or further away from 1/2 than μ. The intuition is the same as for Proposition 1, with the difference that μ^o need not correspond with the true μ. Similarly, we could allow for players to think that the observer's belief about type-h probability is different from her true belief. Let this second-order belief be denoted by μ^p such that $\mu^p \neq \mu^o$. Again, we can replace μ with μ^p to get the equilibrium prize vector: as long as players have the same second-order belief, the model can still provide predictions about their effort provision.

The above modification of the model also helps us in thinking about the policy implications of the result in Proposition 1: manipulating this second-order belief μ^p can then be an effective way to motivate players to exert effort. To do this, second-order beliefs must be changed such that players believe that the observer thinks that the probability of them being a high-ability type is closer to 1/2 than the initial second-order belief.

Looking at the expected effort of players in each contest is more nuanced when it comes to ability asymmetry. ability asymmetry α monotone increases the psychological prize spread, $\bar{v} - \underline{v}$. This is due to a loss being more informative of the player being low ability and a win being more informative of the player being high ability. Increasing α then opens up a wedge between the posteriors, leading to a larger psychological prize spread. In the following, I consider two ways in which an increase in $\alpha = h/l$ is implemented: first, I hold l fixed and increase h. The other case is the reverse, whereby I hold h fixed and decrease l.

Consider first the case of a homogeneous contest of high-ability players: when decreasing ability parameter l, expected effort as given in Theorem 1 would stay the same if Δ were fixed. However, Δ is an increasing function of ability asymmetry $\alpha = h/l$. Decreasing l, thereby increasing ability asymmetry then increases expected effort provision. Similarly, increasing h holding l fixed increases expected effort in a homogeneous contest between two low-ability players.

It is easy to see from Theorem 1 that expected effort also increases when we increase h in a homogeneous contest of two high-ability players as well: this is intuitive as the marginal prize increases again, and the players also become more productive. A more interesting case is when one increases ability asymmetry by decreasing l in a contest between two low-ability players. Were Δ fixed, this would decrease expected effort provision as the competing players become less productive: exerting any fixed effort level $x_i > 0$ becomes more expensive in terms of effort cost. However, when l is sufficiently high such that $h < 2l$, the motivating effect from increasing the psychological prize spread dominates the negative effect of decreasing productive ability on effort provision.

In heterogeneous contests, increasing ability asymmetry has a direct negative effect on expected effort provision through the well-known "discouragement effect" (Baye, Kovenock, and Vries 1993; Baik 1994; Brown 2011; Dechenaux, Kovenock, and Sheremeta 2015).[10][11] The basic mechanism is as follows: when ability asymmetry grows larger, the player with the lower ability face a reduced probability of winning. This leads to him exerting less effort in equilibrium. The high-ability player responds to this by also exerting less effort: total expected effort is decreasing in ability asymmetry (Drugov and Ryvkin 2022).

We can identify this same mechanism in this model as well by differentiating the expressions for expected individual effort implied by the equilibrium effort distribution functions in Theorem 1 with respect to h and l—this is equivalent to increasing ability asymmetry α. Total expected effort in heterogeneous contests equal

$$\frac{1}{2}\Delta\left(l + \frac{l^2}{h}\right) \tag{17}$$

10. See also related work in psychology by Rogers and Feller (2016) for example.

11. Fang, Noe, and Strack (2020) identifies a discouragement effect from prize inequality and contest scale with homogeneous players. While the terminology is the same, here I refer to mechanism that requires player heterogeneity to induce players to exert less effort in equilibrium.

with

$$\Delta = \kappa \frac{2(h-l)(1-\mu)\mu}{h} \tag{18}$$

when $M = 0$ and replacing α with h/l.

When we assume Δ to be fixed, differentiating (17) with respect to h and l yields

$$\frac{\partial \mathbb{E}[X_i^* + X_{-i}^* \mid a_i = h, a_{-i} = l]}{\partial h} = -\frac{1}{2}\Delta \frac{l^2}{h^2} < 0, \tag{19}$$

$$\frac{\partial \mathbb{E}[X_i^* + X_{-i}^* \mid a_i = h, a_{-i} = l]}{\partial l} = \frac{1}{2}\Delta \left(1 + \frac{2l}{h}\right) > 0. \tag{20}$$

This is the direct effect of increasing ability asymmetry, which corresponds to the "discouragement effect" discussed above: increasing h holding l constant decreases total expected effort, similarly to when asymmetry grows by decreasing l holding h constant.

There is, however, an indirect effect of increasing ability asymmetry, which operates through the marginal prize Δ, which is a function of h and l. These indirect effects are given by

$$\frac{\partial \mathbb{E}[X_i^* + X_{-i}^* \mid a_i = h, a_{-i} = l]}{\partial \Delta} \frac{d\Delta}{dh} = \kappa(1-\mu)\mu \frac{l^2}{h^2}\left(1 + \frac{l}{h}\right) > 0 \tag{21}$$

$$\frac{\partial \mathbb{E}[X_i^* + X_{-i}^* \mid a_i = h, a_{-i} = l]}{\partial \Delta} \frac{d\Delta}{dl} = -\kappa(1-\mu)\mu \left(\frac{l}{h} + \frac{l^2}{h^2}\right) < 0, \tag{22}$$

In other words, when increasing h holding l constant (or when decreasing l holding h constant), the increase in the psychological prize spread motivates competing players to exert more effort. This is the "encouraging effect" of increasing ability asymmetry.

Given that the direct and indirect effects push expected effort provision in opposite directions, we already see that having contestants who care about their perceived ability at the very least dampens the magnitude of the discouraging effect. In fact, when we increase ability asymmetry from an initial level that is small enough, this indirect positive effect—the "encouragement effect"—, will dominate the discouragement effect. This is the second main result in this section, which is summarized in Proposition 2.

Proposition 2 (Encouragement)**.** *Assume there are no monetary prizes ($M = 0$).*

1. *Fixing h, increasing ability asymmetry by reducing l,*
   - *Always increases total expected effort in homogeneous contests of high-ability players. It also increases total expected effort in homogeneous contests of low-ability players if $h < 2l$.*
   - *Increases total expected effort in heterogeneous contests if $h < \sqrt{3}l$.*
2. *Fixing l, increasing ability asymmetry by increasing h, increases total expected effort in both homogeneous and heterogeneous contests.*

It is intuitive why the encouragement effect can only dominate the discouraging effect for sufficiently low ability asymmetry: the marginal prize Δ is increasing and concave in ability asymmetry α and expected effort is also increasing and concave in Δ. Taken together, this implies that the motivating effect of larger relative gains from winning become smaller as we drive a larger wedge between player's effort costs. On the other hand, fixing Δ the direct negative effect becomes stronger in magnitude as α increases, and eventually starts to dominate the encouragement effect. While I assume that the contest has no monetary prizes in Proposition 2, the results are qualitatively the same: growing ability asymmetry can motivate players to exert more effort (see Appendix A.7). The encouragement effect dominates when the monetary prize is small enough.

This result on the reversal of the discouragement effect relates to recent work by Drugov and Ryvkin (2022) on how ability asymmetry impacts effort provision in noisy contests with an interior pure strategy equilibrium (for example the Tullock contest (Tullock 1980) or the Lazear-Rosen tournament (Lazear and Rosen 1981)). In these contests, both encouragement and discouragement effect can occur depending on how heterogeneity in ability is introduced to the model. I show that an encouragement effect of different kind exists in a complete information two-player all-pay contest. The mechanism behind the encouragement effect is different from that in Drugov and Ryvkin (2022), as it is driven by the prizes being endogenously determined in the model, and crucially, the prize spread increasing in ability asymmetry.

The analysis in this section connects to the literature on how non-monetary incentives in organizations affect the behavior of agents who have social image concerns (see for example Frey (2007)). I show that when observers form beliefs about ability based on relative performance, this creates implicit incentives that are perfect substitutes of monetary rewards or other form of more explicit incentives. An important distinction between these implicit incentives and monetary rewards are the impossibility for the observer to directly manipulate these perception-based incentives, if we maintain the assumption that she is Bayesian and has correct prior about the distribution of ability types. While I do not model the observer as an active player, it is nonetheless interesting to entertain such possibility: an observer (or now a principal) may find it beneficial to persuade players that she has flat priors for example. If the principal could achieve this, she could maximize effort provision. Similarly, she may persuade players that she believes that ability asymmetry is large, motivating them to exert more effort in trying to obtain the positive ability signal. Whether such persuasion is possible, however, is beyond the scope of this paper.

5 Expected Effort from the Observer's Perspective

While the focus was on expected effort in each possible contest separately in the previous section, I now turn to analyze expected effort that does not condition on a given ability match. I will focus on what probability μ of a player being high ability maximizes this unconditional expected effort provision. In this section I take the position of the observer, who does not know the ability of the competing players. The question I answer here is when can the observer expect the competing players to exert the most effort? In particular, what is the type-h probability μ^* that maximizes expected effort? As I show below, this does not correspond to 1/2 that maximizes expected effort in a given fixed ability matchup. To proceed with the analysis, I define the unconditional expected total effort provision as

$$\begin{aligned}\mathbb{E}[X_1^* + X_2^*] = \mu^2 & \left(\mathbb{E}[X_1^* + X_2^* \mid a_1 = a_2 = h]\right) \\ & + (1-\mu)^2 \left(\mathbb{E}[X_1^* + X_2^* \mid a_1 = a_2 = l]\right) \\ & + 2\mu(1-\mu) \left(\mathbb{E}[X_1^* + X_2^* \mid a_1 \neq a_2]\right).\end{aligned} \tag{23}$$

From Proposition 1 we may think that setting $\mu = 1/2$ maximizes the unconditional expected total effort. While it is true that $\mu = 1/2$ maximizes effort in contests between each ability pairs, it also impacts the probabilities with which these ability pairs occur (see Figure 2). This mechanism becomes important when we note that expected effort varies across contests depending on players' abilities. In fact, we can show by inspecting equilibrium expected effort provision for the three ability pairings that the highest expected effort is generated by the contest where both players are high ability, followed by the contest with two low-ability players. The contest where players' abilities differ leads to the least effort in expectation:

$$\mathbb{E}[X_1^* + X_2^* \mid a_i = h, a_{-i} = l] < \mathbb{E}[X_1^* + X_2^* \mid a_i = l] < \mathbb{E}[X_1^* + X_2^* \mid a_i = h]. \tag{24}$$

It is then clear that if a given μ leads to a heterogeneous ability contest too often, it may not maximize expected effort in the contest from the observer's perspective. Figure 2 shows how the probability of a given ability matchup changes with the probability of

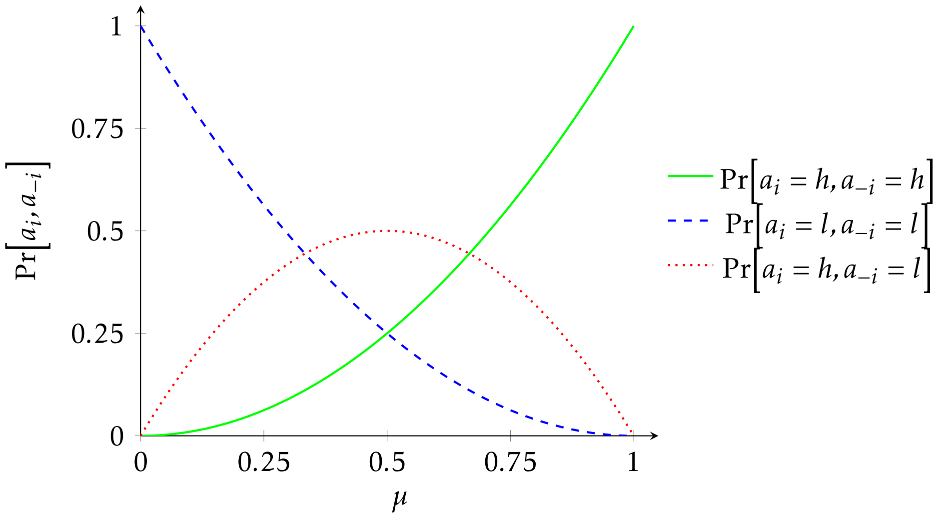

Figure 2: Ability match probabilities as a function of type-h probability μ.

a player being high ability. It is easy to confirm analytically that the probability of a heterogeneous contest $2\mu(1-\mu)$ is maximized at $\mu = 1/2$. This coincides with the prior that maximizes expected effort in each contest. The question then becomes whether one can find an alternative high ability share $\mu \neq 1/2$ that will lead to a higher expected effort without knowing the competing players' abilities.

The probability of a heterogeneous contest occurring is symmetric around its maximum, $\mu = 1/2$.[12] We can also show that the psychological prize spread, $\bar{v} - \underline{v}$, is also symmetric around $1/2$.[13] It follows then that expected effort is never maximized for $\mu < 1/2$. To see this consider setting $\mu = 1/4$. Since $1/4 < 1/2$, we will see a decrease in the psychological prize spread and therefore in expected effort (Proposition 1). On the other hand, moving away from $\mu = 1/2$ reduces the probability of a heterogeneous contest occurring. Since the lowest total expected effort is lowest in such contests—even comparing to the homogeneous contest with two low-ability players—, we may see an increase in expected effort provision even with the higher probability of players having low-ability types. However, we could achieve the same reduction in the chance of the heterogeneous contest occurring by increasing μ to $3/4$. This would yield the same decrease in the psychological prize spread $\bar{v} - \underline{v}$ as setting μ to $1/4$, but now the probability of two high-ability players competing will be higher compared to $\mu = 1/4$. Since we know that holding all else constant, the homogeneous contest of two high-ability players leads to the highest expected effort provision, we then have that $\mu < 1/2$ can never be the expected effort maximizing type-h probability.

12. Let $\mu_1 = 1/2 - b$ and $\mu_2 = 1/2 + b$ with $b \in (0, 1/2)$. The probability of a heterogeneous contest occurring, $2\mu(1-\mu)$, is symmetric around $1/2$ if $2\mu_1 - 2\mu_1^2 = 2\mu_2 - 2\mu_2^2 \iff b^2 = b^2$. which holds for all $b \in (0, 1/2)$.
13. Again, let $\mu_1 = 1/2 - b$ and $\mu_2 = 1/2 + b$ with $b \in (0, 1/2)$. The psychological prize spread, $\bar{v} - \underline{v}$, is symmetric around $1/2$ if $\frac{2(1-\alpha)\kappa}{\alpha}(\mu_1 - 1)\mu_1 = \frac{2(1-\alpha)\kappa}{\alpha}(\mu_2 - 1)\mu_2 \iff -1/2 = -1/2$. This equality of course holds independent of b.

We can also show that μ^* must be strictly above 1/2. While the psychological prize spread $\bar{v} - \underline{v}$ is maximized at $\mu = 1/2$, this is not sufficient for maximizing unconditional expected effort from the observer's perspective. This is because μ influences not only the size of psychological incentives, but also the probability with which different ability pairings occur.

To show this formally, let $\mathbb{E}_{hh}$, $\mathbb{E}_{ll}$, and $\mathbb{E}_{hl}$ denote the expected effort in the three respective matchups (these correspond to the expectation in (23)). Differentiating total expected effort in (23) with respect to μ yields:

$$\begin{aligned}\frac{d}{d\mu}\mathbb{E}[X_1^* + X_2^*] &= 2\mu\mathbb{E}_{hh} - 2(1-\mu)\mathbb{E}_{ll} + 2(1-2\mu)\mathbb{E}_{hl} \\ &\quad + \mu^2 \frac{d\mathbb{E}_{hh}}{d\mu} + (1-\mu)^2 \frac{d\mathbb{E}_{ll}}{d\mu} + 2\mu(1-\mu)\frac{d\mathbb{E}_{hl}}{d\mu}.\end{aligned} \tag{25}$$

From Proposition 1, we know that each conditional effort term is maximized at $\mu = 1/2$, which implies that the derivatives of $\mathbb{E}_{hh}$, $\mathbb{E}_{ll}$, and $\mathbb{E}_{hl}$ with respect to μ are zero at this point. Thus, at $\mu = 1/2$,

$$\frac{d}{d\mu}\mathbb{E}[X_1^* + X_2^*]\bigg|_{\mu=1/2} = \mathbb{E}_{hh} - \mathbb{E}_{ll}. \tag{26}$$

Since $\mathbb{E}_{hh} > \mathbb{E}_{ll}$ by Theorem 1, this derivative is strictly positive, and hence the effort-maximizing prior must satisfy $\mu^* > 1/2$: the observer should expect to see the most intense competition when the type-h probability is greater than 1/2.

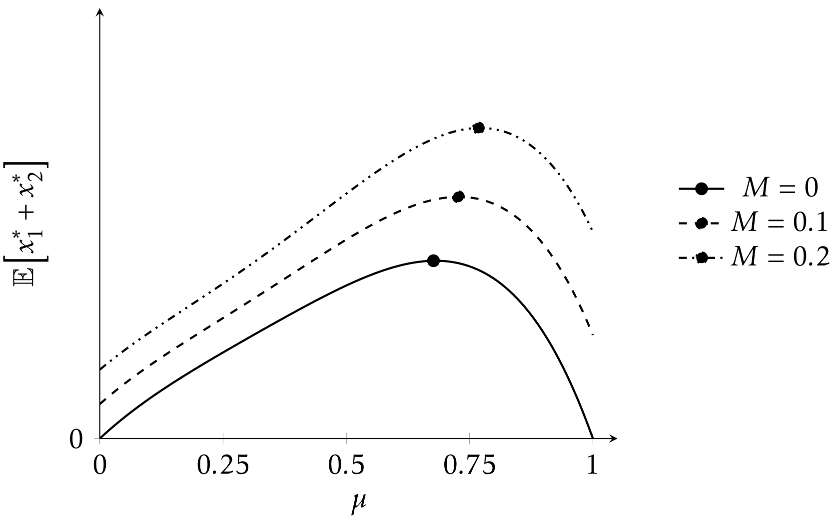

Figure 3: Expected effort for different values of high ability share μ and monetary prize M. The maximum is marked with a solid black circle. The parameter assumptions are: $\alpha = 3, l = 1, \kappa = 1$.

Increasing the monetary prize M does not affect the psychological prize spread. However, as M grows, monetary incentives become more important in motivating effort, compared to psychological payoffs. This shift makes it less consequential—from an effort-maximization perspective—if psychological incentives weaken, as they do when μ increases. As a result, maximizing the probability of a high-ability matchup becomes more important relative to sustaining motivation from *psychological* payoffs. Hence, μ^* increases with M.

I show above that it is not enough to maximize the psychological prize spread to maximize expected effort provision from the observer's perspective, as the probability of a player being a high-ability type μ affects both the probability of different ability pairings, and the psychological prize spread. I summarize this result in Proposition 3.

Proposition 3 (Maximizing Unconditional Expected Effort)**.** *The unconditional expected effort maximizing type-h probability is in the open interval $\mu^* \in (1/2, 1)$, and therefore it is strictly greater than $1/2$, the type-h probability that maximizes motivation in any given ability matchup. μ^* is increasing with monetary prize M.*

6 Heterogeneous Priors

Until now, I assumed that the observer has the same prior belief about both players' abilities. While this assumption allowed us to analyze interesting comparative statics within a tractable framework with sufficient generality, in many real-world contexts, observers have different priors for agents depending on their observable characteristics (Campos-Mercade and Mengel 2024; Charles and Guryan 2011; Hilton and Hippel 1996). For example, women are often stereotyped to be less capable in many tasks than men (Reuben, Sapienza, and Zingales 2014; Ellemers 2018), while black students are stereotyped to perform worse academically (Steele and Aronson 1995). These stereotypes may also explain part of the wage and employment gap between black and white Americans (Lang and Spitzer 2020; Lang and Lehmann 2012) and between women and men (Olivetti and Petrongolo 2016). It is, therefore, important that we consider how competing players expecting an observer to think of their abilities to differ ex-ante may affect their behavior in the contest. To do so, I first generalize the framework with homogeneous priors and allow the observer to have different priors about players' abilities.[14] I then analyze how expected effort provision and win probabilities are affected by prior beliefs.

The basic structure of the game remains the same: there are two players, who can be of high- or low-ability type. However, the probability that ability parameter $a_i \in \{l, h\}$ takes the value h now depends on the player's identity. For Player i, the probability that $a_i = h$ is now $\mu_i \in (0, 1)$. As before, the observer knows μ_i but can only observe the outcome of the subsequent contest. While I generalize along the dimension of observer's beliefs, I assume that there are no monetary prizes in the contest ($M = 0$) to keep the analysis tractable.

14. Allowing players to believe that observer have different priors about their ability is an alternative interpretation of the model that will follow.

Player i's expected utility is then

$$EU_i = w_i(x_i, x_{-i})\,\overline{v}_i + (1 - w_i(x_i, x_{-i}))\,\underline{v}_i - x_i/a_i. \tag{27}$$

Finding the equilibrium requires similar steps to those under homogeneous priors: first I impose that players make conjectures about psychological prizes,[15] choose strategies that are mutual best responses given these conjectures and, finally, impose that that the conjectured prizes must be consistent. To proceed with the first step, define $\Delta_i^c := \bar{v}_i^c - \underline{v}_i^c$ as the difference in conjectured prizes for Player i, and denote a consistent conjecture with Δ_i. Importantly, Δ_1 may not equal Δ_2 under heterogeneous priors.

I proceed with deriving the equilibrium effort choice and implied win probabilities, taking the conjectured prizes as given. Player i has no incentive to exert effort above $a_i\Delta_i^c$. To see this, consider the two extremes, whereby Player i is guaranteed to win the contest versus when she is guaranteed to lose. When she wins with probability one and chooses effort x_i above $a_i\Delta_i^c$, her expected utility is given by

$$\bar{v}_i^c - x_i/a_i. \tag{28}$$

This may be positive even if $x_i > a_i\Delta_i^c$ $(= a_i(\bar{v}_i^c - \underline{v}_i^c))$, as $\underline{v}_i^c$ could be strictly positive. Importantly, an expected utility of $\underline{v}_i^c$ can be trivially guaranteed by exerting no effort. It follows then that if choosing effort x_i yields positive, but nonetheless lower expected payoff than $\underline{v}_i^c$, then this effort level x_i will never be chosen by Player i. We can find the effort level $\bar{x}_i$ for which a guaranteed win in the contest provides the same expected utility as $\underline{v}_i^c$ by rearranging $\bar{v}_i^c - \bar{x}_i/a_i = \underline{v}_i^c$ for $\bar{x}$, which yields $a_i\Delta_i^c$.

Under heterogeneous priors, $a_1\Delta_1^c$ may not equal $a_2\Delta_2^c$. Let $k \in \{1,2\}$ be the index of the player with lower maximum effort, formally

$$k = \arg\min_{i\in\{1,2\}} (a_i\Delta_i^c). \tag{29}$$

Her opponent, Player $-k$ can threaten to exert effort just above $a_k\Delta_k^c$, thereby guaranteeing a win for Player $-k$ with strictly positive expected payoff. Exerting more effort than just above $a_k\Delta_k^c$ decreases Player $-k$'s expected payoff, hence we know that neither player has an incentive to choose effort level above $a_k\Delta_k^c$. It is important to note that under homogeneous priors, who can guarantee a win for herself while capturing positive expected payoff depends on which player has higher ability. In contrast, under heterogeneous priors, also the individual psychological prizes captured by Δ_i^c matters.

The equilibrium properties of the game with heterogeneous priors are otherwise similar

15. Under heterogeneous priors, players make conjectures about both their own, and their opponent's psychological prizes. I assume that players make the same conjecture about every psychological prize.

to the case with homogeneous priors: equilibrium is in mixed strategies, with players randomizing on common support between zero and $a_k\Delta_k^c$ as described in Proposition 4.

Proposition 4 (Equilibrium characterization for given conjectured prizes)**.** *There is a unique equilibrium in mixed strategies. Let k denote the index of the player with lower effort incentive $a_i\Delta_i$, as defined in (29). Players randomize over the same support $x \in [0, a_k\Delta_k^c]$ according to cumulative distribution functions*

$$F^*_{-k}(x_{-k}) = \frac{x_{-k}}{a{-}k\Delta^c_{-k}} \tag{30}$$

$$F^*_k(x_k) = 1 + \frac{x_k - a_k\Delta_k^c}{a_{-k}\Delta^c_{-k}}. \tag{31}$$

The next steps in finding the consistent prizes under heterogeneous priors are (1) use the equilibrium strategies given the conjectured prizes in Proposition 4 above to calculate player's probabilities of winning the contest, (2) use these win probabilities to derive the posteriors using Bayes' rule and finally (3) confirm whether these posteriors correspond to the conjectured prizes. The last step introduces further complications in the analysis, as it involves finding a fixed point. This fixed point problem is, however not tractable when the identity of Player k—the player with the smallest $a_i\Delta_i^c$—varies with the realized abilities of the two players. To keep the analysis tractable, I assume that the identity of Player k remains the same, irrespective of her or her opponent's realized ability. In other words, $a_k\Delta_k^c < a_{-k}\Delta^c_{-k}$ irrespective of the values a_k and a_{-k}. Without loss of generality, let $k = 2$ such that

$$h\Delta_2^c < l\Delta_1^c, \tag{32}$$

which implies that Player 1 has higher effort incentive $a_1\Delta_1$ than Player 2 irrespective of the realized ability pair. With this assumption, I can go through the above steps to find the consistent prizes. Using the equilibrium strategies and noting that Player 1 captures positive expected payoff in equilibrium irrespective of ability realizations, we have that Player 1's probability to win the contest is

$$\Pr[X_2 < X_1] = 1 - \frac{1}{2}\frac{a_2\Delta_2^c}{a_1\Delta_1^c}. \tag{33}$$

Let $\lambda := \Delta_1^c/\Delta_2^c$. Using Bayes' rule, Player 1's win valuation is given by

$$\begin{aligned} \bar{v}_1 &= \kappa \times \Pr\Big[a_1 = h \,|\, \text{Player 1 wins}\Big] \\ &= \kappa \frac{\mu_1\left[\left(1 - \frac{1}{2\lambda}\right)\mu_2 + \left(1 - \frac{l}{2h\lambda}\right)(1-\mu_2)\right]}{\mu_1\left[\left(1 - \frac{1}{2\lambda}\right)\mu_2 + \left(1 - \frac{l}{2h\lambda}\right)(1-\mu_2)\right] + (1-\mu_1)\left[\left(1 - \frac{1}{2\lambda}\right)(1-\mu_2) + \left(1 - \frac{h}{2l\lambda}\right)\mu_2\right]} \end{aligned} \tag{34}$$

As before, $\kappa > 0$ captures how much the player values being perceived as high ability by the observer. The numerator gives the probability that Player 1 wins the contest, conditional on her being high ability, with the relevant win probabilities (in round brackets) defined in (33) and rewritten using the simplification $\lambda := \Delta_1^c/\Delta_2^c$. The denominator is the unconditional probability that Player 1 wins the contest, accounting for all possible ability combinations. The loss valuation for Player 1, as well as the win and loss valuations for Player 2, are calculated analogously using Bayes' rule.

The win and loss valuations are written as a function of the ratio conjectured marginal valuations, λ. This allows us to simplify the system of equations to a single equation with

$$\lambda := \frac{\bar{v}_1^c - \underline{v}_1^c}{\bar{v}_2^c - \underline{v}_2^c} = \frac{\Pr\left[a_1 = h \,|\, \text{Player 1 wins}\right] - \Pr\left[a_1 = h \,|\, \text{Player 1 loses}\right]}{\Pr\left[a_2 = h \,|\, \text{Player 1 wins}\right] - \Pr\left[a_2 = h \,|\, \text{Player 1 loses}\right]} = f(\lambda). \tag{35}$$

The consistent prize vectorλ^* is such that $\lambda^* = f(\lambda^*)$ in (35). Simplifying the RHS of (35) results in all the λ terms dropping out, which then gives us λ^* immediately as a function of parameters μ_i and a_i. Plugging back λ^* in the win and loss valuations allows us to write the win and loss valuations as a function of exogenous parameters a_i, μ_i and κ (see Appendix A.10 for the explicit expressions). Finding λ^* leads us to Theorem 2, which is the main result of this section.

Theorem 2 (Equilibrium)**.** *Define*

$$\Delta_1 = \frac{-2\kappa(h-l)hl(1-\mu_1)^2\mu_1^2}{-2hl(1-\mu_1)\mu_1(h(1-\mu_1)-l\mu_1)+(h-h\mu_1+l\mu_1)^3\mu_2-(h-h\mu_1+l\mu_1)^3\mu_2^2}, \tag{36}$$

$$\Delta_2 = \frac{-2\kappa(h-l)hl(1-\mu_1)\mu_1(1-\mu_2)\mu_2}{(l(1-\mu_2)-h\mu_2)\left[h^2(1-\mu_1)^2(1-\mu_2)\mu_2+l^2\mu_1^2(1-\mu_2)\mu_2-2hl(1-\mu_1)\mu_1(1+(1-\mu_2)\mu_2)\right]}. \tag{37}$$

Suppose

$$h\Delta_2 < l\Delta_1 \tag{38}$$

holds. This condition is equivalent to

$$h < \bar{h} := \frac{l\mu_1(\mu_2-\mu_1)}{2(\mu_1-1)(\mu_2-1)} + \frac{1}{2}\sqrt{\frac{A(\mu_1,\mu_2)}{(\mu_1-1)^2(\mu_2-1)^2\mu_2}}, \tag{39}$$

where $A(\mu_1,\mu_2) =$

$$= l^2\left[\mu_2\mu_1^4 + 4\mu_1^3 + 2\mu_2^2\mu_1^3 - 8\mu_2\mu_1^3 + \mu_2^3\mu_1^2 - 8\mu_1^2 - 8\mu_2^2\mu_1^2 + 16\mu_2\mu_1^2 + 4\mu_1 + 4\mu_2^2\mu_1 - 8\mu_2\mu_1\right].$$

There exists an equilibrium in which Player i's marginal valuation of winning equals Δ_i *and players randomize over the same support* $x \in [0, a_2\Delta_2]$ *according to cumulative distribution* *functions*

$$F_1^*(x_1) = \frac{x_1}{a_1\Delta_1}, \tag{40}$$

$$F_2^*(x_2) = 1 + \frac{x_2 - a_2\Delta_2}{a_1\Delta_1}. \tag{41}$$

The ratio of marginal prizes in this equilibrium is given by

$$\lambda^* = \frac{\Delta_1}{\Delta_2} = \frac{(1-\mu_1)\mu_1}{(1-\mu_2)\mu_2}\frac{\mu_2 h + (1-\mu_2)l}{\mu_1 h + (1-\mu_1)l}\left[1 - \frac{(1-2\mu_1)(h-l)}{h-\mu_1(h-l)}\right]. \tag{42}$$

To see why h needs to be bounded from above for $h\Delta_2 < l\Delta_1 \iff h/l < \Delta_1/\Delta_2$ to hold, take the limit of Δ_1/Δ_2 as $h \to \infty$ to get $\mu_1/(1-\mu_2)$. The limit of the LHS of the inequality is clearly $\lim_{h\to\infty} h/l = \infty$. We can then immediately see that for some priors μ_i, the inequality will not hold if h is too large.

It is also instructive to discuss the expression for λ^*, restated below in (43). The first multiplicative term is the ratio of variances in player's abilities[16] and the remaining two terms are the ratio of expected abilities and an adjustment term that increases in μ_1:

$$\lambda^* = \underbrace{\frac{(1-\mu_1)\mu_1}{(1-\mu_2)\mu_2}}_{\mathbb{V}[a_1]/\mathbb{V}[a_2]}\underbrace{\frac{\mu_2 h + (1-\mu_2)l}{\mu_1 h + (1-\mu_1)l}}_{\mathbb{E}[a_2]/\mathbb{E}[a_1]}\left[1 - \frac{(1-2\mu_1)(h-l)}{h-\mu_1(h-l)}\right]. \tag{43}$$

As under homogeneous priors, uncertainty about players' abilities influences (the ratio of) marginal valuations. To see how, first consider that the variance of a player's realized ability a_i is largest at $\mu_i = 1/2$. Consequently, the closer μ_1 is to 1/2, the higher λ^* will be driven by the uncertainty effect we identified in the homogeneous prior case. On the other hand, the closer μ_2 is to 1/2, the stronger is the downward pressure from the uncertainty effect on the equilibrium ratio of marginal valuations, $\lambda^* = \Delta_1^*/\Delta_2^*$. While uncertainty was one dimensional in the homogeneous prior case, with the observer having different priors associated with the players it is the relative variance of ability that matters for how λ^* is affected through the *relative* uncertainty effect.

Last, the condition on h in Theorem 2 implies a necessary condition on the relative size of priors stated in Remark 1 below—see Figure 4 for an illustration of the admissible pairs of priors. A brief examination of this condition can provide useful intuition for the analysis that follows.

Remark 1. *For $h\Delta_2 < l\Delta_1$ to hold, the observer's prior about Player 2 must be sufficiently extreme relative to that of Player 1 such that*

16. Ability a_i has a Bernoulli distribution with the two events being $a_i = h$ and $a_i = l$.

1. *if $\mu_1 \leq 1/2$, $\mu_2 < \mu_1$ and $1 - \mu_1 < \mu_2$ must hold,*

2. *and if $\mu_1 > 1/2$ then $\mu_2 < 1 - \mu_1$ and $\mu_1 < \mu_2$ must hold.*

These restrictions are intuitive given the results we have from the model with homogeneous priors: players' effort incentives $a_i\Delta_i$ increase in Δ_i. When the observer's uncertainty about a player's ability type vanishes—in other words, her prior $\mu_i \in (0,1)$ is approaches the extremes—the prize spread Δ_i goes to zero. This implies then that for fixed μ_1 and a_i, we need the prior for Player 2 to be sufficiently extreme in order for the inequality $a_2\Delta_2 < a_1\Delta_1$ to be satisfied.

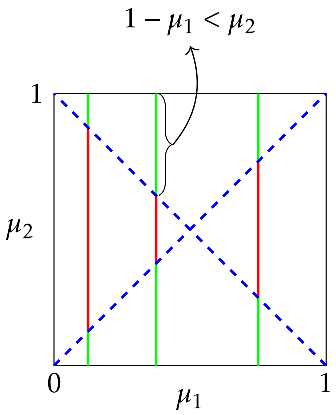

Figure 4: Restrictions on μ_2 for fixed μ_1 under the assumption that $a_2\Delta_2 < a_1\Delta_1$ for all ability pairs (a_1, a_2). The green line segments show the values μ_2 can take for a given μ_1.

Expected effort in the limit. Allowing for heterogeneous priors introduces significant complexity in the model, however, I can extend a key result obtained under homogeneous priors to the case when the observer is allowed to have different priors about players' abilities: the observer being too certain about a player's ability reduces her motivation to exert effort. We can confirm this by taking the limit of players' expected effort with respect to μ_i approaching its extremes at zero and one. Using the equilibrium strategies in Proposition 4 and the consistent psychological prizes, we can calculate expected effort for the two players as

$$\mathbb{E}[X_1^*] = \frac{1}{2}a_2\Delta_2, \tag{44}$$

$$\mathbb{E}[X_2^*] = \frac{1}{2}\frac{(a_2\Delta_2)^2}{a_1\Delta_1}. \tag{45}$$

Players randomize their bids independently and, therefore, expected total effort is simply the sum of expected individual efforts $\mathbb{E}[X_1^*] + \mathbb{E}[X_2^*]$. Since μ_i only enters expected effort through Δ_i, knowing that $\lim_{\mu_i \to 0} \Delta_i = \lim_{\mu_i \to 1} \Delta_i = 0$ provides us with the limit result stated in Proposition 5 (note that we can rewrite the expected effort of Player 2 as a function of Δ_2 and $\lambda^* > 1$). A further implication of this result is that as Δ_i approaches zero, so does the opponent's, Player $-i$'s expected effort. This is intuitive: when one player has no incentive to exert effort, her opponent can win by exerting an infinitesimal amount of effort.

Proposition 5. *Total expected effort approaches zero as uncertainty about players' abilities vanishes ($\mu_i \to 0$ or $\mu_i \to 1$).*

Win probability. It is also important to consider how win probabilities are affected by changing priors, especially in light of the management science literature on how observers' expectations of success probability affects performance (Lount Jr, Pettit, and Doyle 2017; Chen and Klimoski 2003) and how facing low expectations—being an "underdog"—can increase performance in particular (Nurmohamed 2020; Nurmohamed, Kundro, and Myers 2021). In this model, we can interpret the observer having a low prior about a player's ability relative to her opponent as that player facing low expectations. To look at the effects of facing low expectations on performance—measured here by a player's probability to win the contest in which abilities are already realized—, I first rewrite (33) to get Player 1's probability to win the contest as a function of λ and ability parameters a_i:

$$\Pr[X_2 < X_1] = 1 - \frac{1}{2}\frac{a_2}{a_1}\frac{\Delta_2}{\Delta_1} = 1 - \frac{1}{2}\frac{a_2}{a_1}\frac{1}{\lambda} > \frac{1}{2}. \tag{46}$$

The inequality follows from $1/\lambda < l/h < 1$ under the assumption that $a_2\Delta_2 < a_1\Delta_1$.

Taking the derivatives of (46) with respect to μ_1 and μ_2 and analyzing the sign of these derivatives yields the results in Proposition 6: an observer having a more extreme prior belief about one's ability relative some threshold value decreases the player's probability to win the contest.[17] Conversely, an observer having a more extreme prior belief about the opponent's ability relative some threshold increases the player's probability to win.

Proposition 6. *Assume $h < \bar{h}$. Player 1's probability of winning the contest monotone increases in the observer's prior about her ability μ_1 until the threshold $\bar{\mu}_1$, and monotone decreases when $\bar{\mu}_1 < \mu_1$. The threshold is given by*

$$1/2 < \bar{\mu}_1 = \frac{h - \sqrt{lh}}{h - l} < 1. \tag{47}$$

Player 1's win probability monotone decreases in the observer's prior about her opponent's ability μ_2 up until the threshold $\bar{\mu}_2$, and monotone increases thereafter. The threshold is given by

$$0 < \bar{\mu}_2 = \frac{l - \sqrt{lh}}{l - h} < 1/2. \tag{48}$$

17. With the caveat that for this result to hold for μ_1 we must impose that there exists a feasible $\mu_1 > \bar{\mu}_1$.

Note that $h < \bar{h}$ implies that the restrictions on the relative size of priors μ_1 and μ_2 in Remark 1 must also hold. It is, therefore, possible that when μ_2 is close enough to 1/2, such that $1 - \bar{\mu}_1 < \mu_2 \leq \bar{\mu}_1$, then Player 1's win probability never decreases in μ_1 for the feasible parameter values μ_1 can take. Nonetheless, there always exists a feasible $\mu_2 < 1 - \bar{\mu}_1$ or $\bar{\mu}_1 < \mu_2$ for which $\bar{\mu}_1$ is an interior point of the interval of parameter values μ_1 can take. On the other hand, there exists for any μ_1 an interval of feasible μ_2 values close to zero on which Player 1's win probability decreases in μ_2 and another interval close to one on which her win probability increases in μ_2.

It may appear puzzling at first that μ_2 can get closer to 1/2 but this may still result in an increase of Player 1's win probability. We may expect that increasing uncertainty about a player's ability type increases her marginal win valuation and, therefore, her probability to win the contest. However, as we have already seen with expected effort, changing priors affect marginal valuations asymmetrically given the power imbalance between players: for some initial parameter values, an increase in the uncertainty about a player's ability can tilt the ratio of marginal valuation in the opponent's favor.

Corollary 1 (The "Underdog Effect")**.** *For priors $\mu_i < \mu_{-i}$, there exists some interval of μ_i on which Player i's win probability increases as μ_i decreases. There also exists an interval on which decreasing μ_i decreases Player i's win probability.*

The result in Corollary 1 shows that there indeed is an underdog effect as in Nurmohamed (2020): there exists some prior belief by the observer whereby a player is an underdog—meaning that the observer's belief about her ability is lower than that of her opponent—, and further reducing the observer's prior about her ability *increases* her probability to win the contest. This result, however, comes with a significant caveat: the underdog effect reverses when the prior μ_i gets closer to zero. We can interpret this as a complacency effect, briefly mentioned by Grant and Shandell (2022). As priors are becoming more extreme, the observer's posterior will respond less to the ability signal of winning in the contest. This makes the marginal value of winning the contest smaller for the player, which induces low effort and leads to a decreased probability of winning. The same argument holds when the observer's prior approaches one: the complacency effect also operates for players believed to be high ability with high confidence.

While it is clear that social image concerns are not the only mechanism at play when analyzing how agents respond to facing high or low expectations, the model presented in this section allows one to identify a novel mechanism that could (partially) drive the underdog effect, and at the same time could also generate a complacency effect when the observer is too confident in an agent's ability before observing the contest outcome.

7 Concluding Remarks

This paper develops a theoretical model that helps explain competitive behavior even in contexts without explicit monetary rewards by studying how social image concerns shapes effort provision in contests. The model shows how introducing an observer who updates her beliefs about competitors' abilities creates implicit prizes that motivate costly effort, even in the absence of explicit, instrumental rewards.

The model provides novel insights into how social image concerns drive competitive behavior by deriving prizes in equilibrium, rather than fixing exogenously. First, competition is most intense when observers have the greatest uncertainty about competitors' abilities. Second, the model identifies a novel "encouragement effect" of increasing heterogeneity in productive ability between player types, contrasting with the standard discouragement effect in contest theory. This occurs because larger ability gaps make winning more informative about being high ability, potentially offsetting the demotivating effect of facing a stronger opponent. Third, I show that the type-h probability that maximizes unconditional expected effort from the observer's perspective differs from the one that maximizes effort in any given ability matchup. Last, I extend the model to allow the observer to have different priors about players' abilities. This framework allows me to study the "underdog effect" discussed in the management science literature (Nurmohamed 2020). I show that social image concerns can also generate this effect, whereby decreasing the prior belief about the ability of the player with the already lower prior of the two competing players can in fact increase the probability that she wins the contest. On the other hand, this increase in win probability reverses once a threshold prior belief is reached and a "complacency effect" starts to dominate whereby the observer's increasingly sticky prior decreases the player's motivation to exert effort.

The paper provides several testable hypotheses on how agents are motivated by social image concerns, which may motivate future experimental work studying behavior in contests for social image.

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A Appendix

A.1 Proof of Lemma 1.

Proof. The equilibrium strategies are derived in Kawamura and Moreno de Barreda (2014).

For the heterogeneous contest win probabilities, take the general expression in Kawamura and Moreno de Barreda (2014) given for the probability of a high-ability player to win against a low-ability player

$$1 - \frac{1}{\Delta^c} c_h\left(c_l^{-1}(\Delta^c)\right) + \frac{1}{(\Delta^c)^2} \int_0^{c_l^{-1}(\Delta^c)} c_l'(x) c_h(x)\, dx, \tag{49}$$

where the cost function c_{a_i} of a player with ability a_i satisfies $c_{a_i}(0) = 0$, $c_{a_i}'(\cdot) > 0$, $c_{a_i}''(\cdot) \geq 0$, $c_{a_j}(x) \geq c_{a_i}(x)\ \forall x \geq 0$ if $a_i \geq a_j$ and the prize valuation is $\Delta^c > 0$ with loss valuation normalized to zero.

Using the functional form assumption for the cost function (which satisfied the above restrictions in Kawamura and Moreno de Barreda (2014)), we can simplify (49) to get

$$1 - \frac{l}{h} + \frac{1}{(\Delta^c)^2} \int_0^{\Delta^c l} \frac{x}{lh}\, dx =$$

$$= 1 - \frac{l}{2h} = w_1(h, l, \mathbf{v}).$$

We then have $w_1^*(l,h) = \frac{l}{2h} = \frac{1}{2\alpha} = 1 - w_1^*(h,l)$. When players are homogeneous then the above expression becomes $1 - \frac{h}{2h} = 1 - \frac{l}{2l} = 1/2$. □

A.2 Proof of Theorem 1.

Proof. For $M > 0$ players will not remain inactive and therefore the contest outcome is informative of players' abilities: the posterior beliefs conditional on contest outcome are not equal.

Assuming that the prize conjectures are symmetric, we can use the win probabilities in Lemma 1 to calculate the win and loss valuations in Equation 10 and Equation 11. The resulting posteriors are (1) derived via Bayes' rule and (2) are consistent in the sense of Definition 1 since win probabilities are invariant to the marginal prize. The invariance of win probabilities to the marginal prize also ensures that the psychological prizes are indeed unique for fixed μ, α and κ. Given that prizes are unique the two-player all-pay contest has a unique equilibrium in mixed strategies. To get the marginal prize Δ, simply plug in the expressions for the posteriors calculated above into $\Delta = M + \bar{v} - \underline{v}$.

Equilibrium expected effort is derived by integrating the respective equilibrium cumulative distribution function in Lemma 1, replacing the conjectured marginal prize Δ^c with Δ.

When $M = 0$, players may make a symmetric conjecture that correspond to the case of $M > 0$, hence the same equilibrium survives.

To confirm that staying inactive is also an equilibrium, suppose that the conjecture is that the posteriors equal the prior belief. We then have that the marginal prize Δ is zero. From Lemma 1 we know that the equilibrium is then in pure strategies such that both players remain inactive. When both players are inactive the contest carries no information about their abilities, hence the posteriors can be derived with Bayes' rule. It follows that the initial conjecture is indeed consistent. Remaining inactive while expecting that the observer does not update from the contest outcome is then indeed an equilibrium. □

A.3 Proof of Lemma 2.

Proof. Differentiate the win valuation with respect to α and μ:

$$\frac{d\bar{v}}{d\alpha} = \kappa\frac{(1-\mu)\mu}{\alpha^2} > 0,$$
$$\frac{d\bar{v}}{d\mu} = \kappa\frac{2\alpha - 2(\alpha-1)\mu - 1}{\alpha} = \kappa\left(\frac{2\mu-1}{\alpha} + 2 - 2\mu\right) > 0.$$

$\frac{d\bar{v}}{d\alpha} > 0$ clearly holds given that $0 < \mu < 1$ and $\kappa > 0$.

To see that $\frac{d\bar{v}}{d\mu} > 0$ is indeed correct, first we need to realize that no minimum exists, however, we can differentiate with respect to α and μ to identify the appropriate limits to take:

$$\frac{d\left(\frac{d\bar{v}}{d\mu}\right)}{d\mu} = \kappa\left(\frac{2}{\alpha} - 2\right) < 0, \qquad \frac{d\left(\frac{d\bar{v}}{d\mu}\right)}{d\alpha} = \kappa\left(\frac{1-2\mu}{\alpha^2}\right).$$

The derivative with respect to μ is always negative given that $\alpha > 1$ and $\kappa > 0$, while the sign of the derivative with respect to α changes depending on μ (positive if $1/2 > \mu$). We should then take the limits as $(\mu,\alpha) \to (1,1)$ and $(\mu,\alpha) \to (1,\infty)$:

$$\lim_{(\mu,\alpha)\to(1,1)} \frac{d\bar{v}}{d\mu} = \kappa > 0, \qquad \lim_{(\mu,\alpha)\to(1,\infty)} \frac{d\bar{v}}{d\mu} = 0.$$

Therefore, $\frac{d\bar{v}}{d\mu}$ is indeed bounded by zero from below.

□

A.4 Proof of Lemma 3

Proof. First, differentiate $\underline{v}$ with respect to μ and α:

$$\frac{d\underline{v}}{d\mu} = \kappa\left(\frac{2(\alpha-1)\mu+1}{\alpha}\right) > 0, \qquad \frac{d\underline{v}}{d\alpha} = \kappa\left(\frac{(\mu-1)\mu}{\alpha^2}\right) < 0.$$

$\frac{d\underline{v}}{d\mu}$ is clearly always positive as $\alpha - 1 > 0$ and $\kappa > 0$. $\frac{d\underline{v}}{d\alpha} < 0$ holds since $\kappa > 0$ and $0 < \mu < 1$ and therefore the numerator of the fraction is strictly negative. $\square$

A.5 Proof of Proposition 1

Proof. Recall that the marginal value of winning the contest, Δ , is given by

$$\kappa\frac{2(\alpha-1)(1-\mu)\mu}{\alpha} + M.$$

It is easy to confirm that $\bar{v} - \underline{v}$ is concave in μ. The first order condition with respect to μ then pins down the effort maximizing share of high-ability players

$$\begin{aligned} 2\kappa\frac{\alpha-1}{\alpha}(1-2\mu) &= 0 \\ \mu &= \frac{1}{2}. \end{aligned}$$

Differentiating Δ with respect to α yields

$$\kappa\frac{2(1-\mu)\mu}{\alpha^2} > 0.$$

The derivative is always positive as $0 < \mu < 1$ and $\kappa > 0$.

For the second part of the proposition, calculate the expected effort for the given ability pairings. First, for fixed l whereby changing α is equivalent to changing h we have that the expected effort in a heterogeneous contest is given by

$$\mathbb{E}[X_h^*] = \frac{1}{2}l\Delta \qquad \mathbb{E}[X_l^*] = \frac{1}{2\alpha}l\Delta,$$

and in a homogeneous contest by

$$\mathbb{E}[X_i^*] = \frac{1}{2}a_i\Delta.$$

All of these expressions are clearly increasing in Δ.

Given that μ affects expected effort provision through only Δ, maximizing with respect to μ is equivalent to maximizing Δ with respect to μ. From the first part of the proposition we have that $\mu = 1/2$ maximizes Δ, hence we have that $\mu = 1/2$ maximizes expected total effort as well in both homogeneous and heterogeneous contests. □

A.6 Proof of Proposition 2

Proof. It is easy to confirm that the marginal prize for winning the contest is increasing in ability asymmetry α as

$$\frac{d\Delta}{d\alpha} = \frac{2(1-\mu)\mu}{\alpha^2} > 0.$$

To show the result in Part 1) of the proposition, consider the case when α is increased by reducing l.

From the above comparative static we have immediately that the expected effort in homogeneous contests of two high-ability players are increasing in when l is decreased. Next, set the derivative of expected effort in a contest of two low-ability player with respect to l to be greater than zero and rearrange for h to get the inequality provided in part (1) of Proposition 2.

Then differentiate total expected effort in a heterogeneous contest with respect to l to get

$$\frac{d}{dl}\left[\frac{1}{2}\Delta\left(l + \frac{l^2}{h}\right)\right] = \frac{1}{2}\kappa\frac{(1-\mu)\mu}{h}\left(2h - 4l + \frac{4lh - 6l^2}{h}\right)$$

The derivative is positive if

$$2h - 4l + \frac{4lh - 6l^2}{h} = \frac{2h^2 - 6l^2}{h} > 0.$$

This holds if

$$h > \sqrt{3}l.$$

Given that we can increase α by *decreasing* l, we can interpret the above as decreasing l increases expected effort if h is sufficiently low relative to l

$$h < \sqrt{3}l.$$

This is the condition provided in the proposition.

When α is increased by increasing h, as in Part 2) of the proposition, we can see that the partial derivative of the expected effort in a contest between two high-ability players is strictly increasing in h. This observation together with $d\Delta/d\alpha > 0$ yields the result that expected effort is increasing in ability asymmetry also when increasing h holding l constant. Next, from $d\Delta/d\alpha > 0$, we directly have the result that the expected effort in homogeneous contests of two low-ability players are increasing in α.

Finally, differentiate total expected effort in a heterogeneous contest with respect to h to get

$$\frac{d}{dh}\left[\frac{1}{2}\Delta\left(l+\frac{l^2}{h}\right)\right]=\frac{1}{2}\kappa(1-\mu)\mu\left[\frac{2l}{h^2}\left(l+\frac{l^2}{h}\right)-\frac{2(h-l)l^2}{h^3},\right]$$

which simplifies to

$$2\kappa(1-\mu)\mu\frac{l^3}{h^3}>0.$$

□

A.7 Encouragement Effect with Positive Monetary Prizes $M>0$.

In this section, I consider how total expected effort changes with ability asymmetry α when allowing the monetary prize to be strictly positive.

First I assume that α is increased by reducing l. M only affects effort provision through Δ, and hence, when differentiating $\mathbb{E}[X_1^*+X_2^*\mid a_i=h]$ with respect to l we get the same conclusion as when $M=0$.

Differentiation of $\mathbb{E}[X_1^*+X_2^*\mid a_i=l]$ with respect to l yields

$$\kappa\frac{2(h-2l)(1-\mu)\mu}{h}+M,$$

This is negative if

$$h<2l-\frac{Mh}{2\kappa(1-\mu)\mu}$$

holds. This inequality is clearly harder to satisfy when M is large, but holds for small enough M.

The derivative of total expected effort in a heterogeneous contest with respect to l is

$$\frac{d}{dl}\left[\frac{1}{2}(\Delta+M)\left(l+\frac{l^2}{h}\right)\right]=\frac{1}{2}\frac{(1-\mu)\mu}{h}\left[-2\kappa\left(l+\frac{l^2}{h}\right)+2\kappa(h-l)\left(1+\frac{2l}{h}\right)\right]+\frac{1}{2}M\left(1+\frac{2l}{h}\right)$$

This is negative if

$$(1-\mu)\mu\kappa(2h^2-6l^2)<-Mh(h+2l)$$

holds. Note that when $M=0$, this reduces to the inequality we have in Proposition 2. When M is positive, this inequality can still hold but will be harder to satisfy.

Let us now assume that α increased by increasing h. Differentiating $\mathbb{E}[X_1^*+X_2^*\mid a_i=h]$ gives

$$\Delta+\kappa\left(\frac{2l(1-\mu)\mu}{h}\right)>0.$$

Hence the result is the same as with $M=0$: expected effort increases with ability asymmetry. Adding positive monetary prizes in a homogeneous contest with two low-ability players does not effect the sign of the derivative.

Differentiating the expected effort in a heterogeneous contest yields

$$= \frac{1}{2}\left[\kappa\frac{2l(1-\mu)\mu}{h^2}\left(l+\frac{l^2}{h}\right)-\left(\kappa\frac{2(h-l)(1-\mu)\mu}{h}+M\right)\frac{l^2}{h^2}\right]$$

This is positive if

$$M < \frac{l}{h}4\kappa(1-\mu)\mu$$

holds. Note that this inequality becomes harder to satisfy when M is larger.

A.8 Proof of Proposition 3

Proof. I first rank ability matchups with respect to their total expected effort for fixed μ and α. Expected effort in a homogeneous contest is

$$\mathbb{E}[X_i^*] = \frac{1}{2}a_i\Delta.$$

We know that $h > l$ ($a_i \in \{h, l\}$), hence expected effort is clearly larger in a homogeneous contest between high-ability players. Expected effort in a heterogeneous contest is given by

$$\mathbb{E}[X_h^*] = \frac{1}{2}l\Delta \qquad \mathbb{E}[X_l^*] = \frac{1}{2\alpha}l\Delta,$$

for the high ability and the low-ability player, respectively. Here I abuse notation and use a_i in the player index to indicate the ability of the respective player. We know that $1/\alpha < 1$, hence the low-ability player exerts less effort in expectation in a heterogeneous contest than in a homogeneous contest of two low-ability players, while the high-ability player's expected effort is lower in a heterogeneous contest (equals that of a low-ability player's in a homogeneous contest). We then have that

$$\mathbb{E}[X_1^* + X_2^* \mid a_i = h, a_{-i} = l] < \mathbb{E}[X_1^* + X_2^* \mid a_i = l] < \mathbb{E}[X_1^* + X_2^* \mid a_i = h].$$

I have already shown in the text that both the probability of a heterogeneous contest occurring, $2\mu(1-\mu)$, and the psychological prize spread $\bar{v} - \underline{v}$ is symmetric around $\mu = 1/2$.

Given that the high-ability matchup generates the highest expected effort, we want to maximize the probability of this happening. Given the symmetry of $2\mu(1-\mu)$, this does not come at a cost of increasing the probability of a heterogeneous contest occurring: for any symmetric move $-b$ and b (with $b \in (0, 1/2)$) away from $\mu = 1/2$, the decrease in the probability of a heterogeneous match will be identical. However, moving to a lower $\mu = 1/2 - b < 1/2$ will decrease the probability of a contest between two high-ability players, which generates the highest expected effort provision (see Figure 2). We could have the same psychological prize spread if we increase μ by b since $\left(\bar{v}-\underline{v} \mid \mu=1/2-b\right) = \left(\bar{v}-\underline{v} \mid \mu=1/2+b\right)$, whereby we will also increase the chance of a high-ability matchup. It follows then that the effort maximizing share of high-ability players μ^* must be such that $\mu^* \in [1/2, 1)$.

The proof that $1/2 < \mu^*$ holds is provided in the text.

To show that μ^* is increasing in M, note that we already know that $1/2 < \mu^*$. When we increase μ from $1/2$, this leads to two opposing effects: (1) the reduction in the psychological prize spread has a negative effect on expected total effort while (2) increasing the probability of having two high-type players competing has a positive effect on expected total effort. The psychological prize spread

$$\kappa \frac{2(\alpha-1)(1-\mu)\mu}{\alpha}$$

is independent from M. Therefore, the magnitude of the negative effect is invariant to an increase in the monetary prize M. On the other hand, the magnitude of the positive effect increases with M. Thus, and increase in M favors a higher μ to maximize expected total effort.

□

A.9 Proof of Proposition 4.

Proof. The equilibrium is in mixed strategies as the all-pay nature of the contest still cannot support any two fixed effort level in equilibrium even when allowing for different loss valuations: one player would always have an incentive to deviate to an effort level just above that of her opponent, or deviate to exerting no effort with probability one.

Assume $a_k\Delta_k^c < a_{-k}\Delta_{-k}^c$ for all ability pairs (a_k, a_{-k}). Player k earns an expected payoff equal to her conjectured loss valuation $\underline{v}_k^c = \Pi_k$ as Player $-k$ can credibly threaten to exert effort that is higher than any incentive compatible effort level from Player k. Player $-k$ earns what she would if she exerted effort just above Player k's maximum incentive compatible effort level $a_k\Delta_k^c$, guaranteeing a win for herself

$$\Pi_{-k} = \bar{v}_{-k}^c - \frac{a_k\Delta_k^c}{a_{-k}}.$$

Players randomize over the same support, with equilibrium effort in the closed interval $[0, a_k\Delta_k^c]$. Player $-k$ mixes over effort levels using the cumulative distribution function F_{-k}, such that

$$\Pi_k = F_{-k}(x_k)\Delta_k^c + \underline{v}_k^c - x_k/a_k = \underline{v}_k \quad \forall\, x \in [0, a_k\Delta_k^c].$$

Rearranging yields the equilibrium CDF F^*_{-k}:

$$F^*_{-k}(x_{-k}) = \frac{x_{-k}}{a_k \Delta^c_k} \quad \forall\, x \in [0, a_k \Delta^c_k].$$

Similarly, Player k randomizes over effort levels using CDF F_k, such that Player $-k$'s expected payoff equals:

$$\Pi_{-k} = F_k(x_{-k})\Delta^c_{-k} + \underline{v}^c_{-k} - x_{-k}/a_{-k} = \bar{v}^c_{-k} - a_k \Delta^c_k / a_{-k} \quad \forall\, x \in [0, a_k \Delta^c_k].$$

Rearranging yields

$$F^*_{-k}(x_{-k}) = 1 + \frac{x_k - a_k \Delta^c_k}{a_{-k} \Delta^c_{-k}} \quad \forall\, x \in [0, a_k \Delta^c_k].$$

Uniqueness for fixed conjectures Δ^c_i follows from Hillman and Riley (1989) and Baye, Kovenock, and De Vries (1996). $\square$

A.10 Equilibrium when $a_2\Delta_2 < a_1\Delta_1$ holds for all (a_1, a_2).

Endogenous win and loss valuations. When $a_2\Delta_2 < a_1\Delta_1$ holds for all (a_1, a_2), the equilibrium prize vector $\mathbf{v}^* = (\bar{v}^*_1, \bar{v}^*_2, \underline{v}^*_1, \underline{v}^*_2)$ is given by

$$\bar{v}^*_1 = \kappa \frac{l\mu_1\left(\mu_1\left(\mu_2^2(l-h) + \mu_2(h-l) + 2h\right) - 2h\mu_1^2 + h(\mu_2 - 1)\mu_2\right)}{h^2(\mu_2-1)\mu_2 + 2h\mu_1\left(\mu_2^2(l-h) + \mu_2(h-l) + l\right) + \mu_1^2\left(\mu_2^2(h-l)^2 - \mu_2(h-l)^2 - 2hl\right)}$$

$$\underline{v}^*_1 = \kappa \frac{l\mu_1}{\mu_1(l-h) + h}$$

$$\bar{v}^*_2 = \kappa \frac{h\mu_2}{\mu_2(h-l) + l}$$

$$\underline{v}^*_2 =$$

$$\kappa \frac{h\mu_2\left(\mu_1^2\left(\mu_2\left(l^2-h^2\right) + \mu_2^2(h-l)^2 - 2l^2\right) + 2\mu_1\left(\mu_2\left(h^2-l^2\right) + h\mu_2^2(l-h) + l^2\right) + h^2(\mu_2-1)\mu_2\right)}{(l - \mu_2(l-h))\left(h^2(\mu_2-1)\mu_2 + 2h\mu_1\left(\mu_2^2(l-h) + \mu_2(h-l) + l\right) + \mu_1^2\left(\mu_2^2(h-l)^2 - \mu_2(h-l)^2 - 2hl\right)\right)}.$$

One can check that these valuations are indeed always positive by making use of the restriction in this case on $\lambda : 1/\lambda < l/h$, in addition to the assumption imposed on ability parameters l, h.

The threshold $\bar{h}$ is given by

$$\frac{l\mu_1\mu_2 - l\mu_1^2}{2(\mu_1-1)(\mu_2-1)} +$$

$$\frac{1}{2}\sqrt{\frac{l^2\mu_2\mu_1^4 + 4l^2\mu_1^3 + 2l^2\mu_2^2\mu_1^3 - 8l^2\mu_2\mu_1^3 + l^2\mu_2^3\mu_1^2 - 8l^2\mu_1^2 - 8l^2\mu_2^2\mu_1^2 + 16l^2\mu_2\mu_1^2 + 4l^2\mu_1 + 4l^2\mu_2^2\mu_1 - 8l^2\mu_2\mu_1}{(\mu_1-1)^2(\mu_2-1)^2\mu_2}}.$$

A.11 Proof of Theorem 2

Proof. Replace prizes in Bayes' rule with the equilibrium prizes given in Theorem 2, that are for now remain conjectures. Divide the resulting conjectured marginal valuations Δ_i^c to get λ. Simplification yields an expression that is a function of exogenous variables a_i and μ_i only. We have then found λ^*, which we can plug back into the equations for win and loss valuations that we wrote as a function of λ and exogenous variables. The resulting valuations are derived using Bayes' rule, and arise from equilibrium strategies that are mutual best responses given any fixed conjectured Δ_i^c satisfying $h\Delta_2 < l\Delta_1$.

The restriction on ability parameter h is obtained by rearranging the inequality $h\Delta_2 < l\Delta_1$ for h, noting that Δ_i is a function of exogenous parameters κ, l, h, μ_1 and μ_2.

The equilibrium strategies are given by simply replacing Δ_i^c with Δ_i in Proposition 4. $\square$

A.12 Proof of Proposition 5.

Proof. For the first part of the statement in Proposition 5 differentiate expected effort with respect to μ_1 and evaluate at $\mu_1 = 1/2$. Prior $\mu_1 = 1/2$ can only maximize total expected effort provision if the total expected effort derivative is a critical point (equals zero) when $\mu_1 = 1/2$.

$$\left.\frac{d\mathbb{E}[X_1^* + X_2^*]}{d\mu_1}\right|_{\mu_1=1/2} = 0 \iff$$
$$\underbrace{l[a_1(h+l) + 2a_2h]}_{>0} + \underbrace{\mu_2(h+l)[a_1(h-l) + a_2(h+l)] - a_2\mu_2^2(h+l)^2}_{:=A} = 0 \tag{50}$$

The first term of the sum is positive, and therefore it is sufficient for the rest of the sum to be positive as well to show that the derivative cannot equal zero at $\mu_1 = 1/2$.

Expanding the brackets in A and re-arrranging yields

$$a_1(h+l)(h-l)\mu_2 + a_2(h+l)^2(\mu_2 - \mu_2^2) = 0. \tag{51}$$

The first term of the sum is clearly positive since $h > l$, and the second term is also positive since $0 < \mu_2 < 1 \Rightarrow \mu_2 - \mu_2^2 > 0$. This leads to a contradiction as $d\mathbb{E}[X_1^* + X_2^*]/d\mu_1 \neq 0$ when $\mu_1 = 1/2$. This implies that $\mu_1 = 1/2$ does not maximize total expected effort.

To prove the second part of the proposition, write total expected effort as

$$\mathbb{E}[X_1^* + X_2^*] = \frac{1}{2}\left[a_2\Delta_2 + \frac{(a_2\Delta_2)^2}{a_1\Delta_1}\right] = \frac{1}{2}\left[a_2\Delta_2 + \frac{a_2^2}{a_1}\frac{1}{\lambda}\Delta_2\right] \tag{52}$$

with $\lambda = \Delta_1/\Delta_2 > 1$ since $a_2\Delta_2 < a_1\Delta_1$ must hold for all ability combinations, including when the two players have the same ability parameter a_i. Taking the limit of marginal valuations yields

$$\lim_{\mu_i \to 0} \Delta_i = \lim_{\mu_i \to 1} \Delta_i = 0 \tag{53}$$

It is then clear that as $\Delta_2 \to 0$, $\mathbb{E}[X_1^* + X_2^*] \to 0$. While we are dividing with Δ_1 in $\mathbb{E}[X_1^* + X_2^*]$, as $\Delta_1 \to 0$ so does $\Delta_2 \to 0$ and Δ_2 approaches zero faster since $\lambda = \Delta_1/\Delta_2 > 1$ must always be satisfied. We then have that

$$\lim_{\mu_i \to 1} \mathbb{E}[X_1^* + X_2^*] = \lim_{\mu_i \to 0} \mathbb{E}[X_1^* + X_2^*] = 0. \tag{54}$$

□

A.13 Proof of Proposition 6.

Proof. The sign of the derivative of Player 1's win probability (in a contest with realized abilities) with respect to μ_i only depends on $-1/\lambda$. It is therefore sufficient to analyze one of three ability pairs of $(a_1 = a_2)$, $(a_1 = h, a_2 = l)$ and $(a_1 = l, a_2 = h)$ to investigate how win probability is affected by changing the observer's prior about players' abilities.

Looking at the case with homogeneous abilities $(a_1 = a_2)$, differentiating Player 1's win probability with respect μ_1 and some algebra yields the following condition for the derivative to be positive

$$(h-l)\mu_1^2 - 2h\mu_1 + h > 0. \tag{55}$$

The LHS of (55) is a quadratic equation with one positive root $\bar{\mu}_1$ in the $(0,1)$ interval and the other root above 1. Given that $(h-l) > 0$, we know that Player 1's win probability is increasing in the observer's prior about his ability if $\mu_1 < \bar{\mu}_1 = (h - \sqrt{lh})/(h-l)$. The threshold is greater than 1/2 since $\bar{\mu}_1 > 1/2 \iff (h+l)/2 > \sqrt{lh}$, which holds since the arithmetic mean is strictly larger than the geometric mean for any two positive numbers that do not equal each other. Similar steps lead to the condition $d(1 - 1/2\lambda)/d\mu_2 > 0 \iff \bar{\mu}_2 < \mu_2$ with $\bar{\mu}_2 = (-l + \sqrt{lh})/(h-l) < 1/2$. □