Socially responsible multiobjective optimal portfolios

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Abstract

We extend the socially responsible multiobjective problem to (i) estimate optimal portfolios via reward/risk maximization, (ii) include dependence structure between asset returns using vine copulas, and (iii) incorporate enhanced indexation utilizing cumulative zero-order stochastic dominance. In an application of the MOP optimization to a sample of Eurostoxx 50 constituents, we show that the optimal MOPs provide investors with the flexibility of incorporating different objectives. However, there is a trade-off between reward (risk) measures. Although, including social responsibility results in lower portfolio return and economic performance, it reduces the portfolio risk. While the cumulative zero-order SD objective (in most cases) increases the portfolio return when included in socially responsible MOPs, it reduces the portfolio risk. The predictive models lead to MOPs with higher return and reward/risk ratios. In particular, the copula-based MOPs achieve less tail risk.

Keywords: Finance, Multiobjective portfolio, vine copula, multivariate GARCH, multivariate factor stochastic volatility, expectile value at risk, conditional value at risk.

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1. Introduction

The mean–variance framework in Markowitz (1952) is considered as a basis for modern portfolio theory and has been the focus of many extensions and criticisms. For instance, the framework involves a normality assumption through the use of the expected return and variance. However, financial returns (e.g., equity returns) are known to follow non-normal, asymmetric distributions (Fama, 1965; Officer, 1972; Kon, 1984). Furthermore, the framework assumes and suggests investment at the mean–variance efficient frontier. Nevertheless, investors might have other preferences than simply the expected return and variance. In reality, investors might be willing to maximize a multidimensional utility function that incorporates several investment characteristics (Spronk & Hallerbach, 1997; Xidonas et al., 2012). Due to the quadratic form of risk measure, mean–variance analysis also poses computational difficulties when one is dealing with high-dimensional portfolios (Steuer et al., 2011).

Reward/risk ratio maximization is a class of portfolio optimization that originates from the introduction of the Sharpe ratio based on the mean-variance analysis (Sharpe, 1966, 1994). In a reward/risk optimization (also known as optimal portfolio), the risk-averse investor seeks to maximize the risk-adjusted performance of his portfolio. Several extensions have been made to the classical Sharpe ratio maximization based on measuring the reward and the risk of investments. Examples of reward/risk ratios include the stable tail-adjusted return (STARR), Sortino and the Rachev ratios. These risk-adjusted ratios are solely based on one reward and one risk measures, and therefore, they fail to capture multiple decision criteria for the investor. More on the optimal portfolios.

With the advancements in the field of operation research and multicriteria decision making (MCDM), Markowitz's bicriteria portfolio has been extended into a MOP that can incorporate various investor preferences (Steuer, 1986; Martel et al., 1988; Hallerbach & Spronk, 1997; Zopounidis, 1999; Costa & Soares, 2004; Steuer & Na, 2003; Steuer et al., 2005, 2007; Abdelaziz et al., 2007). Ehrgott et al. (2004) combine multiattribute utility theory with Markowitz's mean–variance framework and propose a MOP model with multiple attributes, including the return, volatility, Star Ranking, annual revenue, and 12-months and 3-year performance. Xidonas et al. (2009) propose an integrated multicriteria framework for equity portfolio management. In their MOP problem, they consider several characteristics, namely, the variance, return, beta, capital availability, dividend yield, and marketability. Xidonas & Mavrotas (2014) develop an integrated mixed-integer portfolio model with objectives that include the return, mean-absolute deviation, dividend yield, and the beta coefficient. They also include non-convex policy constraints such as cardinality constraints, buy-in thresholds, and transaction costs. Fliege & Werner (2014) combine a multicriteria problem with robust portfolio optimization and derive a convex parametrization of the robust MOP problem that involves uncertain parameters. Applying the minimax regret and weighted-sum approach, Xidonas et al. (2017) formulate a robust MOP problem based on the return. The literature on MOP problems also includes constructing socially responsible portfolios (Hirschberger et al., 2013; Utz et al., 2014, 2015; Ballestero et al., 2012). See Xidonas et al. (2012); Masmoudi & Abdelaziz (2018) for a review of different programming approaches for solving the MOP problem.

A paragraph on the socially responsible investments.

2. MOP Parametrization

In the remainder of this section, we review and introduce the objective functions included in the multiobjective portfolio optimization problem. We consider five attributes: the expected return, expectile Value-at-Risk, social responsibility, cumulative zero-order stochastic dominance, and portfolio turnover. Combining the weighted-sum approach and reward/risk maximization techniques, we formulate the SR MOOP problem.

2.1. Expected returns

Markowitz (1952) suggested for a d-dimensional portfolio with asset returns $\hat{\mathbf{r}}_t = (\hat{r}_{1t}, \hat{r}_{2t}, ..., \hat{r}_{dt})$, asset weights $\hat{\mathbf{w}}_t = (\hat{w}_{1t}, \hat{w}_{2t}, ..., \hat{w}_{dt})$, and a $d \times 1$ vector of asset means $\hat{\boldsymbol{\mu}}_t = (\hat{\mu}_{1t}, \hat{\mu}_{2t}, ..., \hat{\mu}_{dt})$ at time (out-of-sample iteration) t, the portfolio's expected return is $\hat{\mathbf{w}}_t^\intercal \hat{\boldsymbol{\mu}}_t$.

2.2. Expectile Value at Risk

As regards portfolio risk, the extensions of classical variance used in Markowitz's framework can be divided into two categories. The first includes algorithms that enable fast estimation and the inversion of large covariance matrices, for diagonalization and factorization, for example, (Markowitz & Perold, 1981; Markowitz et al., 1992; Konno & Suzuki, 1992; Takehara, 1993). The second category concerns the application of alternative risk measures, for instance, mean-absolute deviation (Konno & Yamazaki, 1991), Value-at-Risk (Morgan et al., 1996; Jorion, 1997), conditional Value-at-Risk (Rockafellar & Uryasev, 2000a), absolute semideviation (Ogryczak & Ruszczyński, 1999), and expectile Value-at-Risk (Kuan et al., 2009; Bellini et al., 2014).

Although CVaR is a coherent risk measure, in its single form, it is not elicitable. Gneiting (2011) introduce elicitability as a measure for evaluating point forecasts, which follows backtesting a statistical function through a scoring function. Examples of elicitable measures are expected returns, which can be backtested through a scoring function such as root mean squared error, and VaR. As shown by Ziegel (2016), EVaR is a coherent and elicitable risk measure and has recently been utilized in risk management as a measure of downside risk (Delbaen et al., 2016; Bellini & Di Bernardino, 2017).

Perhaps, more details on EVaR-based portfolios.

Newey & Powell (1987) introduced expectiles as one-parameter statistical functions. They are the solutions to the problem of minimizing the expected value of an asymmetric loss function and are defined as:

$$e_\alpha(X) = \underset{\eta\in\Re}{\operatorname{argmin}}\ \mathbb{E}\big[\alpha[(X-\eta)^2]^+ + (1-\alpha)[(X-\eta)^2]^-\big], \tag{1}$$

where $\alpha \in (0,1)$, $[.]^+ = \max(.,0)$, $[.]^- = -\min(.,0)$ and $X \in L^2(\Omega, \mathcal{F}, \mathbb{P},)$.

Among other characteristics, expectiles provide unique solutions that can be obtained by applying the first-order condition

$$\alpha\mathbb{E}\big[[X - e_\alpha(X)]^+\big] = (1-\alpha)\mathbb{E}\big[[X - e_\alpha(X)]^-\big], \tag{2}$$

where $X \in L^1(\Omega, \mathcal{F}, \mathbb{P},)$ (Bellini et al., 2014).

In financial risk management, EVaR is interpreted as the minimum value of an investment that leads to an acceptable and sufficient gain–loss ratio. EVaR is defined as

$$\mathrm{EVaR}_\alpha(X) = -e_{(1-\alpha)}(X) = e_{(\alpha)}(-X),\ \forall\alpha \in [1/2, 1[. \tag{3}$$

Let $\hat{\mathbf{r}}_t = \{\hat{\mathbf{r}}_{mt},\ m = 1, ..., M\}$ be M simulated asset returns obtained from a risk model; by setting $X = -\hat{\mathbf{w}}_t^\intercal \hat{\mathbf{r}}_t$, the portfolio α-level EVaR at time (out-of-sample iteration) t , i.e. $e_{\alpha t}$, is obtained by setting:

$$\alpha\mathbb{E}\big[[-\hat{\mathbf{w}}_t^\intercal \hat{\mathbf{r}}_{mt} - e_{\alpha t}]^+\big] = (1-\alpha)\mathbb{E}\big[[-\hat{\mathbf{w}}_t^\intercal \hat{\mathbf{r}}_{mt} - e_{\alpha t}]^-\big]. \tag{4}$$

2.3. Social responsibility

A common approach to measure social responsibility for investment funds is to use ESG scores Gasser et al. (2017); Hirschberger et al. (2013); Utz et al. (2014, 2015). These scores consist of environmental, social and governance components, and are provided from several rating agencies. Perhaps, more on the ESG scores.

Denoting the ESG scores for individual assets as $\boldsymbol{\theta}_t = (\theta_{1t}, \theta_{2t}, ..., \theta_{dt})$, the portfolio ESG score at time t is given by $\hat{\mathbf{w}}_t^\intercal \boldsymbol{\theta}_t$. As higher ESG scores are favorable, a socially responsible investor seeks to maximize the portfolio ESG scores. Therefore, these scores can be considered as a reward measure. This is in accordance with the reward/risk maximization suggested in Gasser et al. (2017), where the portfolio reward is considered to be the ESG scores and risk is modeled using returns' standard deviation.

2.4. Turnover

Portfolio turnover is a measure of the amount of trading required to implement a portfolio strategy. A high-frequent rebalancing strategy, e.g. daily, can result in higher portfolio turnover compared to that of a low-frequent strategy such as monthly or semi-annually. The portfolio turnover is commonly used to estimate the transaction costs and have already been included in MOP problem (see e.g., Steuer et al., 2005, 2007). Following DeMiguel et al. (2009), we define the portfolio turnover as

$$\vartheta(\hat{\mathbf{w}}_t) = |\hat{\mathbf{w}}_t - \hat{\mathbf{w}}_{t^*}|^\intercal \mathbf{1} \tag{5}$$

where $\hat{\mathbf{w}}_{t^*}$ denotes a vector of asset weights at the end of previous rebalancing period.

We notice for a long-only portfolio strategy, we have $\vartheta(\hat{\mathbf{w}}_t) \in [0, 2]$. When there is no rebalancing, the portfolio has a turnover of zero. However, a turnover of 2 indicates selling all assets with a positive weight and spending 100% of the value of the portfolio on buying the assets that are not included at the previous rebalancing period. [1]

2.5. Cumulative zero-order stochastic dominance

In the Markowitz's mean-variance framework, the investor optimizes his portfolio using reward and risk measures that are basically point estimates of asset returns and uncertainty of these returns. Although the mean-variance framework has been established as the most common approach in asset allocation, due to the convenience and simplicity in estimating the mean-variance feasible portfolios, it does not consider all risk-averse preferences, and consequently, does not follow stochastic dominance rules (see e.g., Ogryczak & Ruszczyński, 1999; Blavatskyy, 2010). An alternative that is suitable for decision making under uncertainty is stochastic dominance approach (see Levy, 1992, and references therein). In particular, the stochastic approach is applied in asset allocation to (i) compare investment strategies (Bawa et al., 1985; Kopa & Post, 2015), and (ii) construct portfolio strategy that stochatiscally dominate a benchmark (e.g., De Giorgi, 2005; Dentcheva & Ruszczyński, 2006; Luedtke, 2008).

Let R and R^I denote random returns for a portfolios and an index market, with distribution functions $F(R;\eta) = \mathbb{P}(R \leq \eta)$ and $F(R^I;\eta) = \mathbb{P}(R^I \leq \eta)$, $\forall \eta \in \mathbb{R}$. The portfolio R dominates the index R^I in the first order, $R \succeq R^I$, if

$$\forall \eta \in \mathbb{R} : F(R;\eta) \leq F(R^I;\eta). \tag{6}$$

For the second order stochastic dominance, $R \succeq_2 R^I$ if

$$\forall \eta \in \mathbb{R} : F_2(R;\eta) \leq F_2(R^I;\eta), \tag{7}$$

where $F_2(.)$ is the second performance function and given by

$$\forall \eta \in \mathbb{R} : F_2(R;\eta) = \int_{-\infty}^{\eta} F(R;\kappa)\mathrm{d}\kappa. \tag{8}$$

[1]In the sense that portfolio turnover can have a value of zero, one has to be careful when including this objective as a risk measure in a reward/risk maximization.

We notice the relations in Eq. (6) and (7) are weak relations of the first order and second order stochastic dominance, and $R \succ R^I$ if and only if $R \succeq R^I$. $R^I \not\succeq R$. Ogryczak & Ruszczyński (1999) show that the second performance function $F_2(.)$ can be expressed as expected shortfall, s.t., $F_2(R;\eta) = \mathbb{E}[(\eta - R)^+]$.

The first and second order stochastic dominance rules are commonly applied in portfolio optimization. In particular, several approaches are suggested for including the second order stochastic dominance constraint in portfolio optimization problems (see Dentcheva & Ruszczyński, 2004, 2006; Luedtke, 2008). Leshno & Levy (2002) suggest that the strict stochastic dominance rules might not capture the preference of any investor, and therefore, introduce the *Almost Stochastic Dominance* that allows small violations from those excessive rules. A review of other relaxations, e.g. the ϵSD, can be found in Kallio & Dehghan Hardoroudi (2019).

A recent approach, suggested in Bruni et al. (2017), is to consider zero-order stochastic dominance (ZSD) s.t.

$$F(R - R^I; 0) = \mathbb{P}(R - R^I \leq 0) = 0. \tag{9}$$

The zero-order stochastic dominance relation, $R \succeq_0 R^I$, means that a portfolio with returns R is preferable to an index benchmark, R^I, in almost every scenario. As shown in Bruni et al. (2017), the zero-order stochastic dominance induce arbitrage opportunities and required relaxations. Let r_t^I be the index return, and $\delta_t(\hat{\mathbf{w}}_t) = \hat{\mathbf{w}}_t^\intercal \hat{\mathbf{r}}_t - r_t^I$ be the excess return of the portfolio w.r.t. the market index at time t. Bruni et al. (2017) suggest that, given a tolerance $\epsilon > 0$, the selected portfolio is preferred to the benchmark index w.r.t. the cumulative zero-order stochastic dominance (CZϵSD) if

$$\forall S \subseteq T : \sum_{t \in S} \delta_t(\hat{\mathbf{w}}_t) \geq -\epsilon, \tag{10}$$

where *epsilon* captures the underperformance of selected portfolio and can be minimized using linear programming.

Bruni et al. (2017) show that

$$\min_{S \subseteq T} \delta_S(\hat{\mathbf{w}}_t) = \sum_{t \in T} [\delta_t(\hat{\mathbf{w}}_t)]^-, \tag{11}$$

where $[.]^- = \min\{0, .\}$. More on the optimization of CZeSD.

2.6. Multicriteria Portfolio Problem and Optimization

In a multicriteria portfolio problem, there are several objective functions to be maximized (or minimized). Let $\Lambda_k(\hat{\mathbf{w}}_t)$ be a reward function and $\psi_q(\hat{\mathbf{w}}_t)$ be a risk measure, a general multiobjective portfolio problem with $K+Q$ attributes is:

$$
\begin{aligned}
\underset{\hat{\mathbf{w}}_t}{\text{maximize}} \quad & \Lambda_1(\hat{\mathbf{w}}_t) \\
& \vdots \\
\underset{\hat{\mathbf{w}}_t}{\text{maximize}} \quad & \Lambda_K(\hat{\mathbf{w}}_t) \\
\underset{\hat{\mathbf{w}}_t}{\text{minimize}} \quad & \psi_1(\hat{\mathbf{w}}_t) \\
& \vdots \\
\underset{\hat{\mathbf{w}}_t}{\text{minimize}} \quad & \psi_Q(\hat{\mathbf{w}}_t) \\
\text{subject to} \quad & \hat{\mathbf{w}}_t^\intercal \mathbf{1} = 1 \\
& \hat{w}_{jt} \geq 0, \forall j \in \{1,2,..,d\}
\end{aligned} \tag{12}
$$

To solve the portfolio optimization problem (12), different approaches have emerged in the MCDM literature including the weighted-sum, ϵ-constraint, goal, and compromise programming (see Masmoudi & Abdelaziz, 2018, for a comparison of different methods). In the weighted-sum approach, the weights assigned to the objective functions, i.e., λ_{Λ_k}, $\lambda_{\psi_q} \in [0,1]$, correspond to the investor's preferences. Furthermore, there are several approaches to normalize the objective functions (see e.g., Cao et al., 2017; Xidonas et al., 2017) Xidonas et al. (2017). One approach is the linear normalization using the so-called utopia and nadir points. Let $\bar{\Lambda}_k$ and $\bar{\psi}_q$ denote the utopia solutions, $\underline{\Lambda}_k$ and $\underline{\psi}_q$ be the nadir points obtained from separate (individual) optimizations. Using the weighted-sum approach, the MOP in Eq. (12) reduces to a convex optimization s.t.

$$
\begin{aligned}
\underset{\hat{\mathbf{w}}_t}{\text{minimize}} \quad & \sum_{q=1}^{Q} \lambda_{\psi_q} \left[\frac{\psi_q(\hat{\mathbf{w}}_t) - \bar{\psi}_q}{\underline{\psi}_q - \bar{\psi}_q} \right] + \\
& \sum_{k=1}^{K} \lambda_{\Lambda_k} \left[\frac{\bar{\Lambda}_k - \Lambda_k(\hat{\mathbf{w}}_t)}{\bar{\Lambda}_k - \underline{\Lambda}_k} \right] \\
\text{subject to} \quad & \hat{\mathbf{w}}_t^\intercal \mathbf{1} = 1 \\
& \hat{w}_{jt} \geq 0, \forall j \in \{1,2,..,d\}
\end{aligned} \tag{13}
$$

where $\sum_{q=1}^{Q} \lambda_{\psi_q} + \sum_{k=1}^{K} \lambda_{\Lambda_k} = 1$. $\bar{\Lambda}_k$ and $\bar{\psi}_q$ denote the utopia solutions, $\underline{\Lambda}_k$ and $\underline{\psi}_q$ be the nadir points obtained from separate (individual) optimizations.

Assume that the investor seeks to maximize a reward/risk ratio with several, i.e., $K+Q$, objective functions. Incorporating the weighted-sum approach, we define the global reward/risk ratio as

$$\frac{\lambda_{\Lambda_1}\Lambda_1(\hat{\mathbf{w}}_t)+\cdots+\lambda_{\Lambda_K}\Lambda_K(\hat{\mathbf{w}}_t)}{\lambda_{\psi_1}\psi_1(\hat{\mathbf{w}}_t)+\cdots+\lambda_{\psi_Q}\psi_Q(\hat{\mathbf{w}}_t)}. \tag{14}$$

We notice in a typical (bi-criteria) reward/risk maximization, the investor maximizes the ratio without any preference for reward or risk. However, for a multiobjective reward/risk ratio, the investor can assign preferences for his reward and risk, i.e., $\sum_{q=1}^{Q}\lambda_{\psi_q}=1$, $\sum_{k=1}^{K}\lambda_{\Lambda_k}=1$. For instance, the investor weights a reward measure only relative to other reward measures. More on the multiobjective reward/risk ratio.

In an optimal portfolio optimization problem the goal is to find the point with highest risk-adjusted ratio (e.g., Max Sharpe ratio) from the efficient frontier. One common approach to solve these portfolios is fractional programming where the non-linear objective function (reward/risk ratio) can be reduced to convex optimization (Charnes & Cooper, 1962; Dinkelbach, 1967; Stoyanov et al., 2007). Incorporating the five objective functions in Sections 2.1-2.5 and the multiobjective problem in Eq. (13)-(14), we formulate the socially responsible multiobjective optimal portfolio problem as

$$\begin{array}{lll}
\underset{\tilde{\mathbf{w}}_t,\nu,e_t,\mathbf{y},\mathbf{g},\boldsymbol{\upsilon}^+,\boldsymbol{\upsilon}^-}{\text{minimize}} & \lambda_{\psi_e}\left[\dfrac{e_t-\bar{\psi}_e}{\underline{\psi}_e-\bar{\psi}_e}\right]+\lambda_{\psi_\delta}\left[\dfrac{\sum_{m=1}^{M}y_m-\bar{\psi}_\delta}{\underline{\psi}_\delta-\bar{\psi}_\delta}\right]+\lambda_{\psi_\vartheta}\left[\dfrac{\sum_{j=1}^{d}g_j-\bar{\psi}_\vartheta}{\underline{\psi}_\vartheta-\bar{\psi}_\vartheta}\right], & \\
\text{subject to} & \lambda_{\Lambda_\mu}\left[\dfrac{\bar{\Lambda}_\mu-\tilde{\mathbf{w}}_t^\intercal\boldsymbol{\mu}_t}{\bar{\Lambda}_\mu-\underline{\Lambda}_\mu}\right]+\lambda_{\Lambda_\theta}\left[\dfrac{\bar{\Lambda}_\theta-\tilde{\mathbf{w}}_t^\intercal\boldsymbol{\theta}_t}{\bar{\Lambda}_\theta-\underline{\Lambda}_\theta}\right]\geq 1, & \\
 & \nu>0, & \\
 & \tilde{\mathbf{w}}_t^\intercal\mathbf{1}=\nu, & \text{full investment,}\\
 & 0\leq\tilde{w}_{jt}\leq\nu,\forall j\in\{1,2,..,d\}, & \text{long positions only,}\\
 & \dfrac{\alpha}{M}\displaystyle\sum_{m=1}^{M}\upsilon_m^+-\dfrac{1-\alpha}{M}\sum_{m=1}^{M}\upsilon_m^-=0, & \\
 & -\tilde{\mathbf{w}}_t^\intercal\hat{\mathbf{r}}_{mt}-e_t-\upsilon_m^++\upsilon_m^-=0, & \forall m\in\{1,2,..,M\},\\
 & \upsilon_{mt}^+,\upsilon_{mt}^-\geq 0, & \forall m\in\{1,2,..,M\},\\
 & y_m+\tilde{\mathbf{w}}_t^\intercal\hat{\mathbf{r}}_{mt}-r_{mt}^I\geq 0, & \forall m\in\{1,2,..,M\},\\
 & \tilde{w}_{jt}-\hat{w}_{jt^*}+g_j\geq 0, & \forall j\in\{1,2,..,d\},\\
 & \hat{w}_{jt^*}-\tilde{w}_{jt}+g_j\geq 0, & \forall j\in\{1,2,..,d\}.
\end{array} \tag{15}$$

where $\hat{w}_{jt}=\frac{\tilde{w}_{jt}}{\nu}$.More on the auxiliary variables and linear transformations.

3. Returns' predictive multivariate distribution

3.1. Vine Copula

In financial econometrics, different approaches have been developed to estimate and model non-normal multivariate financial returns. One of these approaches is copula modeling, in which a joint distribution is estimated using univariate marginal distributions and a copula function (Sklar, 1959, 1973). Copulas have gained popularity in the finance and asset allocation fields (Patton, 2004; Nelsen, 2007; Patton, 2009) because (i) they allow one to model a multivariate distribution using convenient univariate econometric and forecasting models, (ii) they provide flexibility in capturing the non-parametric dependence among nonelliptical random variables, and (iii) they allow one to model asymmetric tail dependence when the underlying assets show different correlations during bearish and bullish market periods. In portfolio optimization, copula modeling is particularly used for tail risk minimization (Low et al., 2013; Boubaker & Sghaier, 2013; Kakouris & Rustem, 2014; Bekiros et al., 2015; Krzemienowski & Szymczyk, 2016; Sahamkhadam et al., 2018; Zhao et al., 2019). Although copula models are well-established in the portfolio management field, to my knowledge, only a small number of studies incorporate them into MOPs (e.g., Babaei et al., 2015; Bilbao-Terol et al., 2016; Goel & Sharma, 2019; Xiao-Li & Xiong, 2020).

According to Sklar's theorem, any multivariate cumulative distribution function F for a random variable set $(Z_1, ..., Z_d)$ consists of a d-dimensional copula C and marginal distributions $F_1, ..., F_d$, such that

$$\forall \mathbf{z} \in \Re^d : F\left(z_1, z_2, \ldots, z_d\right) = C\left(F_1\left(z_1\right), F_2\left(z_2\right), \ldots, F_d\left(z_d\right)\right) = C\left(u_1, u_2, \ldots, u_d\right), \tag{16}$$

where $z_j = F_j^{-1}\left(u_j\right),\ u_j \sim U\left[0,1\right]^d, \forall j \in \{1, 2, \ldots, d\}$. If all the margins F_j are continuous, then C is unique and defined as the joint distribution of $(U_1, ..., U_d) = (F_1(Z_1), ..., F_d(Z_d))$. Let $\mathbf{\Omega}$ be the parameter set of the copula multivariate distribution function $C\left(u_1, u_2, \ldots, u_d|\mathbf{\Omega}\right)$ and f_j be the derivative of the univariate marginal distribution F_j. Then, the density function for the d-dimensional joint distribution is

$$\begin{aligned} f\left(z_1, z_2, \ldots, z_d\right) &= \frac{\partial^d C\left(F_1\left(z_1\right), F_2\left(z_2\right), \ldots, F_d\left(z_d\right)|\mathbf{\Omega}\right)}{\partial z_1, \partial z_2, \ldots, \partial z_d} \\ &= c\left(F_1\left(z_1\right), F_2\left(z_2\right), \ldots, F_d\left(z_d\right)|\mathbf{\Omega}\right) \times \prod_{j=1}^{d} f_j\left(z_j\right), \end{aligned} \tag{17}$$

where c is the copula density function, with log-likelihood function

$$\mathscr{L}((z_1, z_2, ..., z_d)|\mathbf{\Omega}) = \sum_{t=1}^{T}\left[\sum_{j=1}^{d} \log f_j(z_{tj}) + \log\left[c(u_{t1}, u_{t2}, ..., u_{td}|\mathbf{\Omega})\right]\right]. \tag{18}$$

Note that in Eqs. (16)–(17), only one copula function C is used to construct the joint distribution, which means only one copula family is used for the entire set of marginal uniforms $u_1, u_2, \ldots, u_d$. While Joe (1996) suggests a decomposition of c into products of pair-wise densities, Bedford & Cooke (2001, 2002) derive a graphical representation, called a regular vine (Rvine), of the pair-copula construction, in the form of nested trees. Aas et al. (2009) develop maximum likelihood inference and estimation of three vine models with arbitrary pair-copulas (including Archimedean families). More properties and statistical inference for vine copulas have been developed by Joe (2014); Czado (2019).

For a d-dimensional set of continuous random variables, there exist $d\,(d-1)\,/2$ pair-copulas, and the copula density c can be decomposed into a product of these pair-copulas' densities. Using a sequence of $i = 1, 2, \ldots, d-1$ linked trees, the decomposition can be presented in a graphical PCC, known as the regular vine. Let $e \in E_i$ be the edge between two nodes n_e, k_e, representing a pair-copula $c_{n_e,k_e;D_e}$ conditioned on D_e, with copula parameter(s) $\boldsymbol{\Omega}_{n_e,k_e|D_e}$. Let $\boldsymbol{u}_{D_e} = \{u_i | i \in D_e\}$ be the variables in the conditioning set D_e. Let $C_{n_e|D_e}$ be the conditional distribution of $U_{ne}|\boldsymbol{U}_{D_e}$. When the number of trees increases, the conditioning set D_e also grows, and it is common to consider only the dependence of $c_{n_e,k_e;D_e}$ on the indexes in D_e, ignoring the impact of $\boldsymbol{u}_{D_e}$. This is the so-called simplifying assumption (see Acar et al., 2012; Haff et al., 2013). The copula density for a simplified Rvine copula is

$$c\,(u|\boldsymbol{\Omega}) = \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{n_e,k_e;D_e}\left(C_{n_e|D_e}(u_{n_e}|\boldsymbol{u}_{D_e}), C_{k_e|D_e}(u_{k_e}|\boldsymbol{u}_{D_e})|\boldsymbol{\Omega}_{n_e,k_e|D_e}\right), \tag{19}$$

with log-likelihood function

$$\mathscr{L}(\boldsymbol{\Omega}|u) = \sum_{j=1}^{d} \sum_{i=1}^{d-1} \sum_{e \in E_i} \ln\left[c_{n_e,k_e;D_e}\left(C_{n_e|D_e}(u_{j,n_e}|u_{j,D_e}), C_{k_e|D_e}(u_{j,k_e}|u_{j,D_e})|\boldsymbol{\Omega}_{n_e,k_e|D_e}\right)\right]. \tag{20}$$

Although vine copulas are flexible in estimating tail dependency, there is a tradeoff between higher flexibility and an increased computational load in high-dimensional settings. Therefore, truncated and simplified vine structures that allow for high-dimensional dependence estimation have been developed (see e.g., Heinen et al., 2009; Kurowicka, 2011; Brechmann et al., 2012; Brechmann & Joe, 2015). In particular, Brechmann et al. (2012) show how to truncate or simplify the Rvine structure. This truncation is applied to the number of trees in the vine by setting an independence copula at each edge from a specific tree $I \in \{1, 2, \ldots, d-1\}$ to the final tree. The I-level truncated Rvine has density

$$c^{Truncated}\,(u) = \prod_{i=1}^{I} \prod_{e \in E_i} c_{n_e,k_e|D_e}\left(C_{n_e|D_e}\,(u_{n_e}|u_{D_e})\,, C_{k_e|D_e}\,(u_{k_e}|u_{D_e})\,|\boldsymbol{\Omega}_{n_e,k_e|D_e}\right). \tag{21}$$

In a copula-based forecasting approach, one can estimate the returns' conditional multivariate distribution by following a series of steps. First, using a GARCH model, the standardized residuals $\mathbf{z}$ and their marginal densities f_j are obtained. Then, using the marginal uniforms obtained from the probability transformation, a joint distribution is estimated using the truncated Rvine density function. Finally, drawing observations from the joint distribution and utilizing the step-ahead mean and volatility forecasts from the GARCH process, the copula-based multivariate distribution is obtained. Following Nagler et al. (2019), the copula families and truncation level are selected based on the mBICV criterion.[2]

3.2. Multivariate GARCH

Financial returns are known to have time-varying volatility that can be modeled employing GARCH models. In these models, the mean of the model equation follows a recursive heteroscedastic volatility process. The formulation of GARCH models allows taking advantage of the autocorrelation in financial returns. A univariate GARCH process can model and forecast the conditional volatility of the returns of an individual's assets, with some distributional assumptions for the error terms. However, in asset allocation, which requires modeling and forecasting several assets' returns, one can estimate a covariance matrix using a multivariate GARCH process. Let $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{dt})$ be a vector of assets' returns; a general formulation of the multivariate GARCH is

$$\begin{cases} \mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t, \\ \boldsymbol{\epsilon}_t = \boldsymbol{\Sigma}_t^{\frac{1}{2}} \mathbf{z}_t, \\ \mathbf{z}_t \approx (iid), \end{cases} \tag{22}$$

where $\boldsymbol{\Sigma}$ is a $d \times d$ covariance matrix and $\mathbf{z}_t$ denotes the standardized residuals.

Since the seminal paper of Bollerslev et al. (1988), which introduced a vectorized GARCH, many extensions have been suggested. The main contributions include the constant conditional correlation model (Bollerslev, 1990), factor-ARCH model (Engle et al., 1990), BEKK model (Engle & Kroner, 1995), dynamic conditional correlation model (Engle, 2002), generalized orthogonal GARCH model (Van der Weide, 2002), and multivariate realized GARCH model (Hansen et al., 2014). Dynamic conditional correlation is known to be efficient in modeling less biased covariance matrices (see de Almeida et al., 2018). The multivariate realized GARCH is more suitable for high-frequency datasets. Nonetheless, the GOGARCH model of Van der Weide (2002) is the appropriate model for estimating the conditional multivariate distribution of high-dimensional assets' returns. By using unobserved components, the GOGARCH model alleviates the curse of dimensionality.

In the GOGARCH model, the mean equation is driven not only by the recursive volatility process but also by a set of unobserved factors $\mathbf{e}_t$ (also known as the *structural errors*), s.t.

$$\begin{cases} \mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t, \\ \boldsymbol{\epsilon}_t = \mathbf{A}\mathbf{e}_t, \\ \mathbf{e}_t = \mathbf{H}_t^{\frac{1}{2}} \mathbf{z}_t, \\ \mathbf{z}_t \approx (iid), \end{cases} \tag{23}$$

[2] To model both the lower and upper tail dependence, the copula families in the Rvine structure are selected from one of the following distributions: Gaussian, Student$-t$, Clayton, Gumbel, Frank, and Joe.

where $\mathbf{A} = \mathbf{\Sigma}^{\frac{1}{2}}\mathbf{U}$ is a constant and invertible matrix, $\mathbf{U}$ is an orthogonal matrix, and $\mathbf{H}_t = diag(h_{1t}, h_{2t}, ..., h_{dt})$. The factors' conditional variances h_{jt} may be estimated using a GARCH process. The conditional covariance matrix is given by $\mathbf{\Sigma}_t = \mathbf{A}\mathbf{H}_t\mathbf{A}^\intercal$. Among others, the multivariate affine generalized hyperbolic distribution may be considered for the conditional distribution of $\mathbf{z}_t$(see Broda & Paolella, 2009, for further details). The estimation methods for the GOGARCH model include maximum likelihood, the method of moments (Boswijk & Van der Weide, 2011), and independent component analysis (Broda & Paolella, 2009).

3.3. Multivariate Factor Stochastic Volatility

The stochastic volatility model departs from the common GARCH model in that the conditional volatility process is stochastic. Although the conditional volatility process in GARCH-type models is heteroscedastic and deterministic, in factor stochastic volatility models, it is driven by a set of latent variables. The seminal papers on using a stochastic random process for the conditional volatility include Clark (1973), Tauchen & Pitts (1983), Hull & White (1987), and Taylor (1986). Important contributions to multivariate models include quasi-likelihood estimation (Harvey et al., 1994), Bayesian Markov Chain Monte Carlo (MCMC) inference (Kim et al., 1998; Jacquier et al., 2002), and multivariate factor models (Pitt & Shephard, 1999; Chib et al., 2006). More recently, to accelerate the convergence and boost the efficiency of the MCMC method, Kastner et al. (2017) have used interweaving approaches (suggested in Kastner & Frühwirth-Schnatter, 2014). Their approach is appropriate not only for an efficient estimation of the MFSV model but also for high-dimensional datasets. Therefore, we use the model in Chib et al. (2006) and Kastner et al. (2017) to construct MFSV-based portfolios. Let $\tilde{\mathbf{r}}_t = (\tilde{r}_{1t}, \tilde{r}_{2t}, ..., \tilde{r}_{dt})$ be a vector of assets' returns with zero means, and $\boldsymbol{\xi}_t = (\xi_{1t}, \xi_{2t}, ..., \xi_{Nt})$ be a vector of N unobserved latent factors. In the MFSV model, the error terms for both the mean and the state-space equation are allowed to have time-varying variances s.t.

$$\begin{cases} \tilde{\mathbf{r}}_t = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{U}_t(\boldsymbol{h}_t^U)^{\frac{1}{2}}\boldsymbol{\epsilon}_t, \ \boldsymbol{\epsilon}_t \sim \mathcal{N}_d(0, \boldsymbol{I}_d), \\ \boldsymbol{\xi}_t = \mathbf{V}_t(\boldsymbol{h}_t^V)^{\frac{1}{2}}\boldsymbol{\zeta}_t, \ \boldsymbol{\zeta}_t \sim \mathcal{N}_N(0, \boldsymbol{I}_N), \\ \forall j \in [1, d+N]: \ h_{jt} = \mu_j + \phi_j(h_{j,t-1} - \mu_j) + \sigma_j\eta_{jt}, \ \boldsymbol{\eta}_t \sim \mathcal{N}_{N+d}(0, \boldsymbol{I}_{N+d}), \end{cases} \tag{24}$$

where $\mathbf{F}$ is a $N \times j$ loadings matrix, $\mathbf{U}_t(\boldsymbol{h}_t^U)^{\frac{1}{2}} = diag(\exp(h_{1t}), \exp(h_{2t}), ..., \exp(h_{dt}))$ denotes the $d \times d$ matrix of the variances of assets' returns, and $\mathbf{V}_t(\boldsymbol{h}_t^V)^{\frac{1}{2}} = diag(\exp(h_{N+1,t}), \exp(h_{N+2,t}), ..., \exp(h_{N+d,t}))$ is the $N \times N$ matrix with the latent factors' variances. The model-implied conditional covariance matrix is given by $\mathbf{\Sigma}_t = \mathbf{F}\mathbf{V}_t(\boldsymbol{h}_t^V)\mathbf{F}^\intercal + \mathbf{U}_t(\boldsymbol{h}_t^U)$. The Bayesian estimation of the MFSV model includes applying MCMC sampling and selecting prior distributions for μ_j, ϕ_j, and σ_j (see Kastner et al., 2017, for more details on priors).

4. Data

To construct socially responsible multiobjective optimal portfolios, we use a sample of all stocks of the Eurostoxx 50 index. Using this sample has several advantages. First, the constituents of the Eurostoxx 50 index are highly capitalized, providing a proper representation of the Europe market. Second, this sample provides diversification benefits due to the number of included stocks. Finally, we include the Eurostoxx 50 index as the market index when including the CZϵSD objective function. The sample runs from August 2007 to October 2020. This period is selected due to the availability of ESG scores for the constituents of the Eurostoxx 50.

The data include daily adjusted (for splits and dividends) stock prices and the Eurostoxx 50 price index were obtained from Eikon Thompson Reuters' Datastream. The monthly ESG scores are obtained from Sustainalytics.

5. Empirical Analysis

To evaluate the performance of suggested socially responsible portfolio optimization method, we divide our empirical investigation into in-sample and out-of-sample analyses. The former includes a comparison of the multiobjective optimal portfolios based on their resulting efficient frontier sets in an a posteriori approach. The out-of-sample investigation includes portfolio backtesting and robustness analysis on both the risk models. In our robustness analysis, we also consider two alternatives for EVaR, including CVaR and mean absolute deviation (MAD).

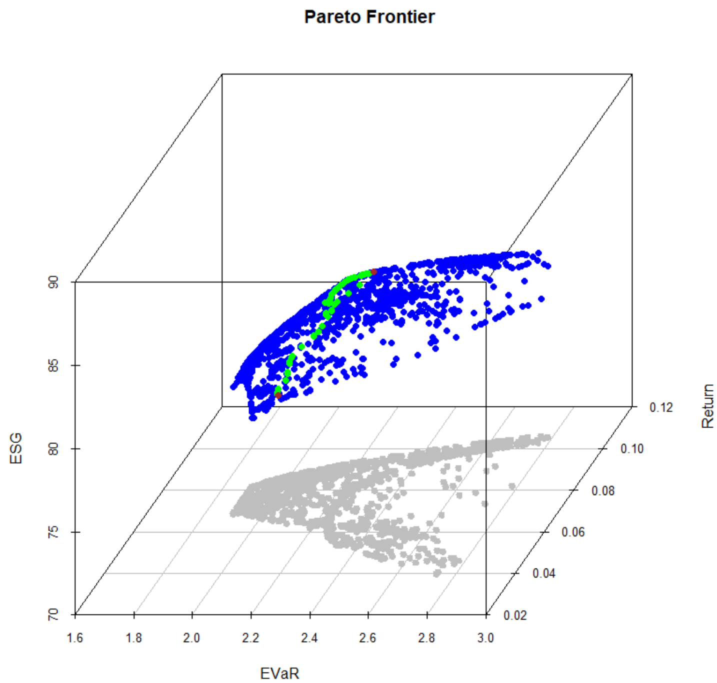

Fig. 1. This figure plots expected return-EVaR-ESG Pareto frontier obtained by changing objective function weights (preference parameters) by 2.5% in the MOP optimization (see Eq. (14)). The optimal MOPs are shown using green points (see Eq. (15)). The brown points represent Max return/EVaR and Max ESG/EVaR portfolios. The portfolios are constructed using all stocks included in Eurostoxx 50 index from December 8, 2016 to October 7, 2020.

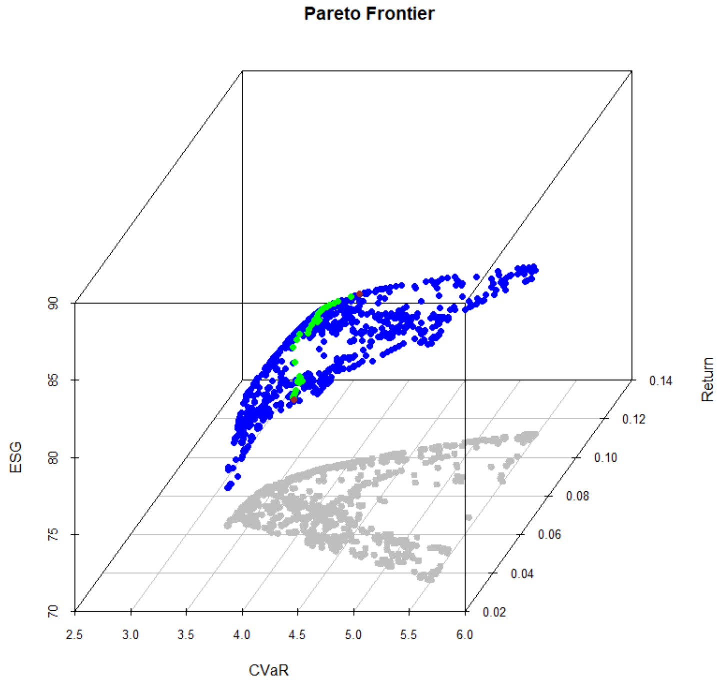

Fig. 2. This figure plots expected return-CVaR-ESG Pareto frontier obtained by changing objective function weights (preference parameters) by 2.5% in the MOP optimization (see Eq. (14)). The optimal MOPs are shown using green points (see Eq. (15)). The brown points represent Max return/CVaR and Max ESG/CVaR portfolios. The portfolios are constructed using all stocks included in Eurostoxx 50 index from December 8, 2016 to October 7, 2020.

Table 1

Single-objective Min Risk portfolio out-of-sample performance

Portfolio Strategy	Av. Return	St. Devation	CVaR	EVaR	CZϵSD	STARR	μ/EVaR	ESG	Av. Turnover	Portfolio Wealth
Panel A: Bechmark										
EQW	0.033	1.37	5.257	2.90	1.19	0.006	0.011	60.9	0.009	197
Panel B: Historical-based portfolios										
Min CVaR	0.039	1.13	4.07	2.25	6.78	0.009	0.017	56.9	0.032	251
Min EVaR	0.04	1.06	3.85	2.11	6.60	0.010	0.019	59.0	0.044	268
Min MAD	0.038	1.02	3.92	2.13	5.95	0.010	0.018	62.7	0.054	256
Min CZϵSD	0.033	1.33	5.07	2.81	1.53	0.006	0.012	61.4	0.022	196
Panel C: Copula-based portfolios										
Min CVaR	0.045	1.05	3.78	2.08	6.39	0.012	0.022	62.9	1.01	312
Min EVaR	0.055	1.01	3.64	2.01	6.46	0.015	0.028	62.9	0.807	421
Min MAD	0.048	0.977	3.66	1.98	6.56	0.013	0.024	62.4	0.474	344
Min CZϵSD	0.046	1.37	5.39	2.91	1.71	0.009	0.016	61.3	0.823	283
Panel D: MGARCH-based portfolios										
Min CVaR	0.021	1.07	4.34	2.30	6.67	0.005	0.009	61.2	1.03	154
Min EVaR	0.037	1.05	4.15	2.21	6.50	0.009	0.017	61.0	0.837	249
Min MAD	0.039	1.04	4.16	2.19	6.30	0.009	0.018	61.1	0.491	260
Min CZϵSD	0.036	1.33	5.07	2.80	0.830	0.007	0.013	61.3	0.479	214
Panel E: MFSV-based portfolios										
Min CVaR	0.032	1.10	4.15	2.28	5.90	0.008	0.014	60.0	1.27	208
Min EVaR	0.033	1.07	4.02	2.20	5.84	0.008	0.015	60.1	1.14	219
Min MAD	0.027	1.04	3.96	2.17	5.77	0.007	0.012	60.3	0.934	185
Min CZϵSD	0.036	1.34	5.10	2.81	1.32	0.007	0.013	61.6	0.852	218

Notes: This table reports out-of-sample performance for Min Risk portfolios. Portfolio returns are obtained using rolling window estimation (with 500 days as the training sample) and portfolio back-testing from August 18, 2009 until October 7, 2020, resulting in 2853 portfolio out-of-sample net returns using 1 basis point proportional transaction cost. Except for CZϵSD, ESG and portfolio wealth, all measures are expressed as percentage. CVaR and EVaR are reported at 1% level. Portfolio wealth is calculated at the end of out-of-sample assuming an initial investment of €100.

Table 2

Bi-objective optimal portfolio out-of-sample performance

Portfolio Strategy	Av. Return	St. Devation	CVaR	EVaR	CZϵSD	STARR	μ/EVaR	ESG	Av. Turnover	Portfolio Wealth
Panel A: Bechmark										
EQW	0.033	1.37	5.26	2.90	1.19	0.006	0.011	61.0	0.009	197
Panel B: Historical-based portfolios										
Max μ/CVaR	0.026	1.57	5.48	3.07	9.93	0.005	0.008	63.9	0.170	148
Max μ/EVaR	0.029	1.56	5.44	3.07	9.90	0.005	0.009	64.8	0.156	161
Max μ/MAD	0.035	1.53	5.37	3.03	9.36	0.006	0.011	64.8	0.135	191
Panel C: Copula-based portfolios										
Max μ/CVaR	0.081	1.62	5.40	3.02	8.42	0.015	0.027	57.4	1.68	697
Max μ/EVaR	0.079	1.65	5.62	3.07	8.68	0.014	0.026	57.5	1.70	645
Max μ/MAD	0.082	1.63	5.55	3.04	8.44	0.015	0.027	58.4	1.70	715
Panel D: MGARCH-based portfolios										
Max μ/CVaR	0.056	1.65	6.35	3.35	9.59	0.009	0.017	60.7	1.73	335
Max μ/EVaR	0.062	1.68	6.41	3.41	9.76	0.010	0.018	59.9	1.73	387
Max μ/MAD	0.061	1.62	6.15	3.29	9.20	0.010	0.019	60.1	1.71	395
Panel E: MFSV-based portfolios										
Max μ/CVaR	0.069	1.59	5.83	3.13	8.28	0.012	0.022	60.4	1.71	494
Max μ/EVaR	0.070	1.65	6.11	3.25	8.66	0.011	0.022	60.5	1.71	502
Max μ/MAD	0.072	1.59	5.95	3.15	8.21	0.012	0.023	60.3	1.69	544

Notes: This table reports out-of-sample performance for optimal portfolios. Portfolio returns are obtained using rolling window estimation (with 500 days as the training sample) and portfolio back-testing from August 18, 2009 until October 7, 2020, resulting in 2853 portfolio out-of-sample net returns using 1 basis point proportional transaction cost. Except for CZϵSD, ESG and portfolio wealth, all measures are expressed as percentage. CVaR and EVaR are reported at 1% level. Portfolio wealth is calculated at the end of out-of-sample assuming an initial investment of €100.

Table 3

Tri-objective optimal portfolio out-of-sample performance

Portfolio Strategy	Av. Return	St. Devation	CVaR	EVaR	CZϵSD	STARR	μ/EVaR	ESG	Av. Turnover	Portfolio Wealth
Panel A: Historical-based portfolios										
Max (μ+ESG)/CVaR	0.024	1.38	4.98	2.78	8.37	0.005	0.008	71.3	0.159	150
Max (μ+ESG)/EVaR	0.026	1.33	4.89	2.71	7.97	0.005	0.01	70.8	0.149	162
Max (μ+ESG)/MAD	0.019	1.27	4.72	2.58	6.89	0.004	0.007	69.8	0.127	137
Panel B: Copula-based portfolios										
Max (μ+ESG)/CVaR	0.056	1.38	4.67	2.66	7.12	0.012	0.021	64.5	1.54	378
Max (μ+ESG)/EVaR	0.057	1.39	4.63	2.64	7.24	0.012	0.022	64.3	1.55	391
Max (μ+ESG)/MAD	0.064	1.36	4.49	2.57	6.81	0.014	0.025	65.9	1.52	483
Panel C: MGARCH-based portfolios										
Max (μ+ESG)/CVaR	0.044	1.47	5.76	3.05	8.17	0.008	0.014	66.2	1.62	257
Max (μ+ESG)/EVaR	0.051	1.49	5.70	3.04	8.16	0.009	0.017	65.5	1.62	309
Max (μ+ESG)/MAD	0.046	1.45	5.60	2.97	7.86	0.008	0.015	66.6	1.57	274
Panel D: MFSV-based portfolios										
Max (μ+ESG)/CVaR	0.046	1.43	5.46	2.92	7.16	0.008	0.016	65.6	1.63	279
Max (μ+ESG)/EVaR	0.044	1.45	5.55	2.96	7.31	0.008	0.015	65.6	1.62	257
Max (μ+ESG)/MAD	0.053	1.40	5.24	2.80	6.80	0.01	0.019	66.5	1.57	347

Notes: This table reports out-of-sample performance for optimal MOP portfolios. Portfolio returns are obtained using rolling window estimation (with 500 days as the training sample) and portfolio back-testing from August 18, 2009 until October 7, 2020, resulting in 2853 portfolio out-of-sample net returns using 1 basis point proportional transaction cost. Except for CZϵSD, ESG and portfolio wealth, all measures are expressed as percentage. CVaR and EVaR are reported at 1% level. Portfolio wealth is calculated at the end of out-of-sample assuming an initial investment of €100. The results are obtained assuming equal preferences for the reward measures.

Table 4

Quad-objective optimal portfolio out-of-sample performance

Portfolio Strategy	Av. Return	St. Devation	CVaR	EVaR	CZϵSD	STARR	μ/EVaR	ESG	Av. Turnover	Portfolio Wealth
Panel A: Historical-based portfolios										
Max (μ+ESG)/(CVaR+CZϵSD)	0.033	1.27	4.67	2.59	5.69	0.007	0.013	69.3	0.119	201
Max (μ+ESG)/(EVaR+CZϵSD)	0.033	1.26	4.56	2.55	5.70	0.007	0.013	69.4	0.12	207
Max (μ+ESG)/(MAD+CZϵSD)	0.027	1.25	4.61	2.56	5.03	0.006	0.011	68.7	0.103	174
Panel B: Copula-based portfolios										
Max (μ+ESG)/(CVaR+CZϵSD)	0.057	1.37	4.94	2.72	5.18	0.011	0.021	64.0	1.47	384
Max (μ+ESG)/(EVaR+CZϵSD)	0.056	1.37	4.92	2.73	5.37	0.011	0.020	64.0	1.48	376
Max (μ+ESG)/(MAD+CZϵSD)	0.055	1.34	4.80	2.68	5.09	0.012	0.021	64.9	1.44	375
Panel C: MGARCH-based portfolios										
Max (μ+ESG)/(CVaR+CZϵSD)	0.050	1.39	5.19	2.83	5.86	0.010	0.018	65.2	1.55	318
Max (μ+ESG)/(EVaR+CZϵSD)	0.053	1.40	5.24	2.85	6.04	0.010	0.019	64.9	1.56	344
Max (μ+ESG)/(MAD+CZϵSD)	0.052	1.38	5.19	2.83	5.75	0.010	0.018	65.6	1.52	337
Panel D: MFSV-based portfolios										
Max (μ+ESG)/(CVaR+CZϵSD)	0.049	1.38	5.23	2.81	5.27	0.009	0.017	65.4	1.55	310
Max (μ+ESG)/(EVaR+CZϵSD)	0.049	1.39	5.32	2.86	5.45	0.009	0.017	65.3	1.55	303
Max (μ+ESG)/(MAD+CZϵSD)	0.053	1.36	5.15	2.76	5.02	0.010	0.019	66.0	1.50	353

Notes: This table reports out-of-sample performance for optimal MOP portfolios. Portfolio returns are obtained using rolling window estimation (with 500 days as the training sample) and portfolio back-testing from August 18, 2009 until October 7, 2020, resulting in 2853 portfolio out-of-sample net returns using 1 basis point proportional transaction cost. Except for CZϵSD, ESG and portfolio wealth, all measures are expressed as percentage. CVaR and EVaR are reported at 1% level. Portfolio wealth is calculated at the end of out-of-sample assuming an initial investment of €100. The results are obtained assuming equal preferences for the reward (risk) measures.

Table 5

Penta-objective optimal portfolio out-of-sample performance

Portfolio Strategy	Av. Return	St. Devation	CVaR	EVaR	CZϵSD	STARR	μ/EVaR	ESG	Av. Turnover	Portfolio Wealth
Panel A: Historical-based portfolios										
Max (μ+ESG)/(CVaR-CZϵSD-PT)	0.035	1.30	4.83	2.63	5.73	0.007	0.013	68.5	0.005	211
Max (μ+ESG)/(EVaR-CZϵSD-PT)	0.032	1.28	4.68	2.57	5.59	0.007	0.013	68.8	0.005	198
Max (μ+ESG)/(MAD-CZϵSD-PT)	0.033	1.26	4.66	2.56	4.90	0.007	0.013	68.9	0.004	206
Panel B: Copula-based portfolios										
Max (μ+ESG)/(CVaR-CZϵSD-PT)	0.058	1.40	5.01	2.79	5.29	0.012	0.021	65.1	0.712	396
Max (μ+ESG)/(EVaR-CZϵSD-PT)	0.056	1.40	5.00	2.80	5.45	0.011	0.020	65.0	0.765	378
Max (μ+ESG)/(MAD-CZϵSD-PT)	0.061	1.37	4.94	2.75	5.09	0.012	0.022	65.8	0.678	430
Panel C: MGARCH-based portfolios										
Max (μ+ESG)/(CVaR-CZϵSD-PT)	0.057	1.39	5.10	2.81	5.82	0.011	0.020	66.7	0.886	390
Max (μ+ESG)/(EVaR-CZϵSD-PT)	0.058	1.40	5.17	2.85	6.06	0.011	0.020	66.8	0.924	398
Max (μ+ESG)/(MAD-CZϵSD-PT)	0.055	1.37	5.11	2.79	5.71	0.011	0.020	67.3	0.814	369
Panel D: MFSV-based portfolios										
Max (μ+ESG)/(CVaR-CZϵSD-PT)	0.061	1.39	5.25	2.80	5.37	0.012	0.022	66.8	0.810	426
Max (μ+ESG)/(EVaR-CZϵSD-PT)	0.060	1.40	5.37	2.84	5.50	0.011	0.021	66.6	0.850	421
Max (μ+ESG)/(MAD-CZϵSD-PT)	0.062	1.35	5.08	2.71	5.02	0.012	0.023	67.0	0.748	454

Notes: This table reports out-of-sample performance for optimal MOP portfolios. Portfolio returns are obtained using rolling window estimation (with 500 days as the training sample) and portfolio back-testing from August 18, 2009 until October 7, 2020, resulting in 2853 portfolio out-of-sample net returns using 1 basis point proportional transaction cost. Except for CZϵSD, ESG and portfolio wealth, all measures are expressed as percentage. CVaR and EVaR are reported at 1% level. Portfolio wealth is calculated at the end of out-of-sample assuming an initial investment of €100. The results are obtained assuming equal preferences for the reward (risk) measures.

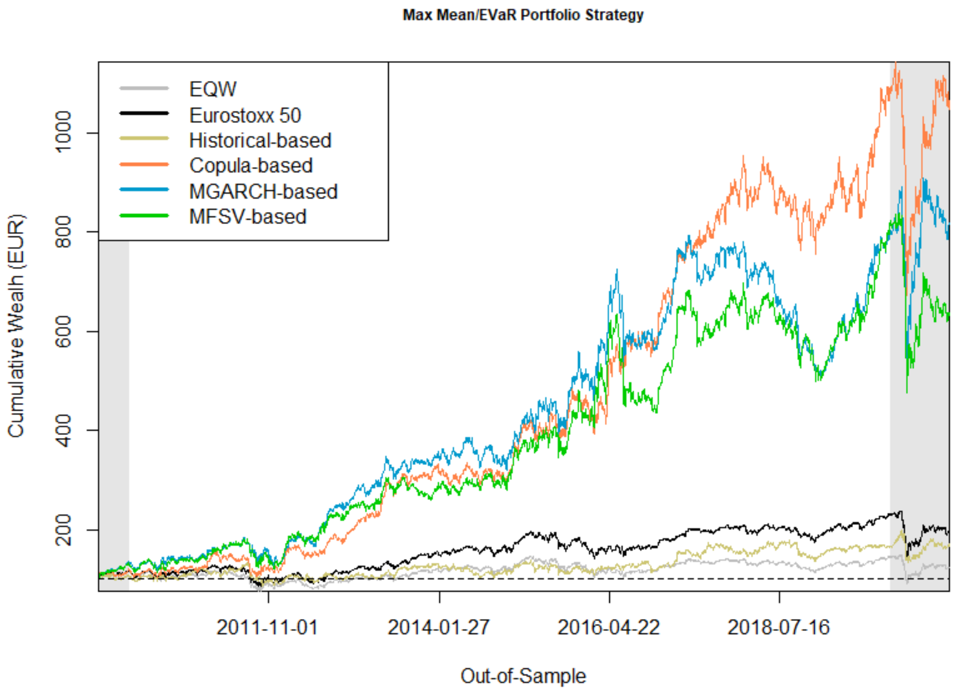

Fig. 3. This figure plots wealth trajectory for Max μ/EVaR portfolios obtained from several risk models with €100 initial investment.

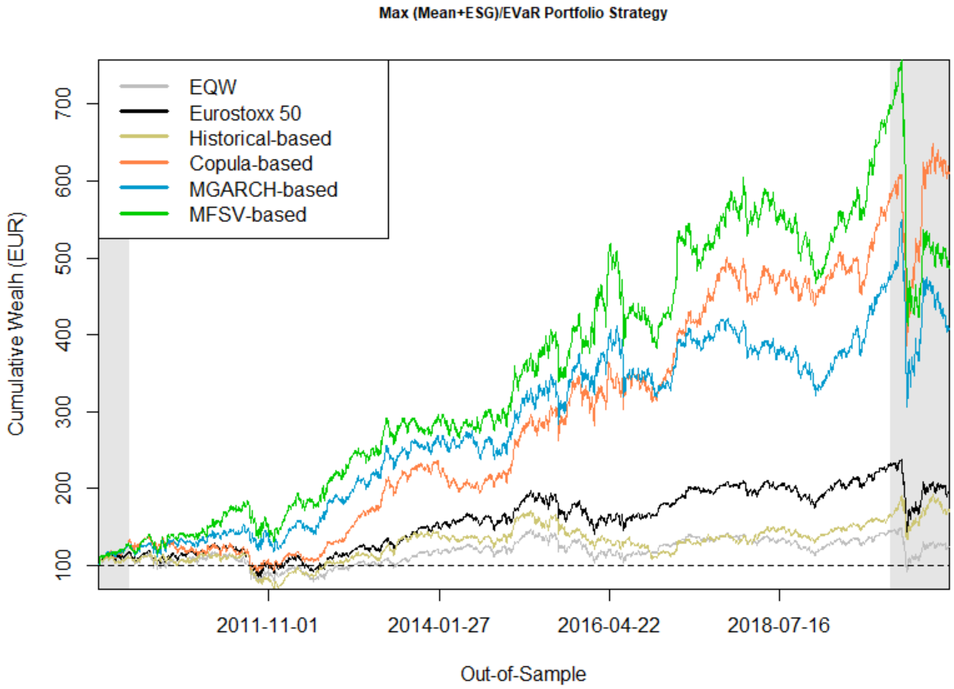

Fig. 4. This figure plots wealth trajectory for Max (μ+ESG)/EVaR portfolios obtained from several risk models with €100 initial investment. The results are obtained assuming equal preferences for the reward measures.

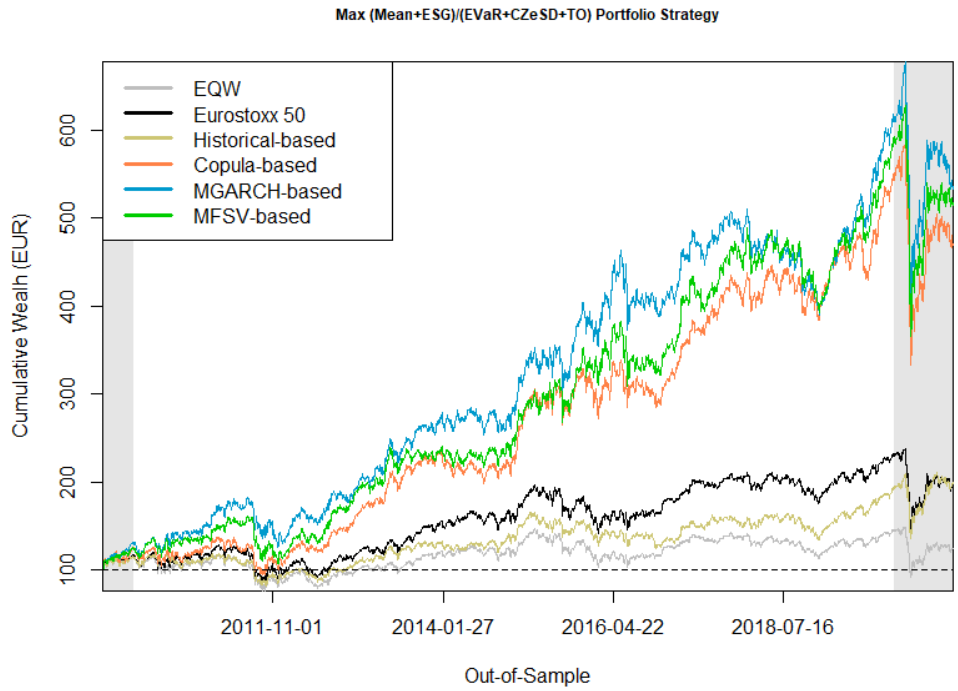

Fig. 5. This figure plots wealth trajectory for Max (μ+ESG)/(EVaR+CZϵSD+PT) portfolios obtained from several risk models with €100 initial investment. The results are obtained assuming equal preferences for the reward (risk) measures.

6. Conclusions

We suggest and study socially responsible multiobjective optimal portfolios. Applying the vine copulas in a first step, we estimate a (step-ahead) multivariate distribution for the assets' returns. In addition to the vine copula, a multivariate GARCH and a stochastic volatility model are used for comparison. Then, drawing observations from the estimated multivariate distribution, the socially responsible multiobjective optimal problem is solved using convex optimization.

The results indicate that optimal MOPs provide investors with the flexibility of incorporating different objectives. However, there is a trade-off between reward (risk) measures. Although, including social responsibility results in lower portfolio return and economic performance, it reduces the portfolio risk. While the cumulative zero-order SD objective (in most cases) increases the portfolio return when included in socially responsible MOPs, it reduces the portfolio risk. The predictive models lead to MOPs with higher return and reward/risk ratios. In particular, the copula-based MOPs achieve less tail risk.

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