Automation, Task Content and Occupations when Skills and Tasks are Bundled

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Preliminary

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Abstract

Automation affects workers because it affects the task content of their occupations. I propose a model which takes two important labor market features into account: (i) automation happens to tasks and (ii) workers with bundled skills work in occupations with bundled tasks. Equilibrium skill returns vary across occupations, and the impact of automation on skill returns is occupation-specific. Using my framework, I find that skill returns in the automated task decline if tasks are gross complements, consistent with much previous literature. Inequality increases in the occupation that is least intensive in the automated task, consistent with the development in Sweden 1985 - 2013. More generally, the model allows exploring how automation of one task affects the task content of occupations, returns to tasks, workers' earnings and inequality.

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1 Introduction

Is automation good or bad for workers? Automation improves productivity, but it displaces labor in automated tasks. However, workers are not employed directly in tasks. Rather, workers are employed in occupations to perform multiple tasks. Furthermore, they bring a bundle of skills – not just one skill – to their occupation. Although a large literature argues that automation affects tasks, which changes the task content of occupations, precisely how this affects workers is yet to be demonstrated. This paper seeks to answer the following question: How does automation affect workers who have bundled skills and who work in occupations which consist of bundles of tasks?

Bundling of skills and tasks are salient features of real-world labor markets. Bundling of skills is due to the fact that workers cannot be in two places at the same time. Bundling of tasks means that a worker is required to perform multiple tasks in her job. A used-cars salesperson uses her interpersonal skills to impel a customer to purchase a car. She uses cognitive skills to compute profit margins, and manual skills when demonstrating the cars to customers. A different occupation – for instance, a teacher or a plumber – also uses some combination of interpersonal, analytical and manual skills, but to varying degrees as they perform a different bundle of tasks compared to the salesperson. Even though skills are useful in multiple occupations, skill bundling means skill returns may not equalize across these occupations. Rather, occupation-specific skill returns depend on, among other things, the task content of occupations. Therefore, it is not obvious how automation of a subset of tasks will affect skill returns and hence wages across occupations. My goal in this paper is to build a general equilibrium model that can speak to these issues.

I provide a unified framework which embeds two important features of the labor market: (i) automation happens at the task level and (ii) workers with bundled skills work in occupations with bundled tasks. Consequently, I connect two important strands of literature: the task-based models of automation[1] and the "tasks-within-occupations" literature, which deals with occupations as (potentially time varying) bundles of tasks.[2]

Automation affects workers because it displaces labor from tasks, implying that the task content of production changes.[3] I consider two large task groups: routine and non-routine tasks. Routine tasks are tasks that can be "accomplished by following explicit rules" (Autor et al. 2003). They can be manual – such as assembling parts along an assembly line – or cognitive – e.g. computing VAT payments on sales. Non-routine tasks, on the other hand, are not easily programmable. They may be analytical, interpersonal or manual – for instance, solving a complicated engineering problem, teaching students or cleaning a hotel room. Up until recently, almost exclusively routine tasks were automated.[4] However, in recent years, new technology such as voice and face recognition and machine learning has enabled and continues to enable automation of many non-routine tasks, ranging from airport passport controls to self-driving vehicles. This development means both the routine and non-routine task groups experience automation of some portions of their content.

Additionally, both routine and non-routine tasks are found in many occupations, since occupations

[1]E.g. Autor et al. (2003), Acemoglu & Autor (2011), Goos et al. (2014), Cortes (2016), Acemoglu & Restrepo (2018*b*, 2018*a*, 2019).

[2]E.g. Spitz-Oener (2006), Gathmann & Schönberg (2010), Autor & Handel (2013), Atalay et al. (2020).

[3]Acemoglu & Restrepo (2019) models this. In their model, labor is pushed out of tasks and into new ones. The production side of my model nest theirs, and while I focus on automation only (they also focus on creation of new tasks), I add to their analysis by considering occupations consisting of *bundles* of tasks.

[4]In fact, Autor et al. (2003) define routine tasks as those susceptible to automation.

consist of packages or bundles of tasks (see e.g. Gathmann & Schönberg 2010 and Autor & Handel 2013).[5] In my model, each occupation produces a distinct good or service by combing tasks at varying intensities. For instance, the in the used-cars sales occupation, interpersonal, routine cognitive and non-routine manual tasks are combined to produce occupational output – the service of selling of used cars.

Apart from the empirical observation that occupations consist of bundles of tasks, there is theoretical justification for this view of occupations and tasks: coordination costs between workers who perform interdependent tasks coupled with a need to respond to current information induce bundling (Dessein & Santos 2006; Becker & Murphy 1992).

Lastly, bundling of skills in workers means workers supply *all* their skills to *one* occupation. The used-cars salesperson has some level of skills in each potential task she might be asked to do. She provides them as a package to one occupation, since she cannot be a used-cars salesperson *and* a teacher *and* a plumber at the same time. This implies skill returns will, in general, vary across occupations, since an occupation can attract workers by compensating relatively low payments to one skill by relatively high payments to another. A worker who maximizes income will choose occupation based on her relative skill endowments and the relative payments to those skills in different occupations, suggesting a Roy model of comparative advantage.[6] The used-cars salesperson has picked that specific occupation because the relative returns to her skills are better there than in any other occupation.

Using my framework, preliminary results indicate that skill returns in the automated task decline if tasks are gross complements, consistent with much previous empirical literature.[7] Inequality increases in the occupation that is least intensive in the automated task, consistent with the development in Sweden 1985 - 2013. More generally, the model allows exploring how automation of one task affects the task content of occupations, returns to tasks, workers' earnings and inequality. In contrast to a model with unbundled skills and tasks, my model allows investigating how automation affects the *within-occupation* inequality.

I plan on calibrating the model to US data to quantify the contribution of automation induced changes in task content and skill returns on inequality.

I add to the several strands of literature: Mainly, I connect to the literature that models and empirically investigates the effects of labor replacing automation in task frameworks, where Autor et al. (2003), Acemoglu & Autor (2011), Cortes (2016), Acemoglu & Restrepo (2018*a*, 2018*b*, 2019) have made substantial advances. While my model is most closely related to that of Acemoglu & Restrepo (2018*a*, 2018*b*), who also consider the effect of automation on the task content, I add to their insights by providing a framework in which workers with bundled skills sort into occupations that are bundles of tasks.

A broader literature on routine-biased technological change looks at reasons for and effects of demand shifts in favor of non-routine workers (Goos et al. 2014). One indication of this demand

[5] Gathmann & Schönberg (2010) find that workers who move between occupations that are close in the "task space" lose less than those who move far, indicating that the task vectors of occupations capture something empirically relevant. Autor & Handel (2013) note that "tasks are a high-dimensional bundle of activities, the elements of which must be performed jointly to produce output" (p.S64).

[6] Autor & Handel (2013) also conjecture that this is an appropriate model for the labor market in the presence of bundled tasks and thus varying skill returns.

[7] E.g. Cortes (2016) find that "routine" workers' wage premia decline. Other work in the routine-biased technological change or job polarization literature find similar results (e.g. Goos et al. 2014). Theoretical literature mainly consider relative wages, for instance between routine and non-routine workers.

shift is the growth in returns to non-cognitive skills, as found for Sweden and the US by Edin et al. (2017) and Deming (2017), respectively. Both papers emphasize that the level and development of returns differ across occupations – something my model speaks to.

Another related literature deals with the changing task content of occupations (e.g. Spitz-Oener 2006, Gathmann & Schönberg 2010, Deming & Kahn 2018 and Atalay et al. 2020). Relatedly, matching multidimensional skills to multidimensional tasks is studied by Yamaguchi (2012), Autor & Handel (2013) and Lindenlaub (2017).

Lastly, I relate to the theory around factor bundling, see e.g. Rosen (1983) and Heckman & Scheinkman (1987). In a simultaneous but independent project, Edmond & Mongey (2020) model workers who apply a bundle of skills to a bundle of tasks. I complement their work by modelling automation.

In the next section (2), I present some empirical observations that motivate my model. Section 3 lays out the model and equilibrium. Sections 4 and 5 are to be completed with an analytical solution and calibration exercise. In section 6, I present the comparative statics, and section 7 contrasts my model to an unbundled version. Section 8 concludes.

2 Automation, occupations and inequality

In this section, I present three observations regarding automation, task content and occupations.

First observation. Routine tasks have been automated to a larger degree than non-routine tasks. Occupations differ in their use of routine and non-routine tasks.

Second observation. Skills are differently rewarded in different occupations. *[To be completed]*

Third observation. Inequality increased more for those occupation that use non-routine tasks intensively. A significant share of this inequality is within narrowly defined occupations.

Firstly, automation varies across tasks. Figure 1 shows how automation levels varies over time for routine and non-routine tasks, relative to the baseline 1950 level. While routine tasks have been heavily automated, the automation level of non-routine tasks have been fairly stable.[8]

At the same time, we know that occupations differ in the intensity of their use of these tasks. Figure 2 shows that high-skilled occupations and services are relatively intensively using non-routine tasks, whilst administrative, manufacturing and elementary occupations, among others, are more intensive in routine tasks. However, both tasks are performed in all occupation, and to a non-trivial degree. Even the least routine-intensive occupations consist of a notable portion routine tasks.

[8]Perhaps a slight regression in automation can be discerned in non-routine tasks, at least from the 1970s until 2000. This is probably due to many *new tasks* being introduced on the non-routine task interval, as argued by Acemoglu & Restrepo (2019). Although I do not explicitly model this, it is not at odds with my model that tasks on the unit interval are exchanged for new tasks.

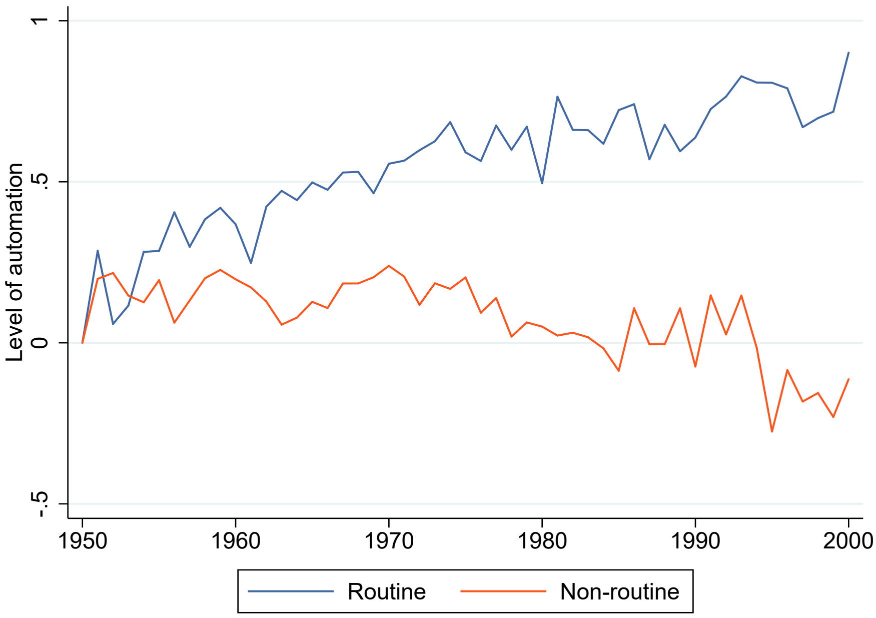


Figure 1: Automation levels of routine and non-routine tasks (b^R and b^{NR}) relative to 1950

Notes: Estimated from the Atalay et al. (2020) data on occupations and task content in US occupations (details in appendix section A.2). Tasks are aggregated according to the classifications used by Spitz-Oener (2006).

Next, I divide occupations into high-routine and low-routine occupations at the mean non-routine-to-routine ratio 0.47. Note how I emphasize that there are no "routine only" occupations and no "non-routine only" occupations. Instead, all occupations perform both tasks, but to varying degree. I compare the returns to two different skills between these occupational groups. Figure 3 shows that low-routine occupations have higher returns to both cognitive and psychological skills. *[To be completed.]*

Finally, in each occupational group, I look at total variance and the within-occupation variance.[9] Occupation, in this context, is a four-digit occupation – for example, Bartenders (5132), Process Operators (pulp) (8172) and Opticians (2284). Panel 4a demonstrates that low-routine occupations exhibit larger inequality than high-routine occupations, and the low-routine occupations have seen variances increase. Panel 4b shows that a significant share of total variance occurs within narrowly defined occupations, and within-occupation inequality also increased somewhat among low-routine occupations.

To explore these patterns in a model, we need to account for the fact that automation happens at the task level, that occupations differ but that all tasks are performed in all occupations. That is, tasks are bundled. Furthermore, to achieve a within-occupation wage distribution and task returns that vary across occupations we need to consider bundled labor. In the next section, I will outline a model that enables discussion of these phenomena.

3 Model setup and equilibrium

Brief overview The economy consists of consumers who are workers, occupational firms and final good firms. The consumers supply labor to occupational firms, who produce a differentiated

[9]The between-variance can be found in Appendix section I.

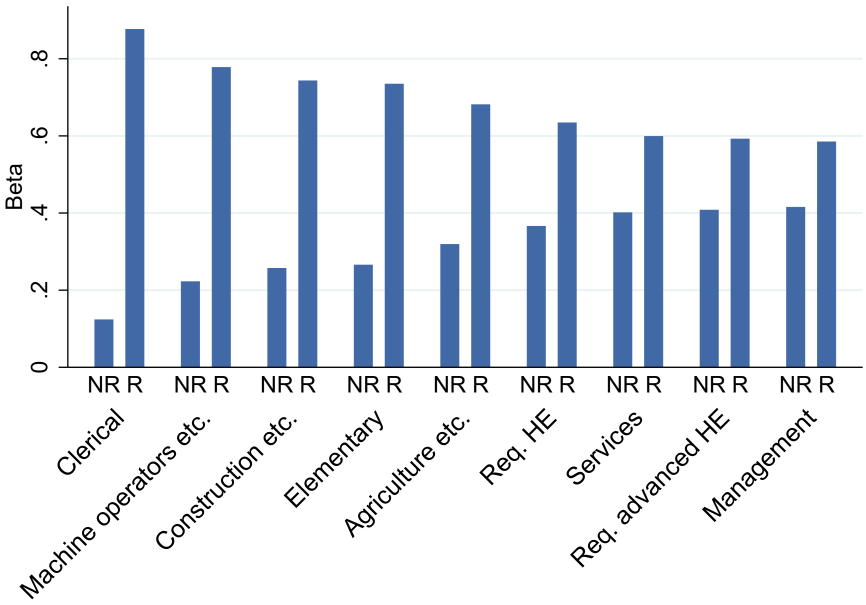


Figure 2: Task weights in different occupations, normalized

Notes: Estimated from the Atalay et al. (2020) data on occupations and task content in US occupations, matched to Swedish occupational classifications (details in appendix section A.2). Task weights are normalized to sum to one within an occupation. Tasks are aggregated according to the classifications used by Spitz-Oener (2006). Occupations are at the 1-digit level in SSYK 2012 classifications, and their labels are slightly shortened and simplified. Occupations are sorted according to their ratio of non-routine to routine tasks. The routine intensive occupations are thus to the left in the panel. The mean non-routine-to-routine ratio is 0.47. The occupational labels are, from left to right (SSYK 2012 1-digit code in parentheses): Administration and customer service clerks (4), Mechanical manufacturing and transport workers, etc. (8), Building and manufacturing workers (7), Elementary occupations (9), Agricultural, horticultural, forestry and fishery workers (6), Occupations requiring higher education qualifications or equivalent (3), Service, care and shop sales workers (5), Occupations requiring advanced level of higher education (2), Managers (1).

good Y_m and pay wages to households. The differentiated good Y_m is purchased by final good firms, who convert it into a final good Y. They convert some of this final good to capital K, which they rent back to occupational firms. The rest is sold as consumption to households. Both types of firms are competitive and owned by households.

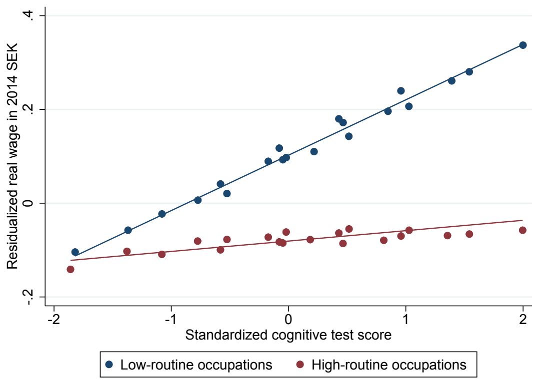
3.1 Worker earnings and occupational choice

Each worker (consumer) is endowed with a vector of skills in T different tasks: $l_i = \{l_i^1, l_i^2, ..., l_i^T\}$.[10] Here, I consider two task groups ($T = 2$), and I call them R and NR – think about routine and non-routine tasks.[11] Skills are bundled, which means a worker cannot supply individual skills to different occupations. Her occupational choice is therefore a discrete choice of an occupation m from the set $\mathbb{M} = \{m_1, m_2, ..., m_M\}$. For ease of exposition, I consider two occupations ($M = 2$). Each unit of skill (or effective labor) is paid its marginal product in each occupation,[12] so worker i's earnings are

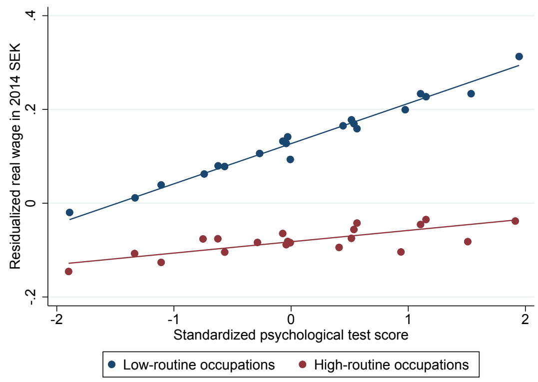
[10] We can think of these skills in tasks as effective labor in tasks: If there are S skills and T tasks, the mapping $\mathcal{L} : \mathbb{R}^S \to \mathbb{R}^T$. l_i^τ thus represents worker i's effective labor in task τ.

[11] These can easily be exchanged for other task groupings, including those with more than two tasks.

[12] See section 3.3 for more on why this is so.



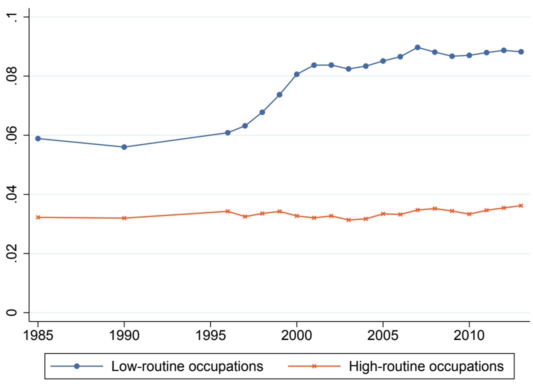
(a) Returns to cognitive skills



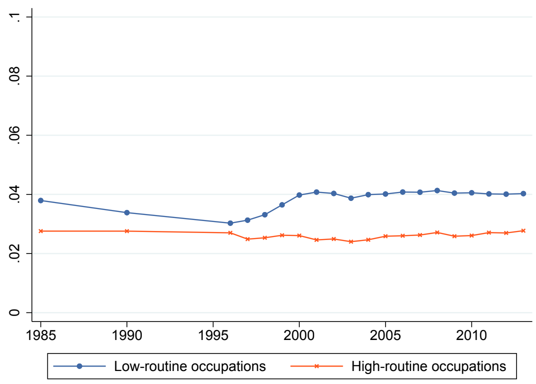
(b) Returns to psychological skills

Figure 3: Returns to skills in different occupations in Sweden

Notes: The figure plots bins of residualized log of real wages (2014 SEK) on test scores for military draft cognitive and psychological tests, normalized by cohort scores. Real wages are residualized using a weighted regression of real wages on gender, whether or not born in Sweden and age. Weights are as recommended by Statistics Sweden. Routine occupations are those with non-routine-to-routine ratio below 0.47, namely (SSYK 2012 in parentheses) Administration and customer service clerks (4), Mechanical manufacturing and transport workers, etc. (8), Building and manufacturing workers (7), Elementary occupations (9), Agricultural, horticultural, forestry and fishery workers (6). Non-routine occupations are the rest: Occupations requiring higher education qualifications or equivalent (3), Service, care and shop sales workers (5), Occupations requiring advanced level of higher education (2), Managers (1). The number of observations is 10,495,545.



(a) Total variance



(b) Within occupation

Figure 4: Residual wage inequality in low-routine and high-routine occupations in Sweden

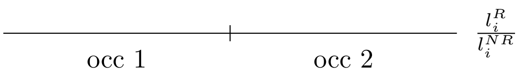
Notes: The figure plots the variance of residualized log of real wages (SEK) for a large, representative sample of the Swedish workforce (Wage Structure Statistics, details to come), divided into high- and low-routine occupations. Real wages are residualized using a weighted regression of real wages on gender, whether or not born in Sweden and age. Weights are as recommended by Statistics Sweden. Routine occupations are those with non-routine-to-routine ratio below 0.47, namely (SSYK 2012 in parentheses) Administration and customer service clerks (4), Mechanical manufacturing and transport workers, etc. (8), Building and manufacturing workers (7), Elementary occupations (9), Agricultural, horticultural, forestry and fishery workers (6). Non-routine occupations are the rest: Occupations requiring higher education qualifications or equivalent (3), Service, care and shop sales workers (5), Occupations requiring advanced level of higher education (2), Managers (1). The within and between variances are computed within and between SSYK 2012 four-digit occupations.

$$W_{im} = \sum_{\tau} w_m^{\tau} l_i^{\tau} \tag{1}$$

Where w_m^τ is the return to worker skills in task τ employed in occupation m. As posited by Autor & Handel (2013), and as explained in Heckman & Scheinkman (1987), there is no single skill price across the economy since labor is bundled. Skill returns depend on the task and occupation in which skills are employed.

For there to be positive employment in all occupations, no occupation can have skill returns that strictly dominate those in another occupation – an occupation with higher returns to routine skills than another occupation must have lower returns to non-routine skills. If it were not so, the occupation that had lower returns to both R and NR would not get any workers.[13] In order to demonstrate the workers' choice graphically below, I assume occupation 1 pays relatively more to non-routine skills (think of a service occupation, like shop attendant) and occupation 2 has high returns to routine skills (think of a machine operator). Assumption 1 in section 3.6 states this assumption in terms of model parameters.

Workers will choose the occupation m in which $W_{im} \geq W_{im'}$ for all $m' \neq m \in \mathbb{M}$. In the case with two tasks, R and NR, workers can be characterized on a line representing their ratio of effective labor l_i^R / l_i^{NR}. Given assumption 1, we can characterize the workers' choice graphically as follows:



The cutoff between occupations 1 and 2 is defined by[14]

$$u \equiv \frac{w_1^{NR} - w_2^{NR}}{w_2^R - w_1^R} \tag{2}$$

since workers on the cutoff are indifferent between occupation 1 and 2, i.e. they have

$$w_1^R l_i^R + w_1^{NR} l_i^{NR} = w_2^R l_i^R + w_2^{NR} l_i^{NR}$$

In accordance with assumption 1, moving rightward along the line, w_m^R increases, but w_m^{NR} decreases. That ensures both numerator and denominator are positive in (2). Workers with relatively high routine skills will choose an occupation that has relatively higher returns to those skills – a "high-routine" occupation. We can think of the high-routine occupation 2 as machine operator, and the "low-routine" occupation 1 as a service occupation, for instance a shop attendant.

The supply of a skill in each occupation is simply the sum of skills of those who choose to work in each occupation[15], namely

$$L_m^\tau = \int_{i \in m} l^\tau(i) di$$

We might call this the "occupational labor supply" of skill τ in occupation m.

[13] This is akin to proposition 1 from Autor & Handel (2013). In proposition 2 of the same paper, we read that there cannot be uniformly positive cross-occupation covariance between task returns for all task pairs, which, for the case of two occupations, means the same as proposition 1.

[14] The lower cutoff for occupation 2 is the same as the upper for occupation 1, and if there were more occupations, these cutoffs would be defined similarly.

[15] To solve the model given some distribution of labor we need to reformulate this integral, see Appendix section B.1 for details.

3.2 Occupational firms: Production

The differentiated good Y_m, which is produced by firms in each occupation ("occupational firms"), is produced by combining task groups in a constant elasticity of substitution (CES) production function.

$$Y_m = \left[\beta_m^{1/\sigma} X_m^{R\ \frac{\sigma-1}{\sigma}} + (1-\beta_m)^{1/\sigma} X_m^{NR\ \frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where σ is the elasticity of substitution between task groups R and NR. The parameter β_m represents the importance of routine tasks in occupation m.

The routine and non-routine task groups consist of a continuum of smaller, intermediate tasks. For instance, the routine task group may include such tasks as typing, operating machinery in a predictable manner, counting, recording, etc. The non-routine task group may include such tasks as cleaning, managing, planning, childminding, etc. Each task group is an aggregate of these smaller, intermediate tasks.

$$X_m^R = \left[\int_0^1 x_m^R(j)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

and similarly for task group NR. A fraction (b_t^R, b_t^{NR}) of task groups R and NR respectively can be performed by machines, and the rest must be performed by labor. For now, let (b_t^R, b_t^{NR}) be constant over time, and drop the time subscript. In the intermediate tasks that can be performed by machines, capital and labor are perfect substitutes. Thus, intermediate task j in task group R is produced as follows:

$$x_m^R(j) = \begin{cases} \lambda k_m^R(j) + l_m^R(j) & \text{if automatable, i.e. } j \in [0, b^R] \\ l_m^R(j) & \text{otherwise, i.e. } j \in (b^R, 1] \end{cases} \tag{5}$$

and similarly for intermediate tasks in task group NR. λ is a capital augmenting productivity factor. Intermediate tasks are symmetric, so $x_m^\tau(j) = x_m^\tau\ \forall j \in [0,1]$. If firms automate all automatable tasks, task group R is produced as follows:

$$X_m^R = \left[b^{R\ 1/\eta}(\lambda K_m^R)^{\frac{\eta-1}{\eta}} + (1-b^R)^{1/\eta} L_m^{R\ \frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where capital letters L_m^R, K_m^R denote the total amount of labor and capital, respectively, that occupation m employs in task group R, i.e.

$$L_m^R = (1-b^R) l_m^R(j) \qquad \forall j \in [b^R, 1] \tag{7}$$

$$K_m^R = b^R k_m^R(j) \qquad \forall j \in [0, b^R] \tag{8}$$

3.3 Occupational firms: Firm problem

There are many firms within each occupation m, meaning that occupational firms are price takers. Capital is supplied by final good firms at the constant rate r.

Labor is – or skills are – provided to occupations in bundles, as in Heckman & Scheinkman (1987). Remuneration to labor will therefore depend on the occupation. However, individual firms within each occupation treat the wage to labor in each task as exogenous.

Now, allow firms to automate *up to* the technological frontier. Denote a firm's optimal automation level as $\tilde{b}_m^\tau$ and the technological frontier as b^τ.

Firms are symmetric within an occupation, so we can treat them as one representative firm. Firms choose capital, labor and the degree of automation, i.e. what share of tasks they want to produce using capital.

$$\begin{aligned}
\max_{K_m^\tau, L_m^\tau, \tilde{b}_m^\tau} \quad & p_m Y_m - r(K_m^R + K_m^{NR}) - w_m^R L_m^R - w_m^{NR} L_m^{NR} \\
\text{s.t.} \quad & Y_m = \left[\beta_m^{1/\sigma} X_m^{R\ \frac{\sigma-1}{\sigma}} + (1-\beta_m)^{1/\sigma} X_m^{NR\ \frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
& X_m^\tau = \left[\int_0^{\tilde{b}_m^\tau} (\lambda k_m^\tau(j))^{\frac{\eta-1}{\eta}} dj + \int_{\tilde{b}_m^\tau}^1 l_m^\tau(j)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \\
& \tilde{b}_m^\tau \leq b^\tau
\end{aligned}$$

where $\tau = R, NR$ and, as before, $l_m^\tau(j) = L_m^\tau/(1-\tilde{b}_m^\tau)$ and $k_m^\tau(j) = K_m^\tau/\tilde{b}_m^\tau$ for all j. The amount of labor and capital in each small task j is the same for all j because of the symmetry of small tasks in equation 4. The last constraint means that firms are free to automate up until the technological limit b^τ, which is the same for the whole economy.

The first order conditions for capital, labor and $\tilde{b}_m^\tau$ are given by

$$r = p_m \left(\frac{\beta_m Y_m}{X_m^R}\right)^{1/\sigma} \left(\frac{\tilde{b}_m^R \lambda^{\eta-1} X_m^R}{K_m^R}\right)^{1/\eta} \tag{9}$$

$$w_m^R = p_m \left(\frac{\beta_m Y_m}{X_m^R}\right)^{1/\sigma} \left(\frac{(1-\tilde{b}_m^R) X_m^R}{L_m^R}\right)^{1/\eta} \tag{10}$$

$$\mu_m^R = p_m \left(\frac{\beta_m Y_m}{X_m^R}\right)^{1/\sigma} \left(\frac{X_m^{R\ 1/\eta}}{\eta-1}\right) \left[\left(\frac{\lambda K_m^R}{\tilde{b}_m^R}\right)^{\frac{\eta-1}{\eta}} - \left(\frac{L_m^R}{1-\tilde{b}_m^R}\right)^{\frac{\eta-1}{\eta}}\right], \tag{11}$$

and similarly for NR. μ_m^τ is the Lagrange multiplier attached to the inequality constraint $\tilde{b}_m^\tau \leq b^\tau$. First, consider the case when $\mu_m^\tau = 0$. This suggests that the chosen level of automation $\tilde{b}_m^\tau$ may be below the constraining b^τ. Solving for $\tilde{b}_m^\tau$ gives

$$\tilde{b}_m^\tau = \frac{\lambda K_m^\tau}{\lambda K_m^\tau + L_m^\tau} \tag{12}$$

Firms will automate up until the point where the automated share of tasks equals the share of effective capital inputs. If instead $\mu_m^\tau > 0$, then the ratio of capital inputs to total inputs will be larger than b^τ. This means occupational firms want to automate *more* than technology allows, and the constraint binds. There will be complete automation of automatable tasks: $\tilde{b}_m^\tau = b^\tau$.

3.4 Consumption

There is a unit interval of consumers i. They derive utility from consumption only, which they purchase from final good firms at price $\tilde{P}$ using their income I_i. Their problem is thus

$$\begin{aligned} \underset{C_i}{max} \quad & C_i \\ \text{s.t.} \quad & \tilde{P}C_i \leq I_i \end{aligned}$$

so that the optimal consumption level for consumer i is $C_i = I_i/\tilde{P}$, meaning that total demand for the consumption good is

$$C = \frac{I}{\tilde{P}}$$

where $C = \sum_i C_i$ and $I = \sum_i I_i$. All consumers are workers, and they own all firms, all of which have zero profits. Their income I_i is therefore $I_i = \sum_\tau w^\tau_{m(i)} l^\tau_i$, where $m(i)$ is the occupation chosen by worker i.

3.5 Final good firms

Final good firms purchase occupational output Y_m from occupational firms at price p_m. They convert it into the homogenous final good Y via the following CES aggregate, where ε is the elasticity of substitution between goods from different occupations:

$$Y = \left[\sum_{m=1}^{M} Y_m^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

The final good firms then convert this final good into capital at the fixed rate γ or to the consumption good C at rate 1. They rent the capital back to occupational firms at rate r and sell the consumption good to consumers at price $\tilde{P}$. Their firm problem is thus

$$\underset{K,C,Y_m}{max}\ rK + \tilde{P}C - \sum_m p_m Y_m \tag{13}$$

$$\text{s.t.} \qquad Y = \left[\sum_{m=1}^{M} Y_m^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{14}$$

$$Y \geq C + \frac{K}{\gamma} \tag{15}$$

Defining the optimal price index as $P = \left[\sum_m p_m^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, the first order conditions imply that the price for the consumption good $\tilde{P} = P$.[16] The first order conditions define the demand for

[16] The first order conditions are as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K} &= r - \mu/\gamma = 0 \\ \frac{\partial \mathcal{L}}{\partial C} &= \tilde{P} - \mu = 0 \\ \frac{\partial \mathcal{L}}{\partial Y_m} &= -p_m + \mu \left(\frac{Y}{Y_m}\right)^{1/\varepsilon} = 0 \end{aligned}$$

where constraint 14 is inserted to 15, and μ represents the Lagrange multiplier attached to that constraint. Now, defining $P = \left[\sum_m p_m^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, it is clear that $\tilde{P} = P$.

occupational goods Y_m and the supply of capital K as follows:

$$p_m = \left(\frac{Y}{Y_m}\right)^{1/\varepsilon} P \tag{16}$$

$$r = \frac{P}{\gamma} \tag{17}$$

Let us set the final consumption good as the numeraire in the model, meaning we define $P = 1$.

3.6 Equilibrium

Given some joint distribution of skills in all different tasks, l_i^τ, an equilibrium is an allocation $\{L_m^\tau, K_m^\tau\}_{m=1,2,\tau\in\{R,NR\}}$ and a set of prices $\{p_m, w_m^\tau\}_{m=1,2,\tau\in\{R,NR\}}$, automation levels $\{\tilde{b}_m^\tau\}_{m=1,2,\tau\in\{R,NR\}}$ and a cutoff $\{u\}$ such that individuals (workers who are also consumers) and firms (final good and occupational) optimize and markets clear subject to the production functions and the bundling constraint. It is characterized by the following equations:

$$r = p_m\left(\frac{\beta_m Y_m}{X_m^R}\right)^{1/\sigma}\left(\frac{\tilde{b}_m^R \lambda^{\eta-1} X_m^R}{K_m^R}\right)^{1/\eta} \tag{18}$$

$$r = p_m\left(\frac{(1-\beta_m) Y_m}{X_m^{NR}}\right)^{1/\sigma}\left(\frac{\tilde{b}_m^{NR} \lambda^{\eta-1} X_m^{NR}}{K_m^{NR}}\right)^{1/\eta} \tag{19}$$

$$w_m^R = p_m\left(\frac{\beta_m Y_m}{X_m^R}\right)^{1/\sigma}\left(\frac{(1-\tilde{b}_m^R) X_m^R}{L_m^R}\right)^{1/\eta} \tag{20}$$

$$w_m^{NR} = p_m\left(\frac{(1-\beta_m) Y_m}{X_m^{NR}}\right)^{1/\sigma}\left(\frac{(1-\tilde{b}_m^{NR}) X_m^{NR}}{L_m^{NR}}\right)^{1/\eta} \tag{21}$$

$$p_m = \left(\frac{Y}{Y_m}\right)^{1/\varepsilon} P \tag{22}$$

$$L_m^\tau = \int_{i\in m} l^\tau(i)di \tag{23}$$

$$u = \frac{w_1^{NR} - w_2^{NR}}{w_2^R - w_1^R} \tag{24}$$

$$\tilde{b}_m^\tau = \begin{cases} \frac{\lambda K_m^\tau}{\lambda K_m^\tau + L_m^\tau} & \text{if this ratio} < b^\tau \\ b^\tau & \text{otherwise,} \end{cases} \tag{25}$$

where I use the equations below to find the quantities $\{X_m^\tau, Y_m, Y, C, K\}$, the price index P (which I set to one to make the final good C the numeraire), and the price of capital r (which is fixed). With two task groups and two occupations, we have 19 unknowns and 19 equations.

$$X_m^\tau = \left[\tilde{b}_m^{\tau\,1/\eta}(\lambda K_m^\tau)^{\frac{\eta-1}{\eta}} + (1-\tilde{b}_m^\tau)^{1/\eta} L_m^{\tau\,\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{26}$$

$$Y_m = \left[\beta_m^{1/\sigma} X_m^{R\,\frac{\sigma-1}{\sigma}} + (1-\beta_m)^{1/\sigma} X_m^{NR\,\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{27}$$

$$Y = \left[\sum_m Y_m^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{28}$$

$$C = \sum_m \sum_\tau w_m^\tau L_m^\tau / P \tag{29}$$

$$K = \sum_m \sum_\tau K_m^\tau \tag{30}$$

$$P = \left[\sum_m p_m^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{31}$$

$$r = P/\gamma \tag{32}$$

$$\tag{33}$$

I compute the equilibrium using the algorithm described in B.2.

I assume:
Assumption 1. $\beta_1 < \beta_2$.

This means that the machine operators (occupation 2) use routine tasks more intensively, while shop attendants (occupation 1) use the non-routine tasks more intensively, and will, in equilibrium, lead to higher returns to routine skills in machine operating, and higher returns to non-routine skills in shop attending. The exception is when returns to skills equalize across occupations, which may happen despite assumption 1 on importance. If returns to skills equalize, workers are indifferent between the two occupations and there is no cutoff defined by equation 2. In the next section, I investigate the possibility of equalizing skill returns.

3.7 Can marginal productivities of labor equalize across occupations?

One of the motivating observations for this paper is that returns to skills are not, in fact, equal across occupation. That is, the law of one price for skills does not hold. Therefore, I introduced skill bundling in my model in order to create a wedge between skill returns in different occupations.

Because as Heckman & Scheinkman (1987) state, if input factors are bundled there is, in general, not equalization of factor productivities and thus factor prices across occupations[17]. However, it is *possible* that marginal productivities of labor (or skills) equalize even when skills are bundled. This happens if the optimal allocation of labor is in the interior of the feasible set. That is, the constraint posed by bundling does not bind at the optimum.

To understand this, consider the first-order conditions for the social planner problem with *unbundled* skills.

$$MPL_m^\tau = \mu^\tau$$
$$MPK_m^\tau = 1/\gamma$$

[17] They use industries rather than occupations.

where μ_τ is the Lagrange multiplier for the total labor supply constraint $L^\tau \geq L_1^\tau + L_2^\tau$. The decentralized problem is analogous, where the prices of labor and capital are $w^\tau = \mu_\tau$ and $r = 1/\gamma$, respectively. There are eight equations and eight unknowns, so it is, in general, possible to find an optimal allocation $\{L_m^\tau, K_m^\tau\}_{m=1,2,\tau=R,NR}$. But if labor is bundled, there are more constraints than unknowns, since the labor in each occupation has to satisfy the bundling constraints:

$$L_1^A = \int_{i \in 1*} l^A(i)di$$
$$L_1^B = \int_{i \in 1*} l^B(i)di$$

where $1*$ is the set of workers placed in (or choosing to be in) occupation 1. The workers who are in occupation 1, and *only those workers*, supply *both* skills to occupation 1. If the unbundled (unconstrained) equilibrium,[18] by chance, satisfies the bundling constraints – i.e. is in the feasible set – marginal productivities and thereby skill returns can equalize across occupations.

Or, equivalently, if the bundled (constrained) equilibrium described in section 3.6 is in the *interior* of the feasible set, then the bundling constraint does not bind, and skill returns can equalize across occupations.

To explore this possibility, I plot the feasible set of allocations in an Edgeworth box, given some distribution of labor (the parameters can be found in Table 1). The left and lower axes represent the amount of R and NR skills allocated to occupation 1, and the right and upper axes represent the corresponding skill allocation to occupation 2. The lens represents the feasible allocations: they satisfy the bundling constraint.

First, note that the $*$ marked allocations are the solutions to the unbundled (unconstrained) problem. They are *outside* the feasible set, indicating that they cannot be an equilibrium allocation in the constrained problem.

Second, the plotted o marked allocations are the equilibrium allocations from the bundled (constrained) problem. These allocations come from my comparative static exercise in section 6 – evidently, all of these are on the upper border of the lens, suggesting that the bundling constraint binds. Consequently, if skills are bundled, and if the parameters are as in table 1, skill returns vary across occupations.

In Appendix section F I provide the computation of the lens (adapted from Edmond & Mongey 2020), as well as an illustrative example of the bundling constraint.

4 Analytical solution

[To be completed. For now, see plan in appendix section G.]

5 Calibration

[To be completed. For now, see plan in appendix section A.3.]

[18] Note that in this section, I consider only unbundled*skills*. If tasks, too, were to unbundle (as in section 7, then all R skills would be employed in the occupation that only produced R tasks, and vice versa for NR.

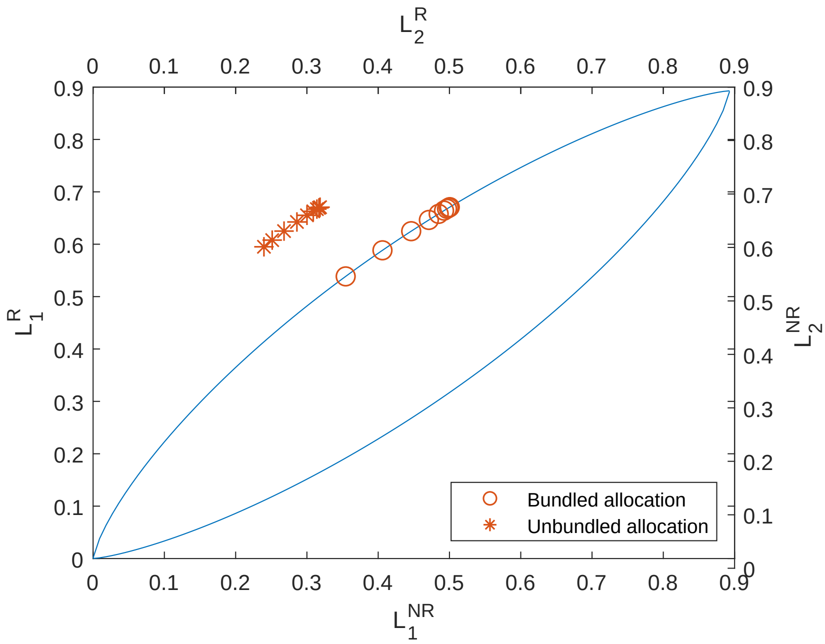


Figure 5: The feasible set in an Edgeworth box

Notes: The total endowment of (L^R, L^{NR}) in the economy is $(0.893, 0.893)$. The o marked allocations are equilibrium allocations depicted in section 6. The $*$ marked allocations are equilibrium allocations for the unbundled problem, as described in section F.3. Parameters are specified as in Table 1 and b^R varies from 0.1 and 0.9.

6 Comparative statics

Comparative statics are computed by varying b^R between 0.1 and 0.9. All other parameters are kept at their initial levels, as given in table 1. First, I investigate the response in skill returns. Thereafter, I compute the implied earnings and mobility responses to automation. Lastly, I look at inequality.

Note that firms are free to automate *less* than technologically feasible. Figure 6 shows that in this parametrization, both occupations automate fully up until routine tasks are 90 % automatable – at that point, occupation 1 chooses to automate slightly less (here, $\tilde{b}_1^R = 0.844$).

6.1 Skill returns and earnings

In figure 7 I plot the log of the skill returns in each occupation as routine tasks are automated. In both occupations, returns to non-routine increase and returns to routine decrease. Routine skill returns fall slightly more in occupation 2 which is intensively using routine tasks. The return to non-routine skills converge when routine tasks become highly automated. This is because the economy is approaching unbundling: as routine skills become almost worthless (the returns approach zero), all workers will choose occupation by comparing non-routine skill returns. Occupations converge in their payment to non-routine skills (that is, occupation 2 converges to occupation 1) in order to keep employment positive. The economy approaches the law of one price for non-routine skills.

To explore these skill returns in detail, consider that they are, in equilibrium, determined by the following product of marginal productivities:

		Routine	Non-routine
Skill distribution family	$l_i^\tau \sim$	Weibull	
Skill distribution scale		1	1
Skill distribution shape		3	3
Importance of routine tasks in occ 1	β_1	0.3	
Importance of routine tasks in occ 2	β_2	0.7	
Automation of non-routine tasks	b^{NR}		0.1
Rental rate	r	0.02	
Capital augmenting factor	λ	4	
Price of final good	P	1	
Elasticity of substitution (EoS) between occupational goods Y_m in final good production	ε	0.2	
EoS between task groups X_m^R, X_m^{NR} in occupational production	σ	0.4	
EoS between smaller tasks in task production	η	0.9	

Table 1: Parameters

Notes: The βs are set so that occupation 1 uses NR intensively, and occupation 2 uses R intensively. The elasticities of substitution are set such that the substitutability increases the "deeper down" in the layers you go. Goods are very complementary (see ε), reflecting that e.g. consumers like variety. Task groups are slightly easier to substitute (see σ), but not much, reflecting the fact that occupations need both types of tasks. The smaller tasks, within task groups, though, are easier to substitute (see η), although they are still gross complements. This reflects the fact that they are less differentiated since they belong to the same task group. Acemoglu & Restrepo (2018*a*) state that the firm-level elasticity of substitution between labor and capital is estimated to be between 0.4 - 0.7, so σ being in the lower span of this seems about right. The two types of labor l_i^R and l_i^{NR} are independently distributed.

$$w_m^\tau = \frac{\partial Y}{\partial Y_m} \frac{\partial Y_m}{\partial X_m^\tau} \frac{\partial X_m^\tau}{\partial L_m^\tau}$$

Therefore, we can decompose figure 7 into these different components. In figure 8 I present one figure for each skill return.

One qualitative difference between the returns to routine skills in occupations 1 and 2 is that $\partial Y/\partial Y_m$ increases in the low-routine occupation 1 (panel 8a), but it decreases in the high-routine occupation 2 (panel 8c). The low-routine occupational good (a shop attendant's services) becomes more productive, and the high-routine occupational good (machine operating services) become less productive. This is because machine operators now produce a lot more of their good (since they can employ machines to perform more and more important routine tasks), so that the good's marginal productivity declines.

Productivity and displacement effects vary across occupations Let us relate these findings to the lessons we learn from Acemoglu & Restrepo (2019), namely that automation brings about two opposing forces: a displacement effect and a productivity effect.

The displacement effect means labor is pushed out of tasks they were previously performing. This "always reduces the labor share" (Acemoglu & Restrepo 2019:10), meaning that labor gets a smaller share of the surplus of production. As the routine task group is automated, routine skills are pressed into a smaller set of routine tasks, meaning their marginal productivity declines – i.e. $\partial X_m^R/\partial L_m^R$ declines.

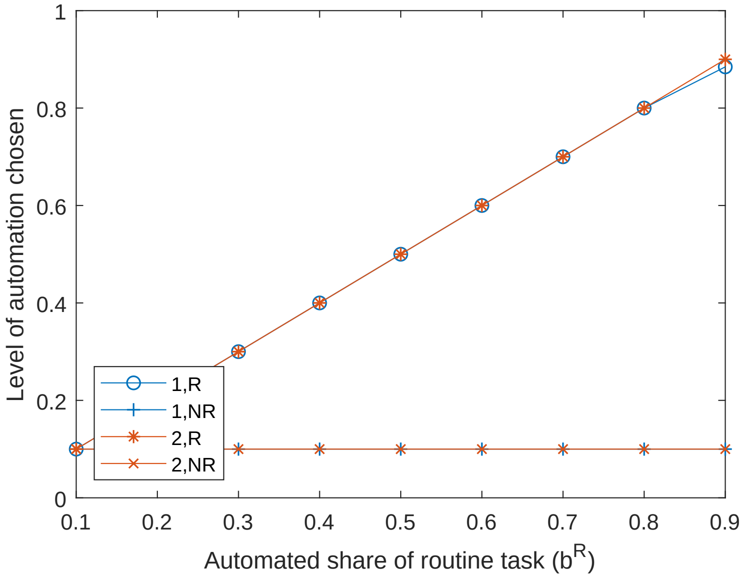


Figure 6: Chosen automation level $\tilde{b}^{\tau}_{m}$

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.Firms are free to set $\tilde{b}^{\tau}_{m} \leq b^{\tau}$. This plots firms' optimal choice of automation level, given the constraining b^{τ}. $b^{NR} = 0.1$ throughout, and firms indeed choose to automate 10% of non-routine tasks throughout. b^{R} goes from 0.1 to 0.9, and firms choose that maximum level of automation, except for occupation 1 (low-routine occupation, shop attendants) who automate slightly less at the final stage ($\tilde{b}^{R}_{1} = 0.844$ when $b^{R} = 0.9$).

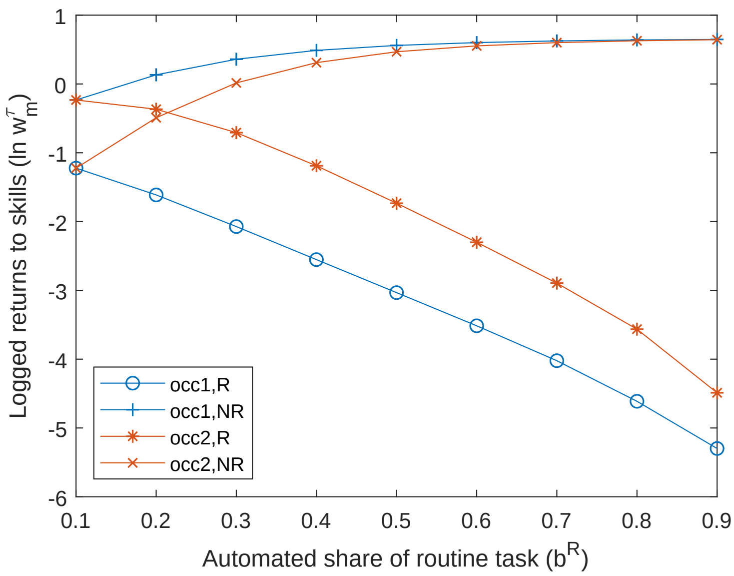
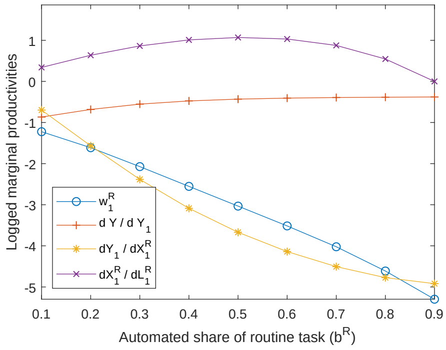


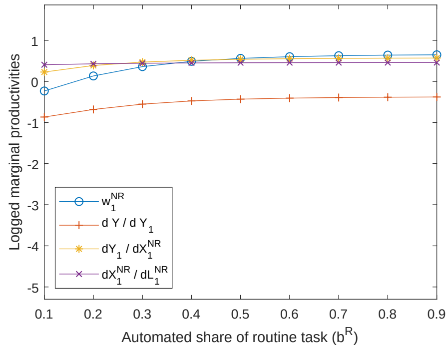
Figure 7: Logged skill returns w^{τ}_{m}

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.

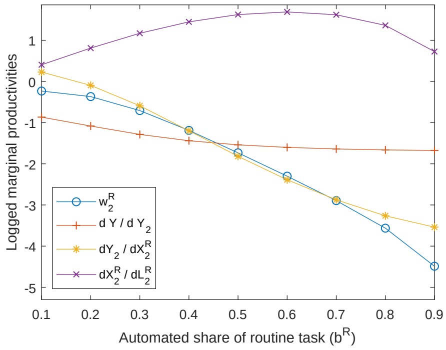
The productivity effect, on the other hand, comes from the increasing value added, which leads firms to demand more labor for the remaining tasks. This affects both tasks: occupational firms



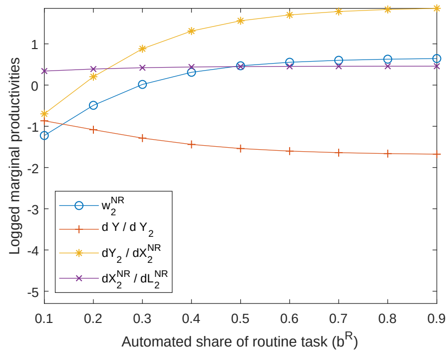
(a) w_1^R



(b) w_1^{NR}



(c) w_2^R



(d) w_2^{NR}

Figure 8: Decomposition of skill returns (all in logs)

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.

can now produce routine tasks cheaper, since they can use relatively cheap capital instead of relatively expensive labor in more tasks. The occupational firm therefore wants to produce more routine tasks, and in total, the output of both occupations will increase. In particular, the high-routine, machine operating occupation 2's output increases relative to the shop attendants' (see panel 20a). This leads the occupational firms to demand more labor, and, in particular, more machine operators.

In figure 8, we note that the w_m^{NR} in panels 8b and 8d experience no changes in labor productivity at the task level ($\partial X_m^{NR}/\partial L_m^{NR}$ is constant). No labor is displaced, but they do not experience higher productivity because of ocucpational firms' cost saving, either. In panels 8a and 8c, however, we see that the productivity effect dominates at the early stages of automation – $\partial X_m^R/\partial L_m^R$ increases, especially in the routine-intensive occupation 2. This is because early on, there are large cost savings to be reaped by occupational firms when they replace high-paid R labor by cheaper machines. At the later stages of automation, however, the displacement effect dominates. Now, labor is so cheap so that increased automation no longer saves large amounts.

In my framework, these effects vary across occupations depending on their initial intensities of task usage. This means that workers close to either side of the border between two occupations, who have very similar skill ratios but work in different occupations, are affected differently by automation.

Earnings To further illustrate how automation, via occupation specific skill returns, affects individual workers, let us follow three types of individuals. I simulate 10,000 workers from the distribution specified in table 1, and I draw three types: The first type has *no* routine skills (1st percentile) but high non-routine skills (75th percentile) – call this type (no R, high NR). The second type is at the median of both skills' distributions: (medium R, medium NR). The third type has high routine skills (75th percentile) and *no* non-routine skills (1st percentile), and is consequently called (high R, no NR). I let them choose occupation according to the model rules, and I apply the resulting skill returns on their skill bundles to produce their earnings.

Figure 9 demonstrates that the higher a worker's non-routine skills are, relative to routine skills, the more they gain from automation of routine tasks.

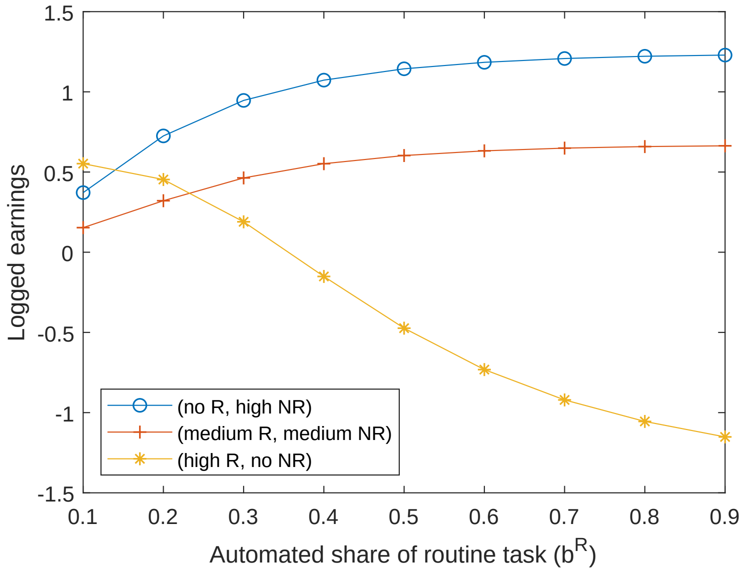


Figure 9: Earnings for three types of workers as R is automated

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.The types [(no R, high NR), (medium R, medium NR), (high R, no NR)] are drawn from the following percentiles of the R and NR skill sample distributions: [(1st, 75th), (50th, 50th), (75th, 1st)].

Mobility is also induced by automation. In this simulation, the (medium R, medium NR) worker switches from occupation 2 to 1 as automation goes from 0.2 to 0.3. (low R, high NR) works in occupation 1 throughout, while (high R, low NR) stays in occupation 2. That is, the middle worker switches from machine operator to shop attendant when the routine task goes from 20 to 30 percent automation.

6.2 Inequality

Using my model, I can explore inequality within and between occupations. To do so, I use the simulated 10,000 workers from section 6.1.

In figure 10, we see that the variances increases in occupation 1 – the low-routine occupation (think about the shop attendants). This reminds us of the pattern we saw in the data in figure 4. The routine-intensive occupation 2 experiences a drop in variance of log earnings at the early stages of automation, but a heavy increase at the later stages of automation. [19]

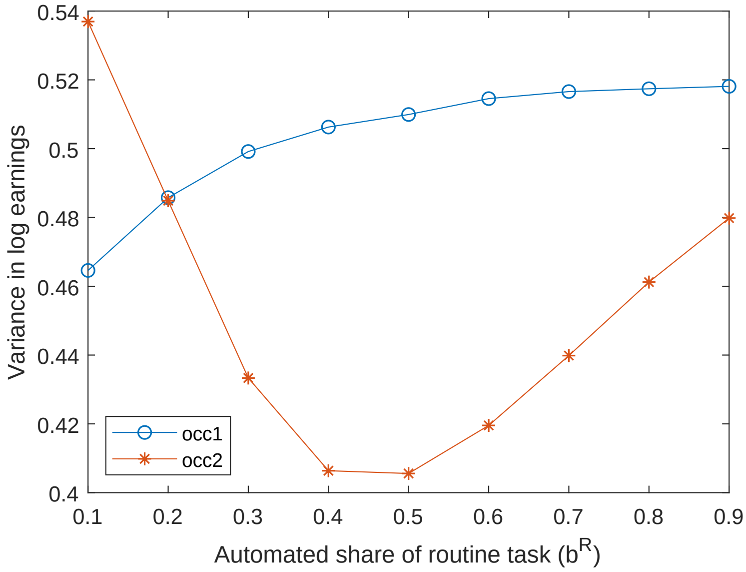


Figure 10: Variance in log earnings within each of the two occupations

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.The number of simulated workers is 10,000.

Earnings for different percentiles in the two occupations can be found in figure 21.

7 Unbundling skills and tasks

My model nests a simpler model with unbundled skills and unbundled tasks. Firstly, removing the bundling constraint 23 means workers are able to supply their different skills to different occupations, since the skills are now *unbundled*. This implies that skill returns will equalize across occupations, such that there is no cutoff 2 between occupations. This version of the model is discussed in section 3.7 and appendix section F.

Secondly, setting $\beta_1 = 0$ and $\beta_2 = 1$ implies that each occupation will consist of one task only. Output from occupation 1 – the service occupation – will be produced by non-routine tasks only, and the output from occupation 2 – the machine operation occupation – will be produced by routine tasks only. Tasks are thus unbundled.

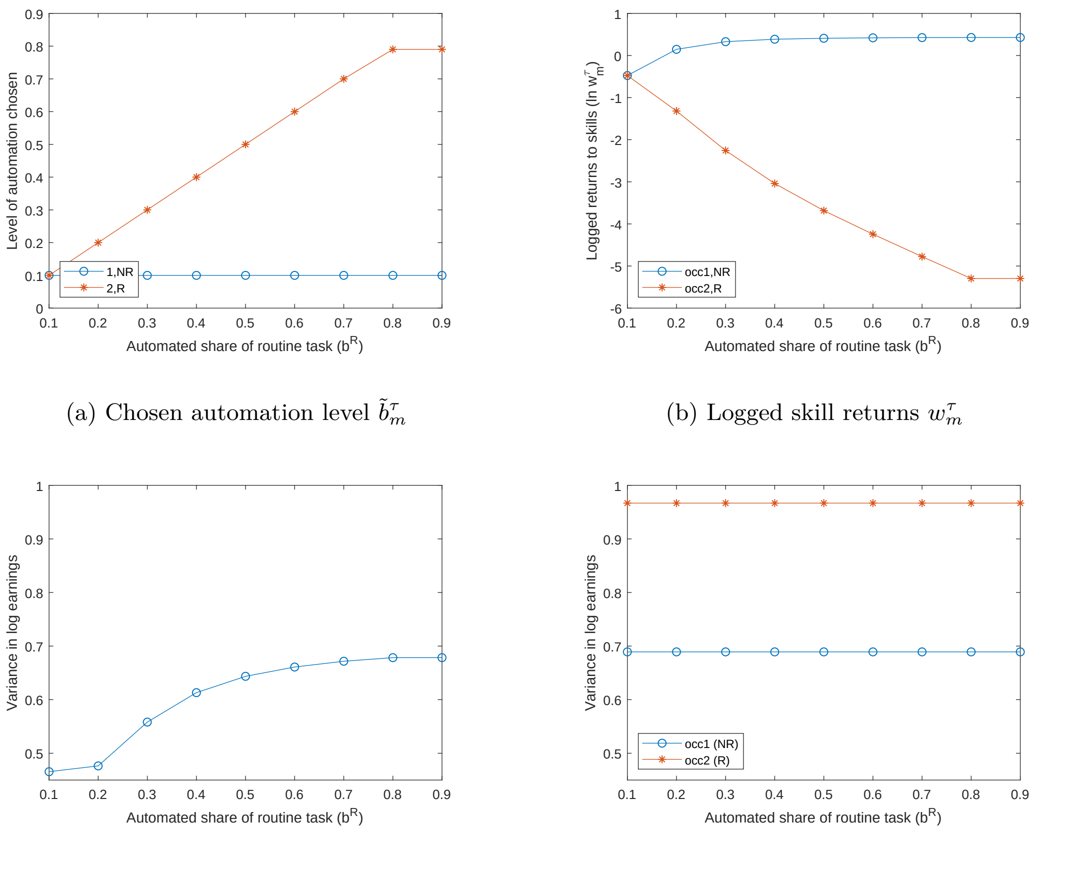
In this equilibrium, automation affects skill returns similarly to in the bundled model. However, while total inequality (as measured by the variance in log earnings) increases, within-occupation inequality is not affected by automation. Consider that inequality can stem from either composition or price effects. By composition effects, I mean the impact from changing skill endowments in the

[19]However, note that the parametrization, and in particular the distribution of skills, is not estimated from data. The reader is advised to take this simply to mean that the model is able to account for varying patterns of inequality both within and between occupations. It is possible to discuss inequality using the model, and a future aim is to relate the model more clearly to data.

workforce, and by price effects, I mean the impact from changing skill prices. In the unbundled model, within-occupation inequality is only affected by composition. Since the same price is paid to the *one* skill applied in each occupation, any skill price changes will proportionally scale earnings in each occupation.

In this comparative statics exercise, I keep total composition fixed, and since there is no mobility (all R skills are applied in occupation 2 and all NR in occupation 1), the within-occupation variance is unaffected by the skill return changes induced by automation.

Total variance increases with automation, since skill returns diverge. Those with routine skills become poorer, and those with non-routine skills become richer.



(a) Chosen automation level $\tilde{b}^{\tau}_m$

(b) Logged skill returns w^{τ}_m

(c) Total variance in log earnings

(d) Within-occupation variance in log earnings

Figure 11: Some comparative statics in the unbundled model

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.

8 Discussion and conclusion

I started from some empirical observations: people have bundles of skills that they supply to one occupation, which, in turn, consists of a bundle of tasks. Automation occurs at the task level, while workers get their earnings from and make mobility decision regarding occupations. I incorporated these two features into a rich but yet standard, nested CES structure, complementing related work such as Acemoglu & Restrepo (2019).[20] Although a similar model is being independently

[20] More on the connection with Acemoglu & Restrepo (2019) in appendix section H

and simultaneously developed by Edmond & Mongey (2020), my model differs in that I allow for labor replacing technologies in tasks within occupations. As a result, I can discuss how occupation specific task returns and inequality are affected by automation.

I find that skill returns are indeed occupation specific. I also find that they evolve differently across occupations in response to automation, although in both occupations, routine skill returns decline and non-routine increase when automation happens.

It is also possible to explore inequality, both within and between occupations. For the chosen parametrization, the within-occupation variance in the low-routine occupation 1 (shop attendants) increases while it exhibits a u-shape in the high-routine occupation 2 (machine operators).

The parametrization is, of course, not innocuous. Different types of distributions of labor work and give similar intuitions - e.g. uniform distribution, different versions of truncated extreme value and truncated log-normal.[21] The common, necessary feature is that they have to be bounded.[22] Another set of parameters that may influence results are the elasticities of substitution. A more elaborate description of how I view them can be found in appendix section E. In short, σ seems to be important. When σ is low (as in the baseline model above, where $\sigma = 0.4$), tasks are very complementary in each occupation's production. When σ becomes larger (0.8 or even above one), tasks become better substitutes. This leads to task returns declining less (or increasing) with automation. This makes sense: when tasks are more substitutable, the productivity of routine tasks does not diminish as quickly. Returns to both R and NR benefit from the increased production of task X_m^R. The differences in productivities of tasks between occupations (i.e. the different β_m^τ) are also augmented by a lower exponent $(1/\sigma)$.

Remaining and ongoing work includes calibrating the model to US data, and investigating whether automation is quantitatively important in accounting for observed increases in inequality, both within and between occupations, as documented by Edmond & Mongey (2020).

I also plan on providing some micro evidence that may support some of the predictions of the model – for example, that the labor share declines and then recuperates in routine-intensive occupations, as explained in appendix section D.

Lastly, one extension of the model may be to include a non-employment alternative for workers, enabling exploring the employment responses to automation.

In all, the proposed model is meant to capture important features in the labor market – the bundling of skills and tasks – and relate them to automation. As such, it complements a vivid literature on the impact of automation on workers.

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[21] Although so far, solutions when using truncated log-normal are imprecise.

[22] Exploring different assumptions on the distribution of labor is still work in progress. I also want to incorporate correlation between skills among workers.

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A Empirical work

A.1 Data

The main sample covers the US over the years 1968 - 2000, which is the interval where I have coverage of all variables listed below.

Task importance and automation are estimated from a regression using task content data from Atalay et al. (2020) (data availability 1950 - 2000), see details in appendix section A.2. Atalay et al. (2020) collect 7.8 million job vacancy listings from *New York Times* (1940 - 2000), *Wall Street Journal* (1940 - 1998) and *Boston Globe* (1960 - 1983) and classify them by occupation. They elicit task information by counting task words using various text analysis methods, and they verify their measures, partly by comparing them to existing, cross-sectional measures such as the O*NET and the DOT.

Capital-to-Output ratio is the capital stock divided by the gross domestic product, retrieved from University of Groningen and University of California, Davis (2020*a*, 2020*b*; Feenstra & Timmer 2015) (1950 - 2017).

Rental rate is the annual mean of daily data on the 10-year treasury constant maturity rate from the Board of Governors of the Federal Reserve System (US) (2020) (1962 - 2020).

A.2 Estimating the parameters of the task content of occupations

When we think about the task content of occupations, we think about how much of each task is needed in production, and how much of that task is performed by labour. This naturally lends itself to a nested CES structure as the one presented in section 3. Substituting for X_m^R and X_m^{NR} (as presented in equation (6) in Y_m (equation 3) gives the following equation:

$$Y_m = \left[\left[\gamma_m^{KR\ 1/\eta} (\lambda K_m^R)^{\frac{\eta-1}{\eta}} + \gamma_m^{LR\ 1/\eta} L_m^{R\ \frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}} + \left[\gamma_m^{KNR\ 1/\eta} (\lambda K_m^{NR})^{\frac{\eta-1}{\eta}} + \gamma_m^{LNR\ 1/\eta} L_m^{NR\ \frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where

$$\gamma_m^{KR} = \beta_m^{\frac{\eta-1}{\sigma-1}} b^R \tag{34}$$

$$\gamma_m^{LR} = \beta_m^{\frac{\eta-1}{\sigma-1}} (1 - b^R) \tag{35}$$

and similarly for task group NR. This suggests that the gammas can be interpreted as the importance of capital and labor, respectively, in each task, taking into account the task's importance in production. In short, it is the share of a day's production performed by some task, multiplied by the share of that task that is done by capital or labour.

Our notion of automation is that the b^R, b^{NR} change over time. Putting time subscripts on the b_t^R, we can rewrite equation 35 in logs

$$ln\gamma_{mt}^{LR} = \frac{\eta - 1}{\sigma - 1} ln\beta_m + ln(1 - b_t^R) \tag{36}$$

If we have a time series of task content for several occupations, we can estimate equation 36 by running the logged task content on occupation and time dummies.

$$ln(TC_{mt}^R) = \alpha_m + \delta_t + u_{mt} \tag{37}$$

Meaning that I can construct

$$b_t^R = 1 - exp\{\hat{\delta}_t\}$$
$$\beta_m = exp\left\{\hat{\alpha}_m - \frac{\eta - 1}{\sigma - 1}\right\}$$

for some values of η, σ. I normalize β_m to be in $[0, 1]$.[23] See figures 1 and 2 for the resulting b_t^τ and β_m.

A.3 Calibration plan

My preliminary calibration sets out to match the capital-output ratio from 1968, using mainly the capital-augmenting productivity factor λ. Table 2 lays out the parameters I get from the data, which ones I pick as exogenous and which one I calibrate.

I converge on the values in Table 2 by the following procedure: I visually explore what level of capital-augmenting productivity (λ) will produce a capital-output ratio similar to the one observed in the data. The resulting plots are found in Figure 12. Panel 12a shows the capital-output ratio for a range of values of λ for the initial specification of elasticities of substitution, namely $\sigma = 0.4$ and $\varepsilon = 0.2$. However, aiming for $K/Y = 3.58$ (see Table 3) means λ will have to be low (below one). I anticipate that using $\lambda < 1$ will prevent occupational firms from automating automatable tasks. Think about equation 12. If λ is very low, effective capital levels are low, whilst labor is relatively abundant. Firms will therefore rather use plentiful labor than scarce capital, and choose not to automate very much. I don't like this equilibrium outcome – I believe it is more reasonable that capital is plentiful (in developed economies over the last few decades), so that firms *want* to automate when technology allows.

By raising the elasticities of substitution between occupational goods Y_m and between R and NR tasks, respectively, I enable reaching $K/Y = 3.58$ with $\lambda > 1$, as evident in panel 12b. I confirm that the comparative statics are similar to my baseline comparative statics in section 6 (omitted from this version).

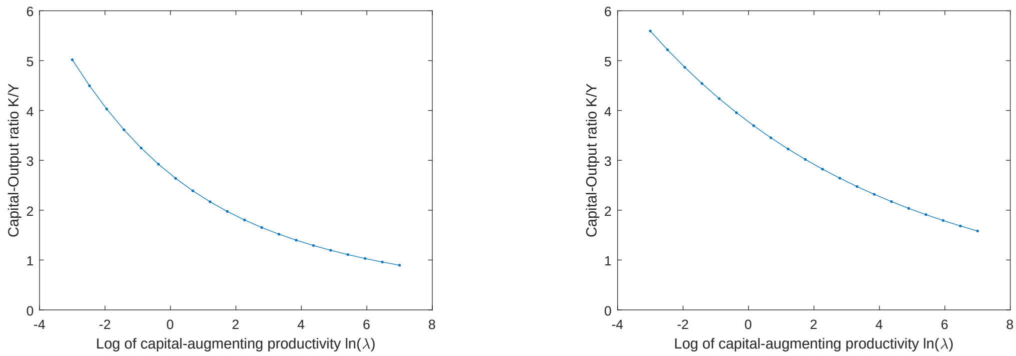
[23] I run regression 37 for both routine and non-routine task content. Then I construct $\tilde{\beta}_m^R$ from the routine task content and $\tilde{\beta}_m^{NR}$ from the non-routine task content. Then I normalize each of these as follows:

$$\beta_m = \frac{\tilde{\beta}_m^R}{\tilde{\beta}_m^R + \tilde{\beta}_m^{NR}}$$
$$1 - \beta_m = \frac{\tilde{\beta}_m^{NR}}{\tilde{\beta}_m^R + \tilde{\beta}_m^{NR}}$$

		Routine	Non-routine
From data (1968)			
Importance of routine tasks in occ 1	β_1	0.33	
Importance of routine tasks in occ 2	β_2	0.62	
Automation	b^τ	0.41	0.11
Rental rate	r	0.056	
Exogenous			
Skill distribution family	$l_i^\tau \sim$	Weibull	
Skill distribution scale		1	1
Skill distribution shape		3	3
Elasticity of substitution (EoS) between occupational goods Y_m in final good production	ε	0.6	
EoS between task groups X_m^R, X_m^{NR} in occupational production	σ	0.7	
EoS between smaller tasks in task production	η	0.9	
Price of final good	P	1	
Calibrated (preliminary)			
Capital augmenting factor	λ	1.50	
Derived from parameters listed above			
Rate of conversion between final good and capital	$\gamma = P/r$	17.86	

Table 2: Parameters (preliminary)

Notes: The importance of each task and automation are estimated from task content data from Atalay et al. (2020), see details in section A.2. Because importance parameters are normalized, they do not depend on parameters but data only. Automation is relative to the level in 1950 – in 1950, both routine and non-routine automation levels are set to zero. Rental rate is is the annual mean of daily data on the 10-year treasury constant maturity rate from the Board of Governors of the Federal Reserve System (US) (2020). The two types of labor (or skills) l_i^R and l_i^{NR} are independently distributed. The low value of ε represents that consumers like variety. The relatively high value of η represents that small tasks are more easily substituted.



(a) $\sigma = 0.4, \varepsilon = 0.2$

(b) $\sigma = 0.7, \varepsilon = 0.6$

Figure 12: Capital-output ratio K/Y and capital-augmenting productivity λ

Notes: The model is solved using the values of b^τ, β_m and r from the 1968 data (see Table 2). Apart from σ, ε and λ, parameters are as presented in Table 2. The difference between the two panels are the values of σ and ε.

Next steps Work in progress includes continuous work on calibrating and estimating parameters. The aim is to evaluate the quantitative contribution of automation in a bundling setting, on the

	Model (preliminary)	Data (1968)
Capital-to-Output ratio	3.58	3.58

Table 3: Matched moment (preliminary)

Notes: The capital-to-output ratio is the capital stock divided by real GDP, both at constant national prices, retrieved from University of Groningen and University of California, Davis (2020*a*) and University of Groningen and University of California, Davis (2020*b*) (Feenstra & Timmer 2015).

evolution of, e.g., inequality (both within and between occupations).

B Solving the model: Details

B.1 Labor supply in each occupation

The effective labor is jointly distributed according to some pdf $f(l_i^R, l_i^{NR})$ with some finite upper bound. Given the cutoff described in section 3.1, we can compute the labor supply of effective task labor to each occupation as the double integrals

$$L_1^R = \int_0^\infty \int_0^{ul_i^{NR}} l_i^R f(l_i^R, l_i^{NR}) dl_i^R dl_i^{NR} \tag{38}$$

$$L_1^{NR} = \int_0^\infty \int_{l_i^R/u}^\infty l_i^{NR} f(l_i^R, l_i^{NR}) dl_i^{NR} dl_i^R \tag{39}$$

$$L_2^R = L^R - L_1^R \tag{40}$$

$$L_2^{NR} = L^{NR} - L_1^{NR}, \tag{41}$$

where there are two occupations and Assumption 1 holds.

B.2 Algorithm for solving the decentralized problem

1. I set parameters in accordance with table 1
2. I guess that firms automate fully – i.e. set $\tilde{b}_m^\tau = b^\tau$ for both tasks and both occupations.
3. I guess skill returns w_m^τ and feed into a function which does the following:
   (a) Given skill returns w_m^τ, constructs cutoff u using equation 2,
   (b) Given cutoff u and labor distribution parameters, constructs occupational labor supply L_m^τ using equations 38 and 39,
   (c) Given skill returns w_m^τ, occupational labor supply L_m^τ, rental rate r, automation b^τ, capital augmenting factor λ, finds capital level in each occupation in each task K_m^τ, using the ratio of first order conditions 9 and 10,
   (d) Given occupational labor L_m^τ, capital K_m^τ, automation $\tilde{b}_m^\tau$, capital augmenting factor λ, finds task output X_m^τ (equation 26), and then sequentially occupational output Y_m (equation 27), final output Y (equation 28), occupational prices p_m (equation 22),
   (e) Writes down first order conditions for labor (equation 10).
4. Given the initial guess, `fsolve` finds the skill returns which sets the first order conditions equal to zero.

5. Now, I check whether the level of automation is consistent with optimal automation. Consider equation 12. If the automation level is *below or equal to* the ratio $\lambda K_m^\tau/(\lambda K_m^\tau + L_m^\tau)$, then firms do indeed automate fully. Intuitively, capital is plentiful, so firms want to automate as much as they can. If so, then I record the solution as the equilibrium.

6. If, instead, automation is *above* the ratio $\lambda K_m^\tau/(\lambda K_m^\tau + L_m^\tau)$, then firms want to reduce automation. Intuitively, capital is scarce, so firms would rather use labor. Then, I set automation levels (for the task and occupation in question) equal to the ratio $\lambda K_m^\tau/(\lambda K_m^\tau + L_m^\tau$, and solve the problem again from 3.

B.3 Algorithm for solving the social planner problem

1. I set parameters in accordance with table 1

2. I guess a cutoff u, capital levels K_m^τ, optimal automation levels $\tilde{b}_m^\tau$. I restrict the guesses to be non-negative, and the automation levels to be feasible, given the technological constraint (i.e. $\tilde{b}_m^\tau \leq b^\tau$). I feed the guess into a function which does the following:

   (a) Given cutoff u and labor distribution parameters, constructs occupational labor supply L_m^τ using equations 38 and 39,

   (b) Given capital levels K_m^τ, finds aggregate capital K (equation 30),

   (c) Given occupational labor L_m^τ, capital K_m^τ and K, automation $\tilde{b}_m^\tau$, capital augmenting factor λ, finds task output X_m^τ (equation 26), and then sequentially occupational output Y_m (equation 27), final output Y (equation 28), and consumption C (equation 15 holding with equality).

3. I minimize negative consumption using `fmincon`.

C Social planner's problem

The social planner seeks to maximize consumption C subject to the available resources and technology in the economy. The social planner also faces the constraint that labor is bundled in workers, and that tasks are bundled in occupations.

The social planner does not have to ensure that workers want to stay in their occupation. She could, in theory, construct multiple cutoffs along the l_i^R/l_i^{NR} line, dividing workers into many small intervals of occupation 1 and 2 workers. However, the social planner wants to put workers where they are most productive. This leads to a single cutoff between the occupations.

Therefore, we can represent the social planner's problem in the following way:

$$
\begin{aligned}
\max_{\{K_m^\tau, \tilde{b}_m^\tau\}_{m=1,2,\tau=R,NR},u} \quad & C \\
\text{s.t.} \quad & C = Y - K/\gamma \\
& Y = \left[\sum_{m=1}^{M} Y_m^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
& Y_m = \left[\beta_m^{1/\sigma} X_m^{R\ \frac{\sigma-1}{\sigma}} + (1-\beta_m)^{1/\sigma} X_m^{NR\ \frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
& X_m^\tau = \left[\tilde{b}_m^{\tau\ 1/\eta} (\lambda K_m^\tau)^{\frac{\eta-1}{\eta}} + (1-\tilde{b}_m^\tau)^{1/\eta} L_m^{\tau\ \frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
& L_1^R = \int_0^\infty \int_0^{u l_i^{NR}} l_i^R f(l_i^R, l_i^{NR}) dl_i^R dl_i^{NR} \\
& L_1^{NR} = \int_0^\infty \int_{l_i^R/u}^\infty l_i^{NR} f(l_i^R, l_i^{NR}) dl_i^{NR} dl_i^R \\
& L_2^R = L^R - L_1^R \\
& L_2^{NR} = L^{NR} - L_1^{NR} \\
& \tilde{b}_m^\tau \leq b^\tau
\end{aligned}
$$

We make assumption 1. The first-order condition for the cutoff u has the following structure:

$$\sum_m \sum_\tau \frac{\partial Y}{\partial Y_m} \frac{\partial Y_m}{\partial X_m^\tau} \frac{\partial X_m^\tau}{\partial L_m^\tau} \frac{\partial L_m^\tau}{\partial \underline{u}} = 0$$

The rest of the first-order conditions follow those of the decentralized problem. Since there are no market imperfections, the social planner and the decentralized solutions are equal. I solve the social planner's problem using the algorithm in section B.3.

D Labor share

The labor share is the total wage bill divided by the total value added in the economy:

$$
\begin{aligned}
\frac{wL}{PY} &= \frac{\sum_m \sum_\tau \frac{\partial Y}{\partial Y_m} \frac{\partial Y_m}{\partial X_m^\tau} \frac{\partial X_m^\tau}{\partial L_m^\tau} L_m^\tau}{Y} && (42) \\
&= \sum_m \sum_\tau \underbrace{\frac{\partial Y}{\partial Y_m} \frac{Y_m}{Y}}_{\substack{\text{weight of occ } m \\ \text{in the economy}}} \underbrace{\frac{\partial Y_m}{\partial X_m^\tau} \frac{X_m^\tau}{Y_m}}_{\substack{\text{weight of task} \\ \tau \text{ in occ } m}} \underbrace{\frac{\partial X_m^\tau}{\partial L_m^\tau} \frac{L_m^\tau}{X_m^\tau}}_{\substack{\text{labor share in} \\ \text{task } \tau \text{ in occ } m}} && (43)
\end{aligned}
$$

Where the second equality is obtained by multiplying and dividing the expression by Y_m and X_m^τ. The labor share is thus computed as the labor share in task τ in occupation m, multiplied by τ's weight in occupation m, multiplied by m's weight in the whole economy. The weights are the elasticities, and are clearly closely related to the marginal productivities. As before, P is normalised to one.

Figure 13, plots the labor share within each occupation, and the total labor share in the economy. The labor share drops in the early stages of automation, after which it stabilizes and slightly increases at the later stages of automation. Figure 14 shows the labor share decomposed into its numerator, the wage bill, and denominator, the value added. In the beginning, there are large benefits to firms when they are able to use cheap capital rather than (expensive) labor in more tasks. This means their value added increases more than proportionally to the wage bill, and the labor share declines. As automation in task R becomes high, returns to R (w_m^R) become small. Thus, the cost saving of using capital instead of labor in more tasks is smaller, and value added does not increase as before.[24]

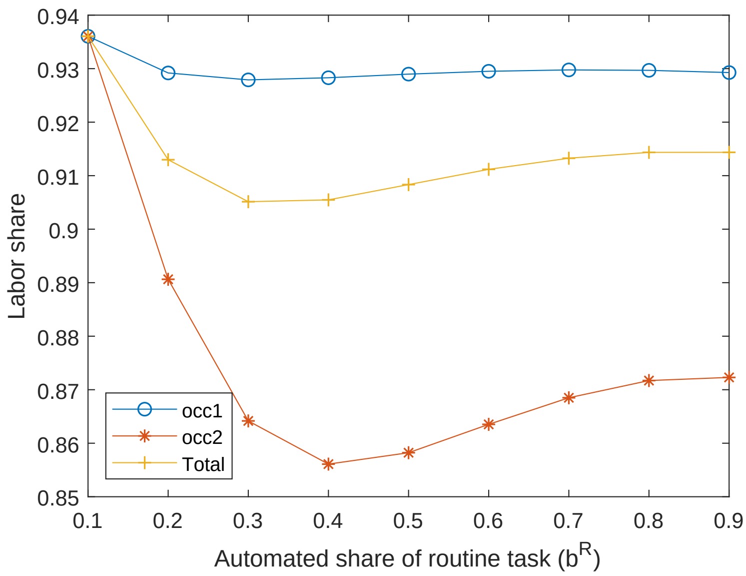


Figure 13: Labor share for both occupations and in total

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.The labor share is computed as wage bill divided by value added.

[24]You might ask why firms in occupation 2 keep automating after $b^R = 0.2$, since their value added shrinks here. But consider that this is a general equilibrium outcome. Firms automate up to the point where $\tilde{b}_m^\tau = \frac{\lambda K_m^\tau}{\lambda K_m^\tau + L_m^\tau}$ so long as this is lower than or equal to the technological limit b^τ. Given that the technological limit increases, and capital and labor are reshuffled between occupations, it might well be that value added decreases for firms in one occupation. Profits are still, of course, zero.

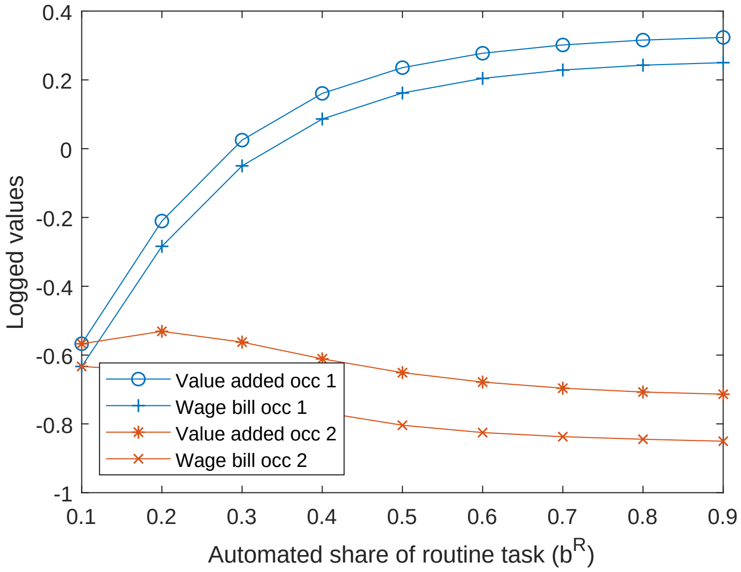


Figure 14: Log wage bill and log value added for both occupations

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.

In occupation 2, which is routine intensive, the labor share falls more than in occupation 1. Figure 15 explores the decompositions of the labor shares (as given in equation 43). In both occupations, the labor share in routine tasks and the weight of routine tasks in the occupation shrinks, because of displacement and complementarity between factors, respectively.[25]

The higher importance of routine tasks in occupation 2 means the impact of automation is larger in this occupation.[26] In occupation 1 – the service occupation – the weight of routine tasks approaches zero from an initially low level. The labor share therefore stabilizes quickly.

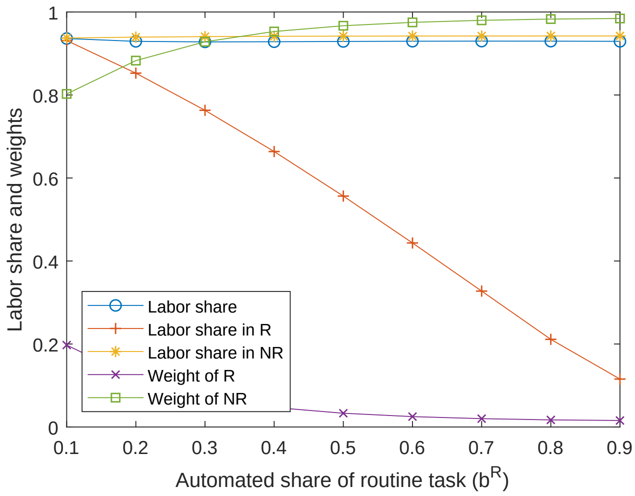
For machine operators – occupation 2 – the fall in the labor share is larger, but it recuperates somewhat after passing 40% automation. This is because NR tasks are now more important than R, as evident from figure 15b. As the weight of NR tasks in occupation 2 continues to rise, more weight is put on the large within-non-routine-task labor share, leading to a slight increase in total labor share in occupation 2.

[25]When R is automated, this unambiguously leads to lower labor share in R. This is the displacement effect. To understand the impact on the *weight* of R in occupation m, consider that it is given by

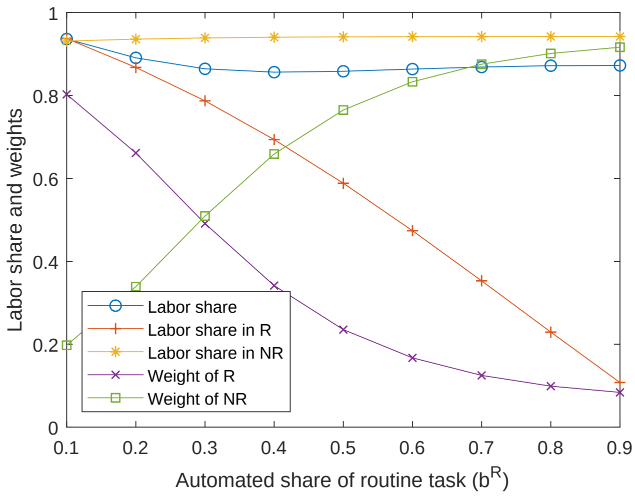
$$\frac{\partial Y_m}{\partial X_m^R}\frac{X_m^R}{Y_m} = \left(\frac{\beta_m Y_m}{X_m^R}\right)^{1/\sigma}\frac{X_m^R}{Y_m}$$
$$= \beta_m^{1/\sigma}\left(\frac{Y_m}{X_m^R}\right)^{\frac{1-\sigma}{\sigma}}$$

As R is automated, the quantity produced X_m^R increases relative to occupational output Y_m (due to complementarity between R and NR). This reduces the weight of R in occupation m, since $\sigma < 1$.

[26]This might be clearer when considering a Cobb-Douglas set up as in appendix section E.1. In the Cobb-Douglas case, the labor share in occupation m is simply $\beta_m(1 - b^R) + (1 - \beta_m)(1 - b^{NR})$. Here, it is clear that the weights of tasks R and NR are fixed at β_m and $(1 - \beta_m)$. Automation only changes the occupational labor share via the *within-task* labor share. The labor share within task R is $(1 - b^R)$, and the reason for the steeper decline in the occupational labor share in occupation 2 as compared to 1 is simply that the (fixed) weight on R is higher in 2. That is, $\beta_2 > \beta_1$.



(a) Decomposition of the labor share in occupation 1



(b) Decomposition of the labor share in occupation 2

Figure 15: Decompositions of labor shares in each occupation

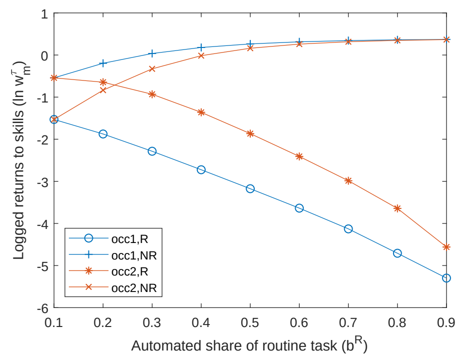
Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.The weight of task τ is the elasticity of occupational output Y_m w.r.t. the task X_m^τ, i.e. $\frac{\partial Y_m}{\partial X_m^\tau}\frac{X_m^\tau}{Y_m}$. labor share in task τ is the elasticity of task X_m^τ w.r.t. labor L_m^τ, i.e. $\frac{\partial X_m^\tau}{\partial L_m^\tau}\frac{L_m^\tau}{X_m^\tau}$.

E Elasticities of substitution

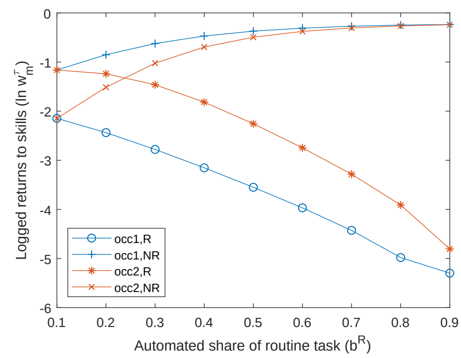
Clearly, the degree of complementarity between factors matters. Recall that they are parametrized to $(\varepsilon = 0.2, \sigma = 0.4, \eta = 0.9)$ in the baseline solutions presented in section 6. If all three elasticities of substitution $(\varepsilon, \sigma, \eta)$ approach unity, production is Cobb-Douglas throughout. This scenario is depicted below in section E.1, and exhibits uniformly increasing returns to both tasks in both occupations.

It seems to be σ that drives the qualitative pattern of the result most. Low values of σ (0.2) produce similar results as the baseline above (i.e. where $\sigma = 0.4$). Higher values of σ means task returns increase more and more. For $\sigma = 1.1$, all task returns increase throughout.

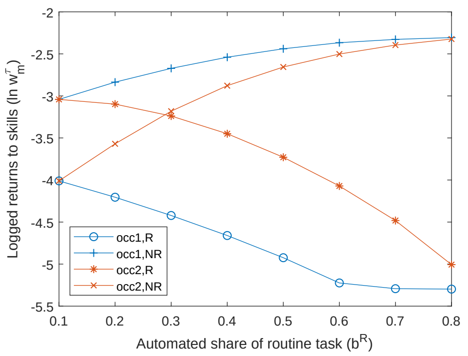
Varying η between 0.2 and 1.1 or varying ε between 0.4 and 0.8, preserves the qualitative pattern of skill returns.



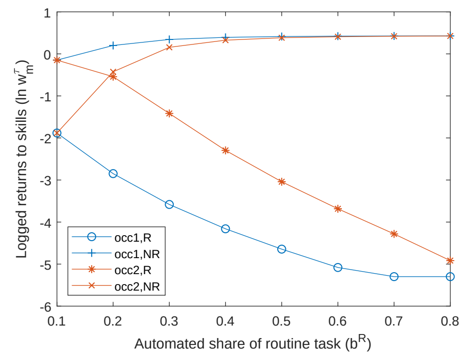
(a) Task returns for $\varepsilon = 0.4$



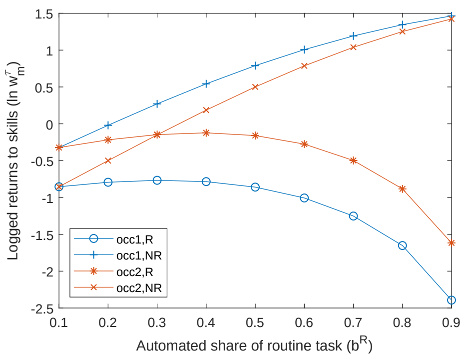
(b) Task returns for $\varepsilon = 0.6$



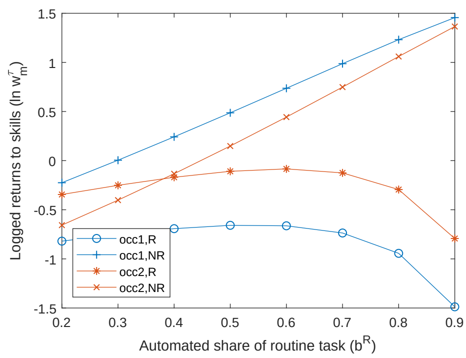
(c) Task returns for $\varepsilon = 0.8$



(d) Task returns for $\sigma = 0.2$

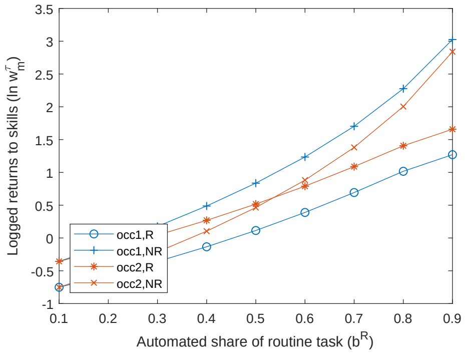


(e) Task returns for $\sigma = 0.8$

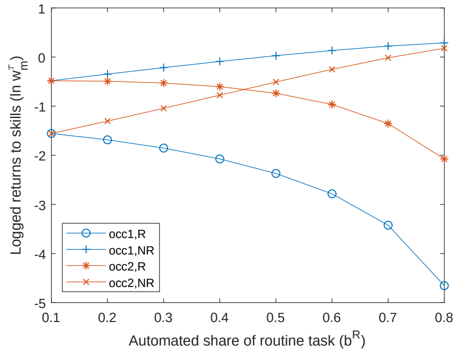


(f) Task returns for $\sigma = 0.95$

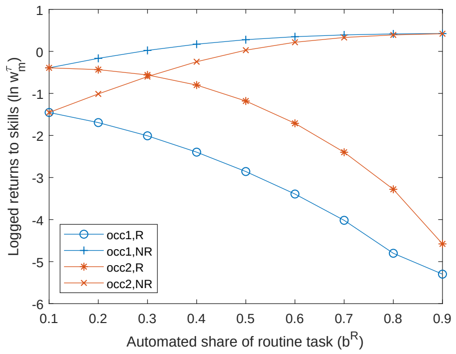
Figure 16: Task returns for different variations of the elasticities of substitution



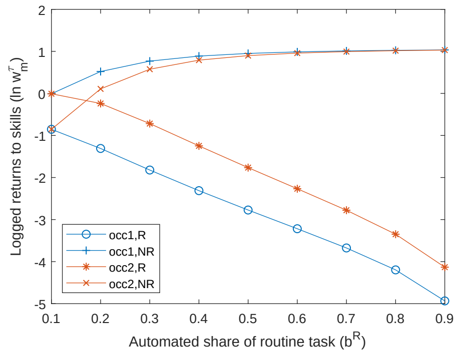
(g) Task returns for $\sigma = 1.1$



(h) Task returns for $\eta = 0.2$



(i) Task returns for $\eta = 0.6$



(j) Task returns for $\eta = 1.1$

Figure 16: Task returns for different variations of the elasticities of substitution

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.Unless otherwise stated in the caption of each panel, ($\varepsilon = 0.2, \sigma = 0.4, \eta = 0.9$). Whenever the scale for b^R ends at 0.8, there were no solutions for the model when $b^R = 0.9$.

E.1 Cobb-Douglas

Here, the model is Cobb-Douglas, meaning $\varepsilon = \sigma = \eta = 1$. Clearly, all task returns increase throughout, which contrasts with the case of non-unity elasticities of substitution in the different layers of production.

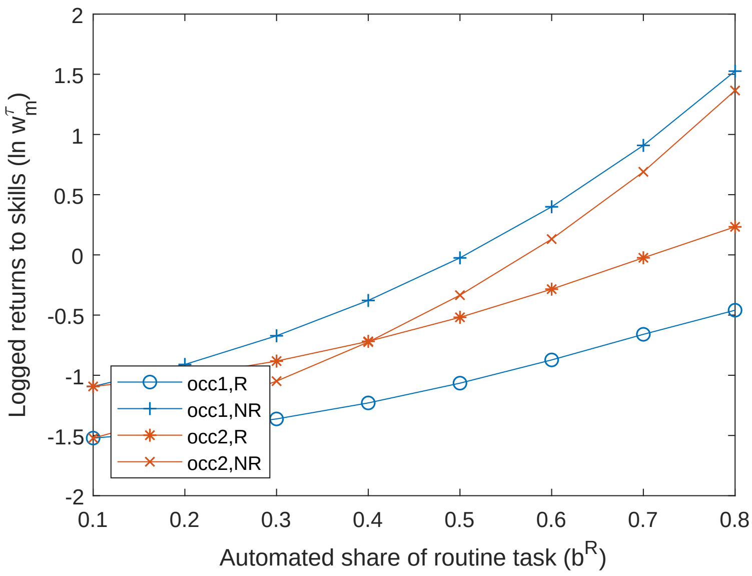


Figure 17: Logged skill returns w^{τ}_{m} in the Cobb-Douglas case

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.The only difference is that all elasticities of substitution ϵ, σ, η go to one. The scale for b^R end at 0.8 since there were no solutions for the model when $b^R = 0.9$.

A note on optimal automation under Cobb-Douglas There is always complete automation under Cobb-Douglas. To see this, consider the firm's first-order conditions presented in section 3.3. Consider 11:

$$\mu^{\tau}_{m} = \frac{\partial Y}{\partial Y_m} \frac{\partial Y_m}{\partial X^{\tau}_{m}} \frac{\partial X^{\tau}_{m}}{\partial b^{\tau}_{m}} \tag{44}$$

Where μ^{τ}_{m} is the Lagrange multiplier on the constraint $b^{\tau}_{m} \leq b^{\tau}$. Say $\mu^{\tau}_{m} = 0$, so that b^{τ}_{m} is free to be anywhere $\in [0, b^{\tau}]$.

None of the first two marginal productivities in the product 44 are zero. In the Cobb-Douglas case, $\frac{\partial X^{\tau}_{m}}{\partial b^{\tau}_{m}} = 0$ only if $K^{\tau}_{m} = L^{\tau}_{m}$. Thus

$$b^{\tau}_{m} \begin{cases} = 0 & \text{if } K^{\tau}_{m} < L^{\tau}_{m} \\ \in [0, b^{\tau}] & \text{if } K^{\tau}_{m} = L^{\tau}_{m} \\ = b^{\tau} & \text{if } K^{\tau}_{m} > L^{\tau}_{m} \end{cases} \tag{45}$$

Consider the first possible case: $b^R_1 = 0$ but all other $b^{\tau}_{m} = b^{\tau} > 0$. From the first-order conditions for capital and labour we know that

$$b^R_1 = \frac{(1/\gamma) K^R_1}{(1/\gamma) K^R_1 + \lambda^R L^R_1} \tag{46}$$

Where λ^R is the Lagrange multiplier attached to the labour supply constraint $L_1^R + L_2^R \leq L^R$. In order for 46 to hold for $b_1^R = 0$, Either $K_1^R = 0$ or $\lambda^R \to \infty$. But if $K_1^R = 0$, its marginal product approaches infinity. Then, K_1^R will increase from zero. If, on the other hand, λ^R approaches infinity, the marginal productivity of L_1^R must approach infinity, meaning that $L_1^R \to 0$. But the same goes for L_2^R – this, too must approach infinite productivity and therefore must be zero. But this is impossible: by complementary slackness, if $L_1^R = 0$ and $\lambda^R \neq 0$, then $L_2^R = L^R > 0$. So we have ruled out both possibilities that could result in $b_1^R = 0$ alone. Similar arguments preclude that any of the $b_m^\tau = 0$ on their own or simultaneously.

Therefore, unless $K_m^\tau = L_m^\tau$ exactly, firms (or the social planner) will always automate up until the technological frontier b^τ.

F Can marginal productivities of labor equalize across occupations? Details

F.1 A simplified example

First, let us look at a simplified example to illustrate the bundling constraint: Here, there is no capital, and all production functions are Cobb-Douglas.[27] Figure 18 depicts four panels, each with a different amount of discrete workers, for illustration.

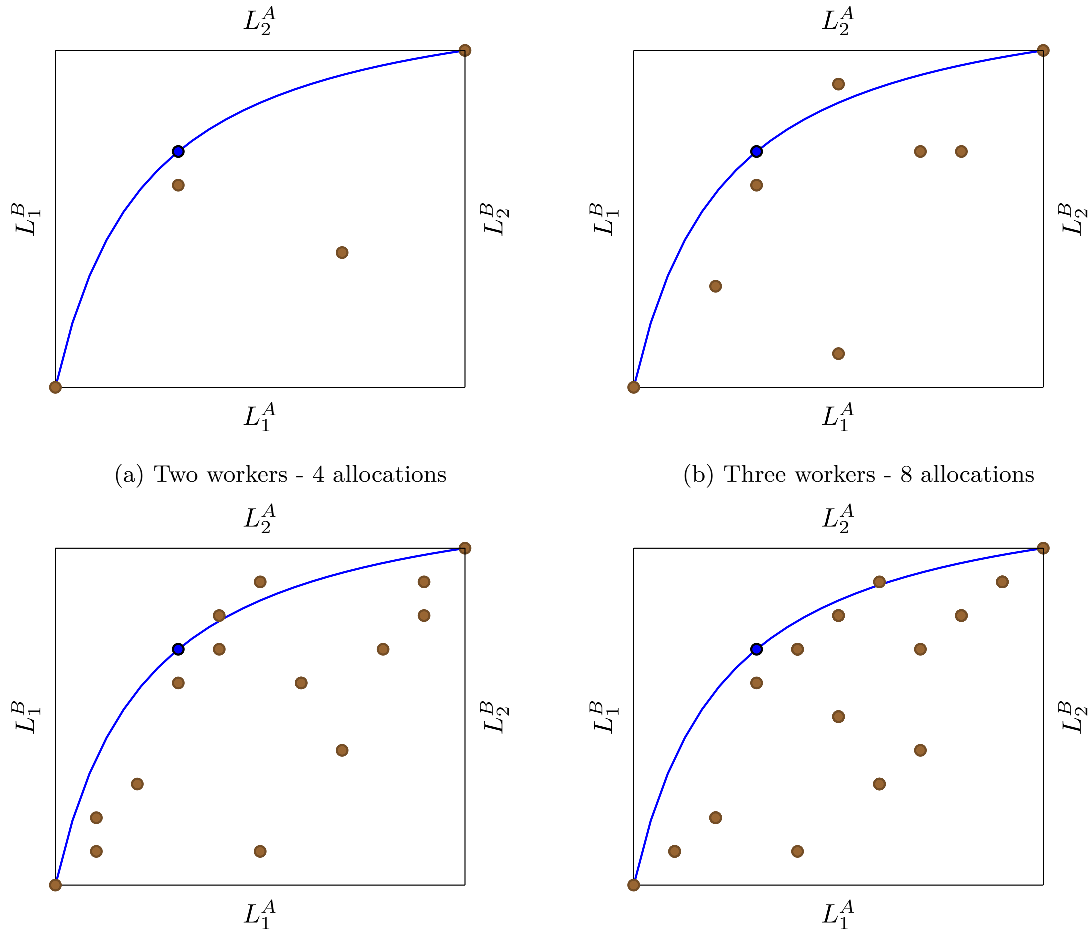
The brown dots show the feasible allocations of labor, given the endowments of each worker. It is clear that the number of allocations increase with number of workers. The set of potential allocations will converge to a convex set as the number of workers goes to infinity.

The blue curve is the contract curve - where the marginal rates of substitution between R and NR labor equalizes across occupations. The social planner optimum,which coincides with the market solution, where marginal productivities of labor are equalized across occupations, is marked with a blue dot along the contract curve.

If the blue, unbundled, optimal allocation is in the set of feasible allocations, then it can obtain as an equilibrium in the case with bundled skills. Whether or not it is in the set of feasible allocations depend on the distribution of labor among workers.

[27] That is, they are special cases of the CES production functions I have described, where all the elasticities of substitution $\varepsilon, \sigma, \eta$ go to one.

$$Y = Y_1^{1/2} Y_2^{1/2}$$
$$Y_m = X_m^{R\ \beta_m} X_m^{NR\ (1-\beta_m)}$$
$$X_m^\tau = L_m^\tau$$



(a) Two workers - 4 allocations

(b) Three workers - 8 allocations

(c) Four workers - 16 allocations

(d) Five workers - 32 allocations

Figure 18: Potential allocations of labor

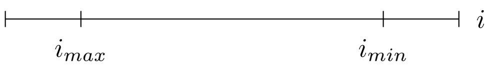
Notes: The brown dots represent the potential allocations of labor when labor is bundled. The blue line represents the contract curve in the problem with unbundled labor. Note that the allocations need not be unique - that is, a single point may have several different combinations of workers, depending on the distribution of labor bundles among the workers.

F.2 Drawing the lens

Edmond and Mongey (2020) use a clever method to draw up this convex lens.

First, solve the unbundled problem to produce a candidate solution.[28] Take the candidate $L_1^{R\ cand}$ and think about what the minimum level of L_1^{NR} that goes with your candidate L_1^R, as follows:

Order the interval of workers i so that l^R/l^{NR} is increasing over the interval. Given $L_1^{R\ cand}$, the minimum amount of L_1^{NR} is achieved by starting from the highest l_i^R/l_i^{NR} and going down to some i_{min}. At this i_{min}, we have achieved $L_1^{R\ cand}$ quickly, without adding much L_1^{NR}, since we added the workers with high R relative to NR labor first.



[28] If you solve a Cobb-Douglas problem, you only have to find the labor L_m^τ since it is independent of capital and automation levels.

Analytically (or numerically), we find this i_{min} by solving[29]

$$L_1^{R\ cand} = \int_{i_{min}}^{1} l^R(i)di \tag{47}$$

for i_{min}, and we use this i_{min} to find the minimum L_1^{NR} associated with our $L_1^{R\ cand}$. Call this $\underline{B}(L_1^R)$.

$$\underline{B}(L_1^R) = \int_{i_{min}(L_1^R)}^{1} l^{NR}(i)di \tag{48}$$

Similary, we find the maximum amount of L_1^{NR} associated with $L_1^{R\ cand}$. Call this $\bar{B}(L_1^R)$.

$$\bar{B}(L_1^R) = \int_{0}^{i_{max}(L_1^R)} l^{NR}(i)di \tag{49}$$

$\underline{B}(L_1^R)$ and $\bar{B}(L_1^R)$ are the borders of the lens, as depicted in figure 5. If the candidate solution is inside the lens, i.e. if $L_1^{NR\ cand} \in [\underline{B}(L_1^{R\ cand}), \bar{B}(L_1^{R\ cand})]$, then the undbundled solution is the optimal one. That is, the bundling constraint does not bind, and solving the bundled problem will give the same solution as solving the unbundled.

F.3 Unbundled problem

As suggested in figure 5, the equilibria I find in my comparative statics, using parameters from Table 1, are all constrained by the bundling constraint. The unbundled problem gives equilibrium allocation outside the feasible set, as evident in figure 5. These allocations are produced by letting the social planner freely choose $(L_m^\tau, K_m^\tau, \tilde{b}_m^\tau)$ subject to the aggregate labor market constraints $L^\tau = \sum_m L_m^\tau$ and the technological constraint $\tilde{b}_m^\tau \leq b^\tau$.

The comparative statics from that exercise are found in figure 22.

G Analytical solution

If routine and non-routine skills are independently, identically distributed according to the Weibull distribution with shape and scale parameters of $[(1,1),(1,1)]$, then the probability density functions of skill τ is $f_\tau = e^{-\tau}$ for $\tau = R, NR$. The resulting labor supplies in each occupation are computed

[29] When we solve this numerically or analytically, we need to reformulate the problem in terms of labor distributions, rather than integrating over persons i. Then, we also reformulate the i_{max} and i_{min} accordingly.

as follows:

$$\begin{aligned}
L_2^R &= \int_0^\infty f_{NR}(l^{NR})\left(\int_{u\times l^N}^\infty l^R f_R(l^R)dl^R\right)dl^{NR} \\
&= \int_0^\infty e^{-l^{NR}}\left(\int_{u\times l^N}^\infty l^R e^{-l^R}dl^R\right)dl^{NR} \\
L_1^R &= L^R - L_2^R \\
&= 1 - L_2^R \\
L_2^{NR} &= \int_0^\infty f_R(l^R)\left(\int_0^{l^R/u} l^{NR} f_{NR}(l^{NR})dl^{NR}\right)dl^R \\
&= \int_0^\infty e^{-l^R}\left(\int_0^{l^R/u} l^{NR} e^{-l^{NR}}dl^{NR}\right)dl^R \\
L_1^{NR} &= L^{NR} - L_2^{NR} \\
&= 1 - L_2^{NR},
\end{aligned}$$

Where u is the cutoff, as above. The resulting labor supplies are

$$L_1^R = 1 - \frac{1}{(1+u)^2} \qquad\qquad L_1^{NR} = 1 - \frac{1}{(1+u)^2}$$

$$L_2^R = \frac{1}{(1+u)^2} \qquad\qquad L_2^{NR} = \frac{1}{(1+u)^2}$$

H Relating to Acemoglu & Restrepo (2019)

My model is, in part, equivalent to the model presented in Acemoglu & Restrepo (2019).

$$Y = \Pi(I,N)\left(\Gamma(I,N)^{1/\sigma}(A^L L)^{\frac{\sigma-1}{\sigma}} + (1-\Gamma(I,N))^{1/\sigma}(A^K K)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where Γ is the economy wide equivalent of my parameter $1-b^\tau$, namely the (labor) task content of production, and the σ used by Acemoglu & Restrepo (2019) is equivalent to my η - the elasticity of substitution between tasks (and the derived elasticity of substitution between capital and labor). In my model, both Π and A^L are set to one, and A^K is represented by λ.

The labor share in **?** is

$$\frac{WL}{Y} = \frac{1}{1 + \frac{1-\Gamma(I,N)}{\Gamma(I,N)}\left(\frac{R/A^K}{W/A^L}\right)^{1-\sigma}}$$

and in my model, the labor share of value added within a task within an occupation is the corresponding expression:

$$\frac{w_m^\tau L_m^\tau}{p_m^\tau X_m^\tau} = \frac{1}{1 + \frac{b^\tau}{1-b^\tau}\left(\frac{r/\lambda^{\frac{\eta-1}{\eta}}}{w_m^\tau}\right)^{1-\eta}}$$

where p_m^τ is the (shadow) price of task A in occupation m.[30] Thus, my model takes the CES structure presented by Acemoglu & Restrepo (2019), but treats it as one occupation, from which individuals can move their bundle of skills. The labor share for Acemoglu & Restrepo's (2019) whole economy is thus the labor share for *one* occupation in my economy.

[30]The labor share of production of task A in occupation m is derived as follows (where it is also clear that this is

I Auxiliary graphs

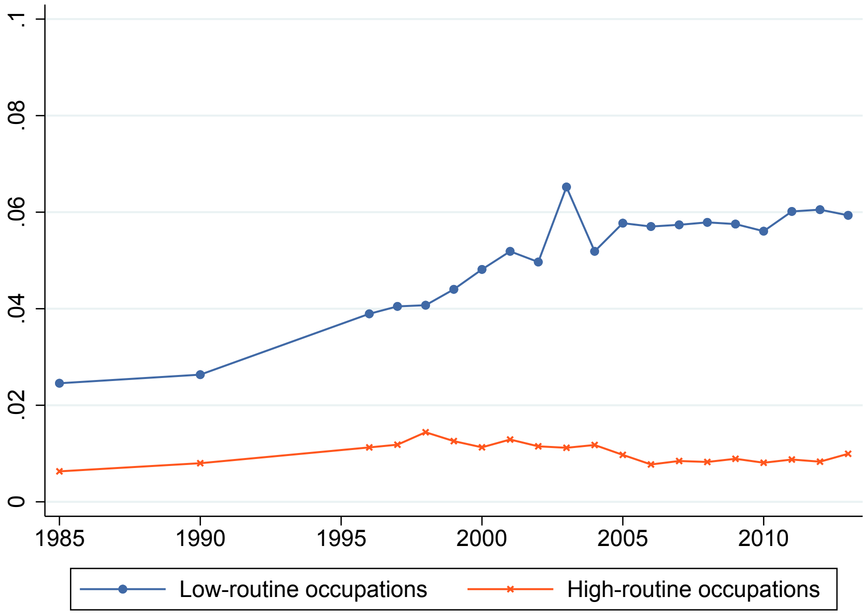


Figure 19: Between-occupation variance in real wages in low-routine and high-routine occupations

Notes: The figure plots the variance of residualized log of real wages (SEK) for a large, representative sample of the Swedish workforce (Wage Structure Statistics, details to come), divided into high- and low-skilled occupations. Real wages are residualized using a weighted regression of real wages on gender, whether or not born in Sweden and age. Weights are as recommended by Statistics Sweden. Wages above the 99.5th percentile are cut. Routine occupations are those with non-routine-to-routine ratio below 0.47, namely (SSYK 2012 in parentheses) Administration and customer service clerks (4), Mechanical manufacturing and transport workers, etc. (8), Building and manufacturing workers (7), Elementary occupations (9), Agricultural, horticultural, forestry and fishery workers (6). Non-routine occupations are the rest: Occupations requiring higher education qualifications or equivalent (3), Service, care and shop sales workers (5), Occupations requiring advanced level of higher education (2), Managers (1). The between variance is computed between SSYK 2012 four-digit occupations.

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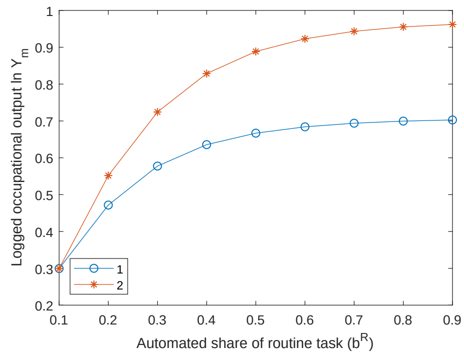
an elasticity):

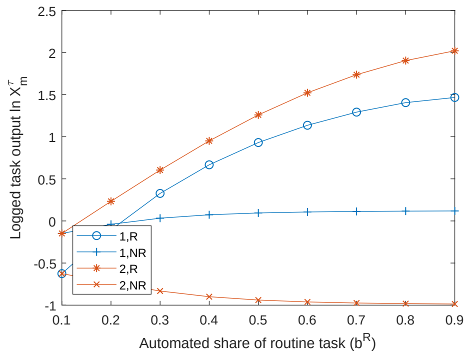
$$\frac{w_m^\tau L_m^\tau}{p_m^\tau X_m^\tau} = \frac{\frac{\partial Y}{\partial Y_m}\frac{\partial Y m}{\partial X_m^\tau}\frac{\partial X_m^\tau}{\partial L_m^\tau}L_m^\tau}{\frac{\partial Y}{\partial Y_m}\frac{\partial Y m}{\partial X_m^\tau}X_m^\tau} = \frac{\partial X_m^\tau}{\partial L_m^\tau}\frac{L_m^\tau}{X_m^\tau}$$

$$= (1-b^\tau)^{1/\eta}\left(\frac{X_m^\tau}{L_m^\tau}\right)^{\frac{1-\eta}{\eta}}$$

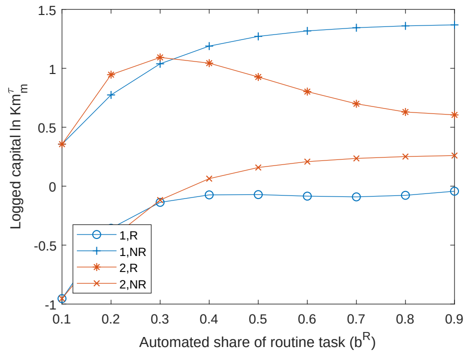
Substitute for $\frac{X_m^\tau}{L_m^\tau} = \left[b^{\tau 1/\eta}\left(\frac{K_m^\tau}{L_m^\tau}\right)^{\frac{\eta-1}{\eta}} + (1-b^\tau)^{1/\eta}\right]^{\frac{\eta}{\eta-1}}$, and then for the combined first-order conditions from the firm problem: $\frac{K_m^\tau}{L_m^\tau} = \left(\frac{w_m^\tau}{r}\right)^\eta \frac{b^\tau}{(1-b^\tau)}\lambda^{\eta-1}$.



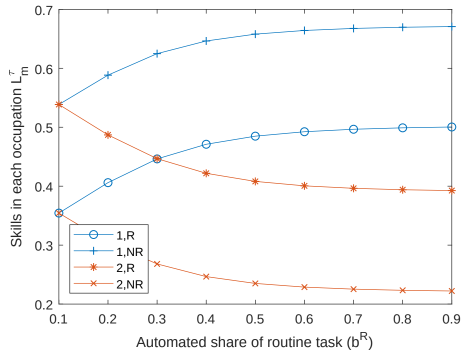
(a) Output Y_m



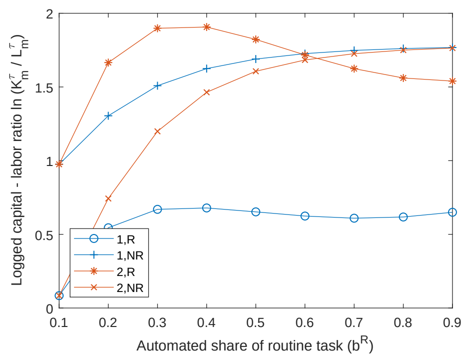
(b) Task output X_m^τ



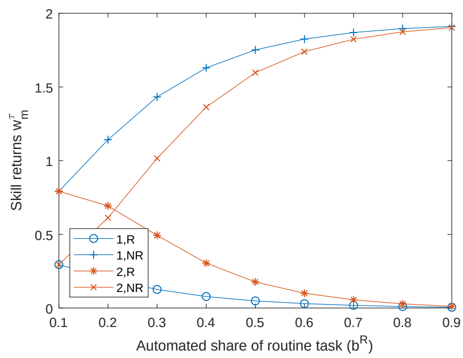
(c) Capital K_m^τ



(d) Labour L_m^τ

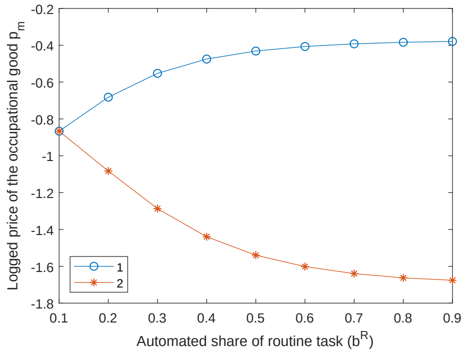


(e) Capital-labour ratios K_m^τ / L_m^τ

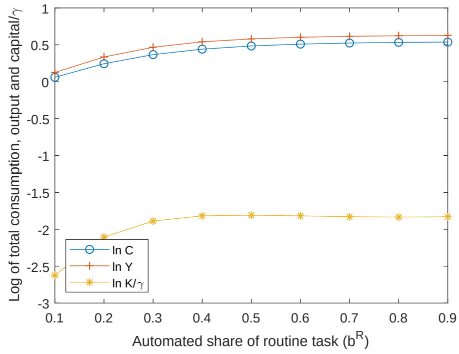


(f) Non-logged skill returns w_m^τ

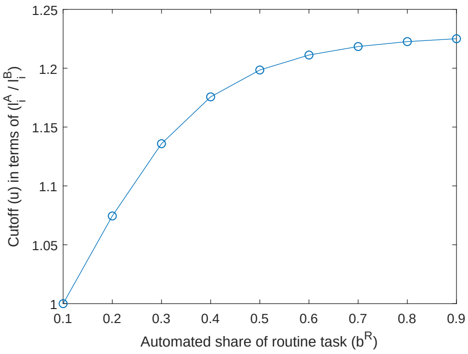
Figure 20: Comparative statics



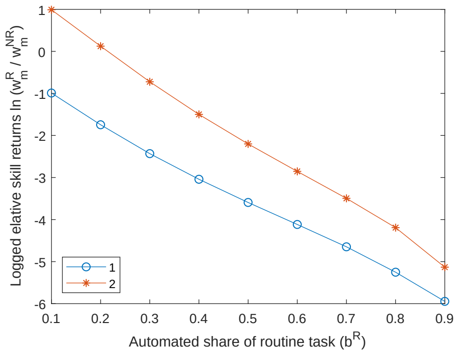
(g) Prices P_m



(h) Total output Y, consumption C and capital K/γ



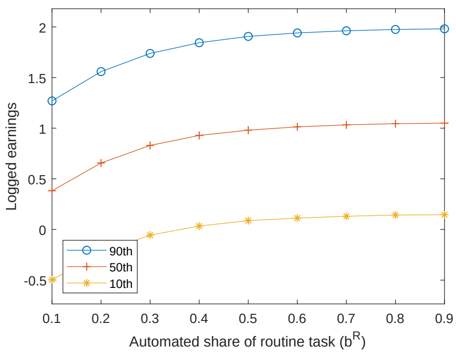
(i) Cutoff in terms of l_i^A/l_i^B. Workers with l_i^A/l_i^B above the cutoff are in occupation 2, and the ones below are in occupation 1.



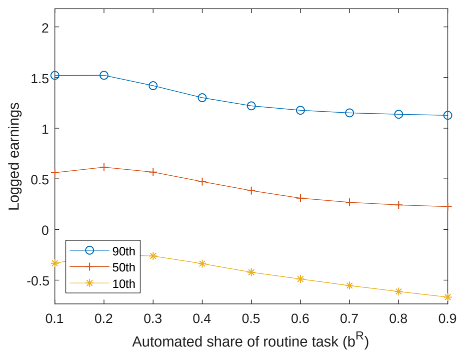
(j) Relative task returns w_m^A/w_m^B within each occupation

Figure 20: More comparative statics from the decentralised problem in section 6

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.



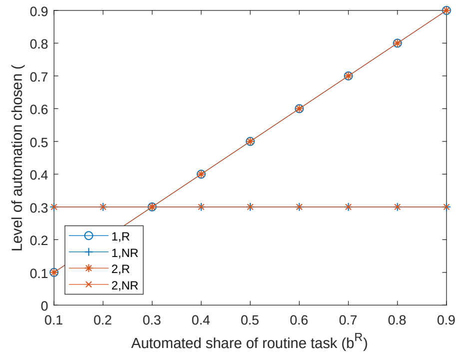
(a) Logged earnings at percentiles 10, 50 and 90 in occupation 1



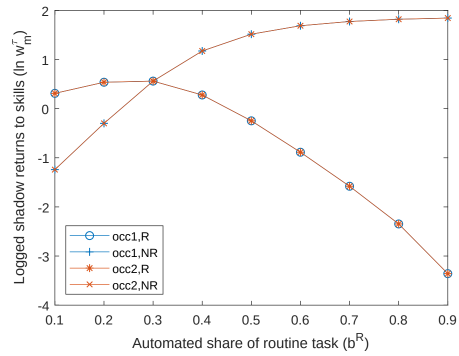
(b) Logged earnings at percentiles 10, 50 and 90 in occupation 2

Figure 21: Earnings at percentiles 10, 50 and 90

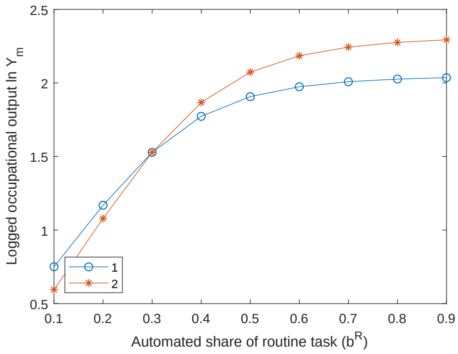
Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.



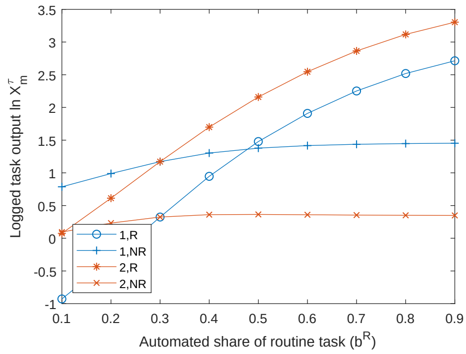
(a) Optimal automation $\tilde{b}_m^\tau$



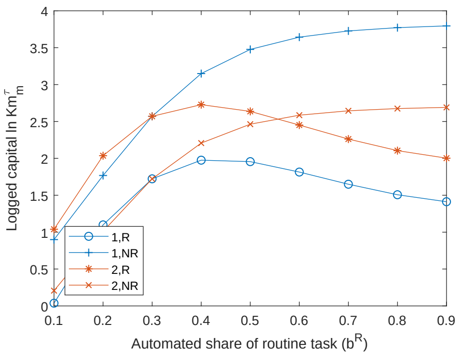
(b) Skill returns w_m^τ



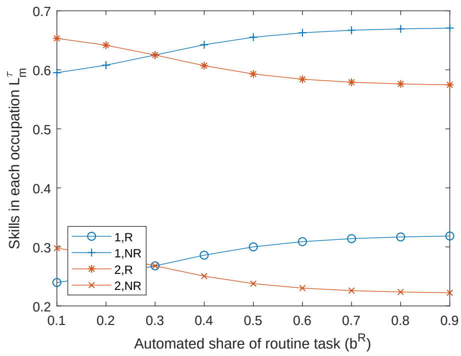
(c) Output Y_m



(d) Task output X_m^τ

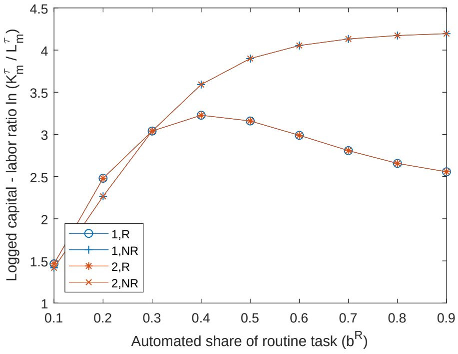


(e) Capital K_m^τ

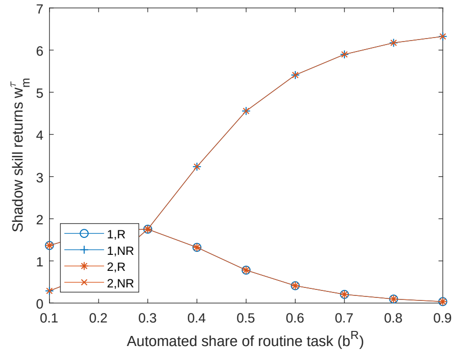


(f) Labour L_m^τ

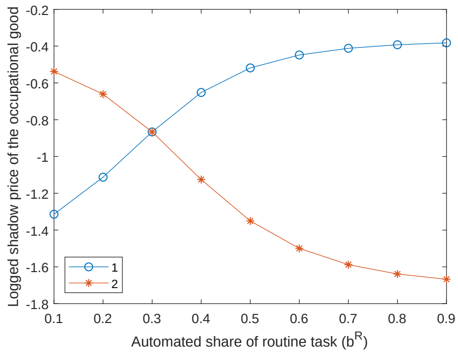
Figure 22: Comparative statics in the undbundled problem in section F.3



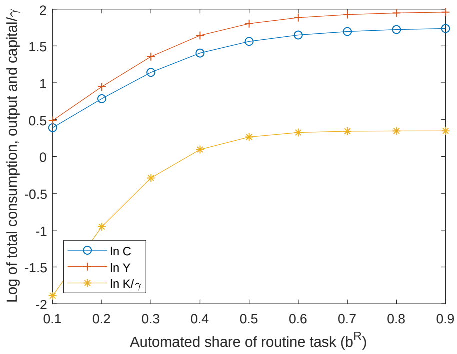
(g) Capital-labour ratios K^{τ}_m/L^{τ}_m



(h) Non-logged skill returns w^{τ}_m



(i) Shadow prices p_m



(j) Total output Y, consumption C and capital K/γ

Figure 22: Comparative statics from the unbundled problem in section F.3

Notes: $b^{NR} = 0.1$ throughout, and the parameters are as specified in Table 1.The bundling constraint is not imposed.