Index Number Theory

Summer School in Statistics 2026
Statistics Sweden and Örebro University

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Contents

Chapter 1

Setting the Stage

- A *price index* is a statistical measure that summarizes the overall differences in the prices of a specified set of goods and services between two or more periods or spatial units.

- Well known uses are:
  - consumer price index (CPI)
  - harmonized index of consumer prices (HICP)
  - producer price index (PPI)
  - import price index
  - export price index.

- The CPI and the HICP are used as approximations of the general rate of inflation.

- Such a measure is required in order to

  1. set anti-inflationary targets for monetary policy,
  2. judge the success of monetary policy,
  3. make international comparisons of inflation rates (e.g., convergence required for entry into the European Monetary Union),
  4. analyze inflationary pressure (CPI/HICP together with other measures of price movement).

- The CPI is also used for the indexation of

    1. wages,
    2. social security benefits (retirement pensions, unemployment benefits, sickness benefits, child allowances, and so on),
    3. taxes (e.g., thresholds of income tax schedules, liability for capital gains tax, and so on),
    4. interest (e.g., government bonds),
    5. rents and other contractual payments (e.g., alimony for the support of children, insurance premiums, and so on).

Table 1.1: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 1	0.5	3	1.0	3
Item 2	1.0	5	2.0	5

- t is the time period
- $t = 0$ (price reference period) and $t = 1$ (price comparison period)
- $i = 1, 2, ..., N$ denotes the item
- p_i^t is the unit price of item i in period t
- x_i^t is the quantity of item i in period t

- Computations:

$$\begin{aligned}
\text{Exp. Ratio} &: \quad \frac{p_1^1 x_1^1 + p_2^1 x_2^1}{p_1^0 x_1^0 + p_2^0 x_2^0} = \frac{13.0}{6.5} = 2 \\
\text{Price Ratios} &: \quad \frac{p_1^1}{p_1^0} = \frac{1.0}{0.5} = 2 \text{ (Item 1)} \quad \frac{p_2^1}{p_2^0} = \frac{2.0}{1.0} = 2 \text{ (Item 2)} \\
\text{Relative Prices} &: \quad \frac{p_1^0}{p_2^0} = \frac{0.5}{1.0} = 0.5 \text{ (Period 0)} \quad \frac{p_1^1}{p_2^1} = \frac{1.0}{2.0} = 0.5 \text{ (Period 1)}
\end{aligned}$$

- Overall price change: 100%.

- Note: *Constant relative prices* is equivalent to *proportional price changes* and, thus, to *identical price ratios*.

- Analogously, *constant relative quantities* is equivalent to *proportional quantity changes* and, thus, to *identical quantity ratios*.

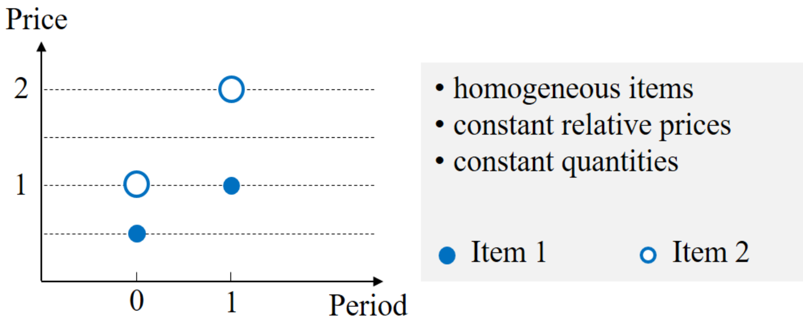


Figure 1.1: Price–quantity scenario in which price change can be measured by comparing expenditures.

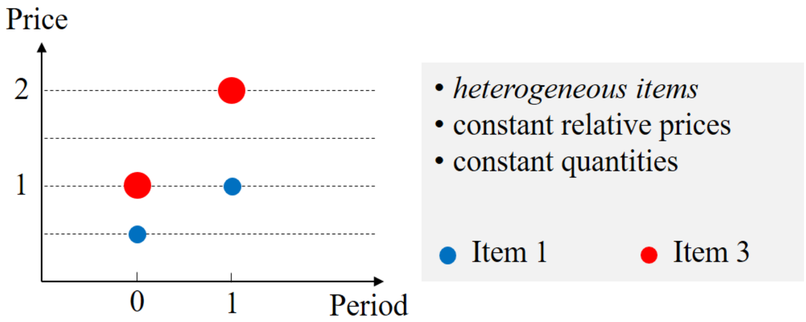


Figure 1.2: Price–quantity scenario in which price change can be measured by comparing expenditures.

- Implicitly, we have applied *axiomatic index theory*: It considers specific price-quantity scenarios and asks what number should a sensible measure of price change yield for this scenario.

- *Weak Proportionality Axiom:* When the quantities and the relative prices of the (homogeneous or heterogeneous) items are constant, the measure should be equal to the expenditure ratio.

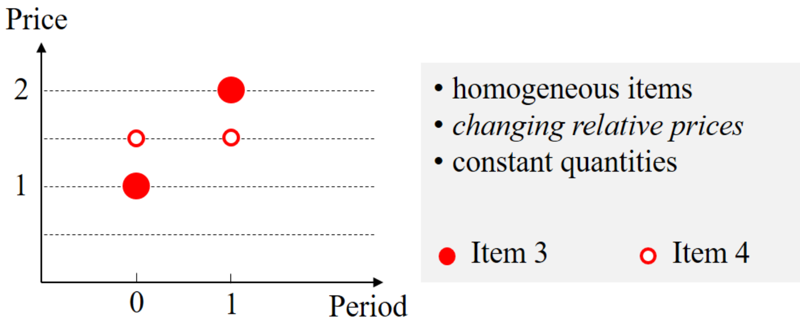


Figure 1.3: Another price–quantity scenario in which price change can be measured by comparing expenditures.

Table 1.2: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 3	1.0	5	2.0	5
Item 4	1.5	3	1.5	3

- Computations:

$$\text{Exp. Ratio} \; : \; \frac{p_3^1 x_3^1 + p_4^1 x_4^1}{p_3^0 x_3^0 + p_4^0 x_4^0} = \frac{14.5}{9.5} = 1.5263$$

$$\text{Price Ratios} \; : \; \frac{p_3^1}{p_3^0} = \frac{2.0}{1.0} = 2 \text{ (Item 3)} \qquad \frac{p_4^1}{p_4^0} = \frac{1.5}{1.5} = 1 \text{ (Item 4)}$$

- Overall price change: 52.63%.

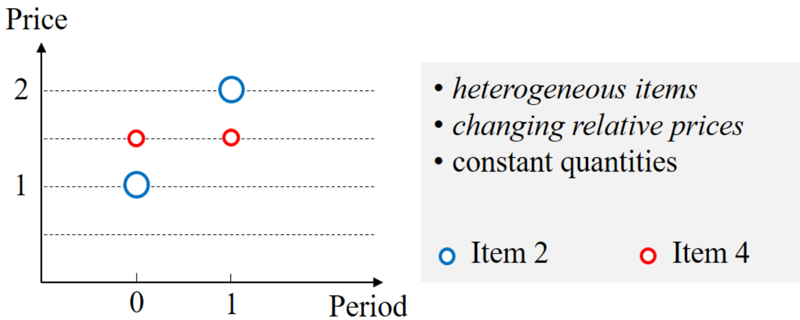


Figure 1.4: Yet another price–quantity scenario in which price change can be measured by comparing expenditures.

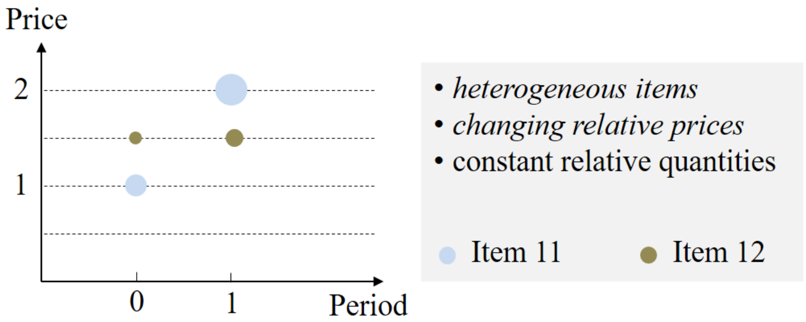


Figure 1.5: Price–quantity scenario in which price change can be measured by comparing quantity-scale-adjusted expenditures.

Table 1.3: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 11	1.0	4	2.0	6
Item 12	1.5	2	1.5	3

- Inappropriate computations: Expenditure ratio

$$\text{Exp. Ratio:} \quad \frac{p_{11}^1 x_{11}^1 + p_{12}^1 x_{12}^1}{p_{11}^0 x_{11}^0 + p_{12}^0 x_{12}^0} = \frac{16.5}{7} = 2.3571$$

- Appropriate Computations: Quantity-scale-adjusted expenditure ratio

  Quantity Ratios λ: $\frac{x_{11}^1}{x_{11}^0} = \frac{6}{4} = 1.5$ (Item 11) $\quad \frac{x_{12}^1}{x_{12}^0} = \frac{3}{2} = 1.5$ (Item 12)

$$
\begin{aligned}
\frac{p_{11}^1 x_{11}^1 + p_{12}^1 x_{12}^1}{p_{11}^0 x_{11}^0 + p_{12}^0 x_{12}^0} \frac{1}{\lambda} &= \frac{p_{11}^1 x_{11}^1 + p_{12}^1 x_{12}^1}{p_{11}^0 x_{11}^0 \lambda + p_{12}^0 x_{12}^0 \lambda} \\
&= \frac{p_{11}^1 \left(x_{11}^1/\lambda\right) + p_{12}^1 \left(x_{12}^1/\lambda\right)}{p_{11}^0 x_{11}^0 + p_{12}^0 x_{12}^0} \\
&= \frac{16.5}{7} \frac{1}{1.5} = 1.5263
\end{aligned}
$$

- Overall price change: 52.63%.

First Insights:

- *When quantities are constant over time, the expenditure ratio is an appropriate measure of overall price change.*

- *As long as the relative quantities do not change over time, the quantity-scale-adjusted expenditure ratio is an appropriate measure of overall price change.*

- *When relative quantities change, a more sophisticated measure is required.*

Chapter 2

Unit Value Index

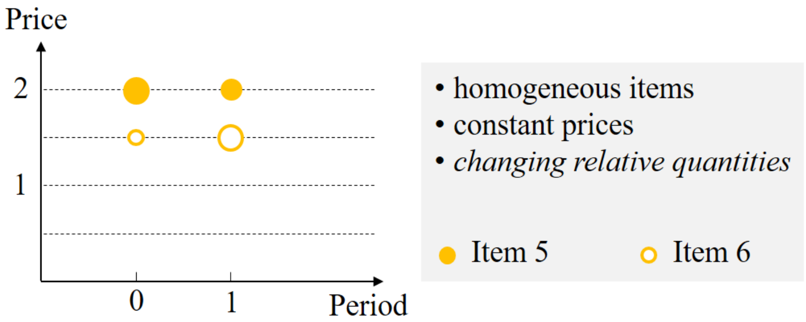


Figure 2.1: Price–quantity scenario in which price change cannot be measured by comparing quantity-scale-adjusted expenditures.

Table 2.1: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 5	2.0	5	2.0	4
Item 6	1.5	3	1.5	5

- Computations:

$$\text{Price Ratios} \quad : \quad \frac{p_5^1}{p_5^0} = \frac{2.0}{2.0} = 1 \text{ (Item 5)} \qquad \frac{p_5^1}{p_6^0} = \frac{1.5}{1.5} = 1 \text{ (Item 6)}$$

$$\text{Exp. Ratio} \quad : \quad \frac{p_5^1 x_5^1 + p_6^1 x_6^1}{p_5^0 x_5^0 + p_6^0 x_6^0} = \frac{15.5}{14.5} = 1.069$$

- Overall price change: 0% or 6.9% or something else?

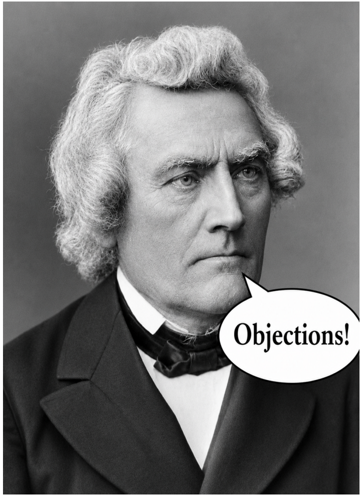
- When the relative quantities change over time, the quantity-scale-adjusted expenditure ratio is not applicable.

- A more sophisticated price index is required.

- Which requirements should such a price index satisfy?

- *Weak Identity Axiom:* When prices and relative quantities are constant, the price index should give the value 1.

- *Strict Identity Axiom:* When prices are constant, the price index should give the value 1, regardless of the quantity movements.



Moritz Wilhelm Drobisch (1802-1894)

- Unit values of homogeneous items:

$$\text{Period 0:} \qquad p_{\mathrm{UV}}^0 = \frac{p_5^0 x_5^0 + p_6^0 x_6^0}{x_5^0 + x_6^0} = \frac{2.0 \cdot 5 + 1.5 \cdot 3}{5+3} = 1.8125$$

$$\text{Period 1:} \qquad p_{\mathrm{UV}}^1 = \frac{p_5^1 x_5^1 + p_6^1 x_6^1}{x_5^1 + x_6^1} = \frac{2.0 \cdot 4 + 1.5 \cdot 5}{4+5} = 1.7222$$

- Note that

$$\frac{p_5^0 x_5^0 + p_6^0 x_6^0}{x_5^0 + x_6^0} = \frac{p_5^0 x_5^0}{x_5^0 + x_6^0} + \frac{p_6^0 x_6^0}{x_5^0 + x_6^0} = \frac{x_5^0}{x_5^0 + x_6^0} p_5^0 + \frac{x_6^0}{x_5^0 + x_6^0} p_6^0$$

- The ratio of unit values is the Unit Value (UV) index:

$$P_{\mathrm{UV}} = \frac{p^1_{\mathrm{UV}}}{p^0_{\mathrm{UV}}} = \frac{\frac{p^1_5 x^1_5 + p^1_6 x^1_6}{x^1_5 + x^1_6}}{\frac{p^0_5 x^0_5 + p^0_6 x^0_6}{x^0_5 + x^0_6}} = \frac{p^1_5 x^1_5 + p^1_6 x^1_6}{p^0_5 x^0_5 + p^0_6 x^0_6} \, \frac{x^0_5 + x^0_6}{x^1_5 + x^1_6}$$

- In the present scenario:

$$P_{\mathrm{UV}} = \frac{p^1_{\mathrm{UV}}}{p^0_{\mathrm{UV}}} = \frac{1.7222}{1.8125} = 0.9502$$

- Overall price change: -4.98%.

- Note the formal similarity to the quantity-scale-adjusted expenditure ratio:

$$P_{\mathrm{UV}} = \frac{p^1_5 x^1_5 + p^1_6 x^1_6}{p^0_5 x^0_5 + p^0_6 x^0_6} \, \frac{1}{\gamma} \,, \qquad \text{with} \quad \gamma = \frac{x^1_5 + x^1_6}{x^0_5 + x^0_6}$$

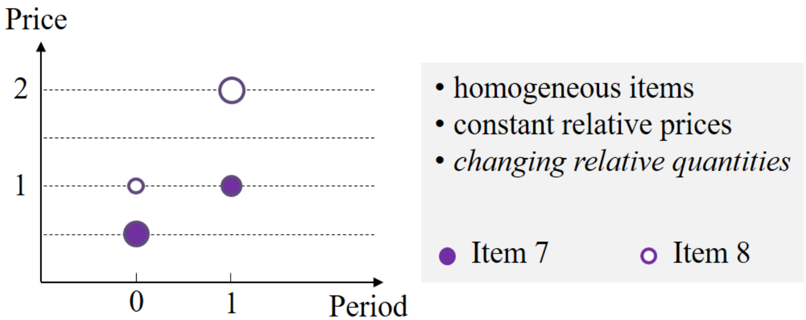


Figure 2.2: Price–quantity scenario in which price change can be measured by the UV index.

Table 2.2: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 7	0.5	5	1.0	4
Item 8	1.0	3	2.0	5

- Unit values:

$$\text{Period 0:} \qquad p^0_{\mathrm{UV}} = \frac{p^0_7 x^0_7 + p^0_8 x^0_8}{x^0_7 + x^0_8} = \frac{0.5 \cdot 5 + 1.0 \cdot 3}{5+3} = 0.6875$$

$$\text{Period 1:} \qquad p^1_{\mathrm{UV}} = \frac{p^1_7 x^1_7 + p^1_8 x^1_8}{x^1_7 + x^1_8} = \frac{1.0 \cdot 4 + 2.0 \cdot 5}{4+5} = 1.5556$$

- UV index

$$P_{\mathrm{UV}} = \frac{p^1_{\mathrm{UV}}}{p^0_{\mathrm{UV}}} = \frac{1.5556}{0.6875} = 2.2627$$

- Overall price change: 126.27%.

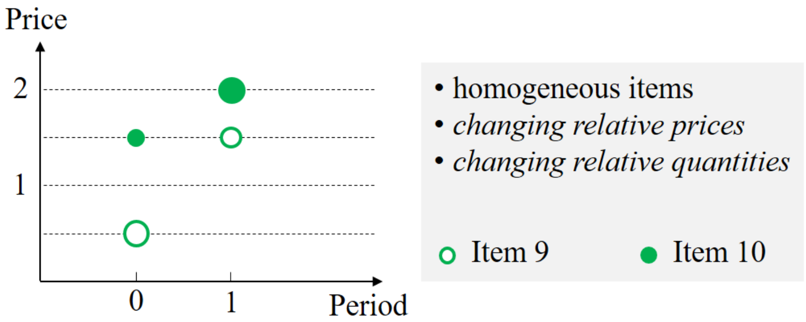


Figure 2.3: Another price–quantity scenario in which price change can be measured by the UV index.

Table 2.3: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 9	0.5	5	1.5	4
Item 10	1.5	3	2.0	5

- Unit values:

$$\text{Period 0:} \qquad p^0_{\mathrm{UV}} = \frac{0.5 \cdot 5 + 1.5 \cdot 3}{5 + 3} = 0.8750$$

$$\text{Period 1:} \qquad p^1_{\mathrm{UV}} = \frac{1.5 \cdot 4 + 2.0 \cdot 5}{4 + 5} = 1.7778$$

- UV index:

$$P_{\mathrm{UV}} = \frac{p_{\mathrm{UV}}^1}{p_{\mathrm{UV}}^0} = \frac{1.7778}{0.8750} = 2.0318$$

- Overall price change: 103.18%.

Summary of Current Insights:

- *As long as the quantities do not change over time, the expenditure ratio is a sensible measure of overall price change.*
- *As long as the relative quantities do not change over time, the quantity-scale-adjusted expenditure ratio is a sensible measure of overall price change.*
- *As long as the items are homogeneous, the unit value (UV) index is a sensible measure of overall price change.*
- *When the items are heterogeneous and relative quantities change, a more sophisticated measure is required.*

Chapter 3

Other Price Indices

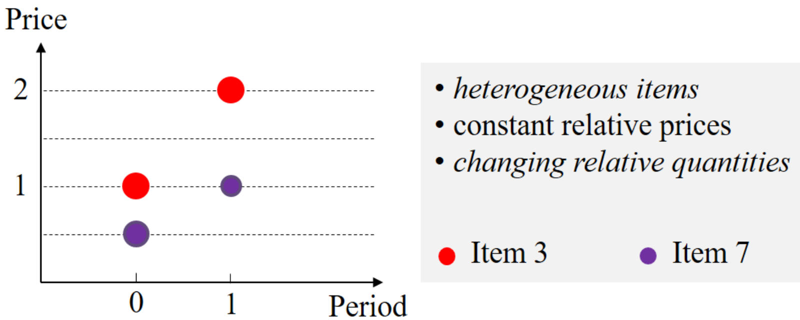


Figure 3.1: Price–quantity scenario in which a price index other than the UV index is required to measure price change.

Table 3.1: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 3	1.0	5	2.0	5
Item 7	0.5	5	1.0	4

- Unit values:

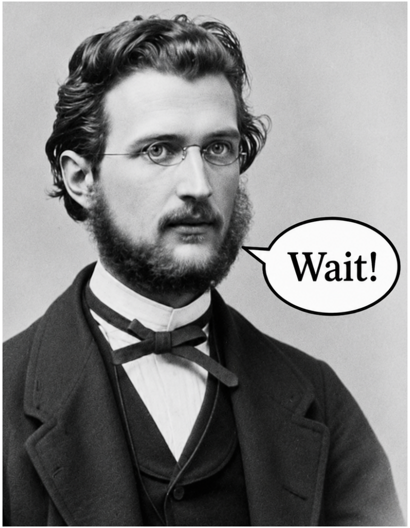
$$\text{Period 0:} \qquad p^0_{\mathrm{UV}} = \frac{1.0 \cdot 5 + 0.5 \cdot 5}{5 + 5} = 0.7500$$

$$\text{Period 1:} \qquad p^1_{\mathrm{UV}} = \frac{2.0 \cdot 5 + 1.0 \cdot 4}{5 + 4} = 1.5556$$

- UV index:

$$P_{\mathrm{UV}} = \frac{p^1_{\mathrm{UV}}}{p^0_{\mathrm{UV}}} = \frac{1.5556}{0.7500} = 2.0741$$

- Overall price change: 107.41%.



Julius Lehr (1845-1894)

- When items are heterogeneous, the UV index sums over different item-identifying units:

$$P_{\mathrm{UV}} = \frac{p^1_{\mathrm{UV}}}{p^0_{\mathrm{UV}}} = \frac{\frac{p^1_3 x^1_3 + p^1_7 x^1_7}{x^1_3 + x^1_7}}{\frac{p^0_3 x^0_3 + p^0_7 x^0_7}{x^0_3 + x^0_7}} = \frac{p^1_3 x^1_3 + p^1_7 x^1_7}{p^0_3 x^0_3 + p^0_7 x^0_7} \, \frac{x^0_3 + x^0_7}{x^1_3 + x^1_7}$$

- *Solution:* Before summation, the item-identifying units must be transformed into "intrinsic-worth units".

- For example, if item 7 is twice as valuable as item 3,
  - the quantities and prices of item 3 can stay unchanged,
  - the quantities of item 7 should be multiplied by 2, and
  - the prices of item 7 should be divided by 2.

  Then, the prices and quantities of item 7 are expressed in units of item 3. The latter represent the "intrinsic-worth units".

- Let z measure the "worth" of item 7 relative to item 3.
- This yields the following generalized unit value (GUV) index:

$$\begin{aligned} P_{\mathrm{GUV}} &= \frac{p_3^1 x_3^1 + (p_7^1/z)\,(x_7^1 z)}{p_3^0 x_3^0 + (p_7^0/z)\,(x_7^0 z)}\;\frac{x_3^0 + (x_7^0 z)}{x_3^1 + (x_7^1 z)} \\ &= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0}\;\frac{x_3^0 + x_7^0 z}{x_3^1 + x_7^1 z} \end{aligned}$$

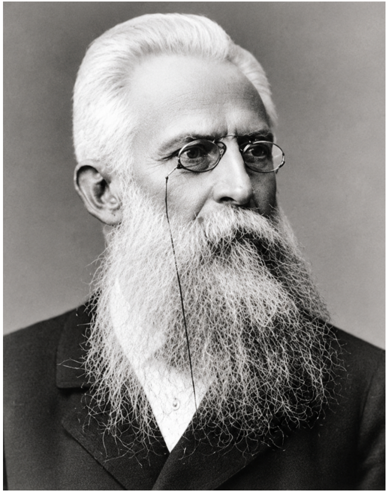
- How to find a plausible value of the relative worth z?

- One could use the items' prices.

- If one uses $z = p_7^1/p_3^1$, the GUV index becomes

$$
\begin{aligned}
P_{\text{GUV}} &= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{x_3^0 + x_7^0 \left(p_7^1/p_3^1\right)}{x_3^1 + x_7^1 \left(p_7^1/p_3^1\right)} \\
&= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_3^1 x_3^0 + p_7^1 x_7^0}{p_3^1 x_3^1 + p_7^1 x_7^1} \\
&= \frac{p_3^1 x_3^0 + p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} = P_{\text{Laspeyres}}
\end{aligned}
$$

- The Laspeyres index, $P_{\text{Laspeyres}}$, measures the change in expenditure that would occur if consumers continued to purchase the price reference period ($t = 0$) basket also in the price comparison period ($t = 1$).



Ernst Louis Étienne Laspeyres (1834-1913)

- The Laspeyres index can equivalently be expressed as

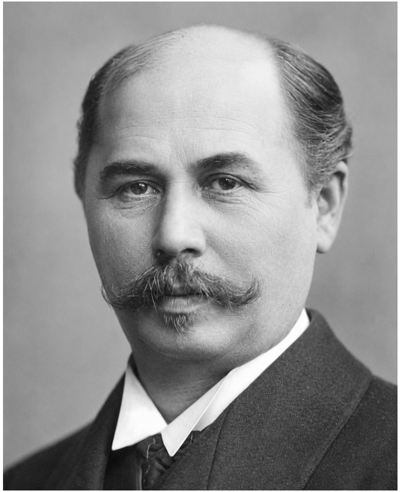
$$
\begin{aligned}
P_{\text{Laspeyres}} &= \frac{p_3^1 x_3^0 + p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \\
&= \frac{p_3^1 x_3^0}{p_3^0 x_3^0 + p_7^0 x_7^0} + \frac{p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \\
&= \frac{p_3^1 x_3^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_3^0}{p_3^0} + \frac{p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_7^0}{p_7^0} \\
&= \frac{p_3^0 x_3^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_3^1}{p_3^0} + \frac{p_7^0 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_7^1}{p_7^0}
\end{aligned}
$$

- This is a weighted arithmetic average of the price ratios p_3^1/p_3^0 and p_7^1/p_7^0, where the weights are the expenditure shares of the price reference period.

- If one uses in the GUV index $z = p_7^0/p_3^0$, the index becomes

$$
\begin{aligned}
P_{\text{GUV}} &= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{x_3^0 + x_7^0 \left(p_7^0/p_3^0\right)}{x_3^1 + x_7^1 \left(p_7^0/p_3^0\right)} \\
&= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_3^0 x_3^0 + p_7^0 x_7^0}{p_3^0 x_3^1 + p_7^0 x_7^1} \\
&= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^1 + p_7^0 x_7^1} = P_{\text{Paasche}}
\end{aligned}
$$

- The Paasche index, P_{Paasche}, measures the change in expenditure that would occur if consumers had purchased the price comparison period ($t = 1$) basket also in the price reference period ($t = 0$).



Hermann Paasche (1851-1925)

- The Paasche index can equivalently be expressed as

$$
\begin{aligned}
P_{\text{Paasche}} &= \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^1 + p_7^0 x_7^1} \\
&= \left(\frac{p_3^0 x_3^1 + p_7^0 x_7^1}{p_3^1 x_3^1 + p_7^1 x_7^1} \right)^{-1} \\
&= \left(\frac{p_3^0 x_3^1}{p_3^1 x_3^1 + p_7^1 x_7^1} + \frac{p_7^0 x_7^1}{p_3^1 x_3^1 + p_7^1 x_7^1} \right)^{-1} \\
&= \left(\frac{p_3^1 x_3^1}{p_3^1 x_3^1 + p_7^1 x_7^1} \left(\frac{p_3^1}{p_3^0} \right)^{-1} + \frac{p_7^1 x_7^1}{p_3^1 x_3^1 + p_7^1 x_7^1} \left(\frac{p_7^1}{p_7^0} \right)^{-1} \right)^{-1}
\end{aligned}
$$

- This is a weighted harmonic average of the price ratios p_3^1/p_3^0 and p_7^1/p_7^0, where the weights are the expenditure shares of the price comparison period.

- Both the Laspeyres index and the Paasche index can be interpreted in three different ways:
  - They measure the change in generalized unit values (GUV indices),
  - They are weighted means of price ratios.
  - They track the cost of purchasing a fixed basket of items.

- Numerical results

$$P_{\text{Laspeyres}} = \frac{p_3^1 x_3^0 + p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} = \frac{2.0 \cdot 5 + 1.0 \cdot 5}{1.0 \cdot 5 + 0.5 \cdot 5} = 2.0$$

$$P_{\text{Paasche}} = \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^1 + p_7^0 x_7^1} = \frac{2.0 \cdot 5 + 1.0 \cdot 4}{1.0 \cdot 5 + 0.5 \cdot 4} = 2.0$$

- Recall that in the current scenario we have $p_3^1/p_3^0 = p_7^1/p_7^0$. Thus, the weighting of these price ratios is irrelevant.

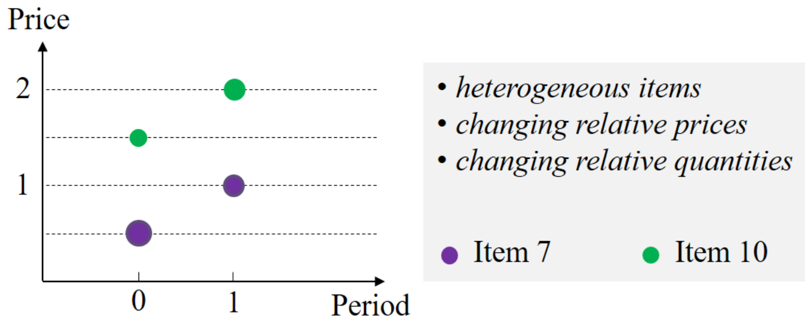


Figure 3.2: Price–quantity scenario in which price change must be measured by a price index.

Table 3.2: Prices and quantities during periods 0 and 1.

	Period 0		Period 1	
	Price	Quantity	Price	Quantity
Item 7	0.5	5	1.0	5
Item 10	1.5	3	2.0	4

- Numerical results:

$$P_{\text{Laspeyres}} = \frac{p_3^1 x_3^0 + p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} = \frac{1.0 \cdot 5 + 2.0 \cdot 3}{0.5 \cdot 5 + 1.5 \cdot 3} = 1.5714$$

$$P_{\text{Paasche}} = \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^1 + p_7^0 x_7^1} = \frac{1.0 \cdot 5 + 2.0 \cdot 4}{0.5 \cdot 5 + 1.5 \cdot 4} = 1.5294$$

- The Laspeyres and Paasche indices appear equally plausible.

- Four different strategies:

  Strategy 1: In
  $$P_{\mathrm{GUV}} = \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \frac{x_3^0 + x_7^0 z}{x_3^1 + x_7^1 z}$$
  compute z on the basis of both price reference and comparison period information.

  Strategy 2: Form some average of $P_{\mathrm{Laspeyres}}$ and P_{Paasche}.

  Strategy 3: Instead of using either quantities from the price reference period (as in $P_{\mathrm{Laspeyres}}$) or from the price comparison period (as in P_{Paasche}), use some average of the two periods.

  Strategy 4: Instead of using either expenditure shares from the price reference period (as in $P_{\mathrm{Laspeyres}}$) or from the price comparison period (as in P_{Paasche}), use some average of the two periods.

- Examples for Strategy 1:

$$P_{\text{Banerjee}} = \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{x_3^0 + x_7^0 \frac{\left(p_7^0 + p_7^1\right)/2}{\left(p_3^0 + p_3^1\right)/2}}{x_3^1 + x_7^1 \frac{\left(p_7^0 + p_7^1\right)/2}{\left(p_3^0 + p_3^1\right)/2}}$$

$$P_{\text{Davies}} = \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{x_3^0 + x_7^0 \frac{\sqrt{p_7^0 p_7^1}}{\sqrt{p_3^0 p_3^1}}}{x_3^1 + x_7^1 \frac{\sqrt{p_7^0 p_7^1}}{\sqrt{p_3^0 p_3^1}}}$$

$$P_{\text{Lehr}} = \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{x_3^0 + x_7^0 \left(\frac{p_7^0 x_7^0 + p_7^1 x_7^1}{x_7^0 + x_7^1} \Big/ \frac{p_3^0 x_3^0 + p_3^1 x_3^1}{x_3^0 + x_3^1} \right)}{x_3^1 + x_7^1 \left(\frac{p_7^0 x_7^0 + p_7^1 x_7^1}{x_7^0 + x_7^1} \Big/ \frac{p_3^0 x_3^0 + p_3^1 x_3^1}{x_3^0 + x_3^1} \right)}$$

- Example for Strategy 2:

$$P_{\text{Fisher}} = \sqrt{P_{\text{Laspeyres}} P_{\text{Paasche}}} = \sqrt{\frac{p_3^1 x_3^0 + p_7^1 x_7^0}{p_3^0 x_3^0 + p_7^0 x_7^0} \, \frac{p_3^1 x_3^1 + p_7^1 x_7^1}{p_3^0 x_3^1 + p_7^0 x_7^1}}$$

- Examples for Strategy 3:

$$P_{\text{Marshall-Edgeworth}} = \frac{p_3^1 \frac{\left(x_3^0 + x_3^1\right)}{2} + p_7^1 \frac{x_7^0 + x_7^1}{2}}{p_3^0 \frac{x_3^0 + x_3^1}{2} + p_7^0 \frac{x_7^0 + x_7^1}{2}} = \frac{p_3^1 \left(x_3^0 + x_3^1\right) + p_7^1 \left(x_7^0 + x_7^1\right)}{p_3^0 \left(x_3^0 + x_3^1\right) + p_7^0 \left(x_7^0 + x_7^1\right)}$$

$$P_{\text{Walsh}} = \frac{p_3^1 \sqrt{x_3^0 x_3^1} + p_7^1 \sqrt{x_7^0 x_7^1}}{p_3^0 \sqrt{x_3^0 x_3^1} + p_7^0 \sqrt{x_7^0 x_7^1}}$$

- Examples for Strategy 4:

$$
\begin{aligned}
P_{\text{Marshall.-Edgeworth}} &= \frac{p_3^0\left(x_3^0+x_3^1\right)}{p_3^0\left(x_3^0+x_3^1\right)+p_7^0\left(x_7^0+x_7^1\right)}\left(\frac{p_3^1}{p_3^0}\right) \\
&\quad + \frac{p_7^0\left(x_7^0+x_7^1\right)}{p_3^0\left(x_3^0+x_3^1\right)+p_7^0\left(x_7^0+x_7^1\right)}\left(\frac{p_7^1}{p_7^0}\right) \\
P_{\text{Walsh}} &= \frac{\sqrt{p_3^0x_3^0\cdot p_3^0x_3^1}}{\sqrt{p_3^0x_3^0\cdot p_3^0x_3^1}+\sqrt{p_7^0x_7^0\cdot p_7^0x_7^1}}\left(\frac{p_3^1}{p_3^0}\right) \\
&\quad + \frac{\sqrt{p_7^0x_7^0\cdot p_7^0x_7^1}}{\sqrt{p_3^0x_3^0\cdot p_3^0x_3^1}+\sqrt{p_7^0x_7^0\cdot p_7^0x_7^1}}\left(\frac{p_7^1}{p_7^0}\right)
\end{aligned}
$$

$$
\ln P_{\text{Törnqvist}} = \frac{\frac{p_3^0x_3^0}{p_3^0x_3^0+p_7^0x_7^0}+\frac{p_3^1x_3^1}{p_3^1x_3^1+p_7^1x_7^1}}{2}\ln\left(\frac{p_3^1}{p_3^0}\right)+\frac{\frac{p_7^0x_7^0}{p_3^0x_3^0+p_7^0x_7^0}+\frac{p_7^1x_7^1}{p_3^1x_3^1+p_7^1x_7^1}}{2}\ln\left(\frac{p_7^1}{p_7^0}\right)
$$

Table 3.3: Price index values for the illustrative price–quantity scenario.

Index formula	Index value	Price change (in %)
Laspeyres	1.5714	57.14
Paasche	1.5294	52.94
Fisher	1.5503	55.03
Marshall–Edgeworth	1.5484	54.84
Walsh	1.5499	54.99
Banerjee	1.5548	55.48
Davies	1.5497	54.97
Lehr	1.5527	55.27
Törnqvist	1.5497	54.97

Summary of Latest Insights:

- *For scenarios with heterogeneous items, changing relative quantities, and constant relative prices, the GUV indices of Laspeyres and Paasche are suitable and give identical index values.*

- *For scenarios with heterogeneous items, changing relative quantities, and changing relative prices, the GUV indices of Banerjee, Davies, and Lehr or the price indices of Fisher, Marshall-Edgeworth, Walsh, Törnqvist, and others appear preferable.*