Estimating Fiscal Multipliers by Combining Statistical Identification with Potentially Endogenous Proxies*

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Abstract

Different proxy variables used in fiscal policy SVARs lead to contradicting conclusions regarding the size of fiscal multipliers. In this paper, we show that the conflicting results are due to violations of the exogeneity assumptions, i.e. the commonly used proxies are endogenously related to the structural shocks. We propose a novel approach to include proxy variables into a Bayesian non-Gaussian SVAR, tailored to accommodate potentially endogenous proxy variables. Using our model, we show that increasing government spending is a more effective tool to stimulate the economy than reducing taxes. We construct new exogenous proxies that can be used in the traditional proxy VAR approach resulting in similar estimates compared to our proposed hybrid SVAR model.

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1 Introduction

There is renewed interest among researchers and policy makers in the effects of fiscal policy on macroeconomic activity. The main challenge to measuring the aggregate effects of changes in tax and spending policy, however, is endogeneity of fiscal measures. Most recently, the so-called proxy VAR approach has become a popular tool for identification of fiscal policy shocks. The main idea is that there exists an external instrument that is correlated with the structural shock of interest (relevance condition) while not correlated with the remaining shocks (exogeneity assumption) (Stock and Watson, 2012; Mertens and Ravn, 2013).

Two prominent examples in this literature are Mertens and Ravn (2014) who use an adjusted version of the narrative Romer and Romer (2010) tax shock proxy and Caldara and Kamps (2017) who rely on non-fiscal proxy variables like the Fernald (2012) adjusted TFP series. Notably, both studies use different potentially valid proxy variables and reach contradicting conclusions regarding the size of fiscal multipliers (Angelini et al., 2023). In particular, Mertens and Ravn (2014) find that the tax multiplier is larger than the government spending multiplier, whereas Caldara and Kamps (2017) provide evidence that the government spending multiplier exceeds the tax multiplier. One potential explanation for these divergent results is that the proxy variables do not fulfill the exogeneity assumption, thereby being related to other structural disturbances. Importantly, within the standard proxy VAR approach the validity of the instrument cannot be investigated because the exogeneity assumption is central to achieve identification.

In this paper we propose a hybrid approach that combines proxy variables with statistical identification through non-Gaussianity. The hybrid approach is able to deal with potentially invalid proxy variables and allows us to provide an explanation for the contradicting results of Mertens and Ravn (2014) and Caldara and Kamps (2017). We show that the commonly used tax proxy is negatively correlated with output shocks and the TFP proxy is negatively correlated with government spending shocks. Accounting for the endogeneity in the proxy variables leads to the robust result of a government spending multiplier being larger than the tax multiplier.

Our econometric contribution consists of two parts. First, we propose a novel proxy weighting approach in a Bayesian framework. The proxy weighting approach weights the likelihood of the SVAR with a set of moment conditions penalizing deviations from exogenous proxy variables. Therefore, the approach bears resemblance to the moment-based frequentist proxy approach outlined by Stock and Watson (2012); Mertens and Ravn (2013) and offers a Bayesian alternative to the augmented proxy VAR proposed by Caldara and Herbst (2019). Unlike the augmented proxy approach and akin to moment-based frequentist proxy estimators, the proxy weighting approach does not necessitate specifying the functional form of the proxy. We show that this is an advantage in non-Gaussian SVAR models, where misspecification of the functional form of the proxy can lead to dependent shocks and to a failure of non-Gaussian identification approaches assuming independent shocks.

Second, we combine the proxy weighting approach with independent and non-Gaussian shocks. Independent and non-Gaussian shocks ensure identification of the SVAR. Therefore, we do not need to assume exogeneity of the proxy variables, instead, we estimate the correlation of the proxy variables with non-target shocks. More precisely, we propose a prior distribution designed to shrink the estimates towards exogenous proxy variables. However, in the non-Gaussian setup these priors are updated by the data. Therefore, we propose a non-dogmatic way to benefit from the information provided by proxy variables and are able to detect endogenous proxy variables.

The proposed estimator builds on a growing literature using stochastic properties to ensure identification, see, e.g., Rigobon (2003), Lanne et al. (2010), Matteson and Tsay (2017), Lütkepohl and Netšunajev (2017), Lanne et al. (2017), Keweloh (2021), Lewis (2021), or Bertsche and Braun (2022). Moreover, our model relates to a recent literature combining statistical identification with traditional economically motivated restrictions and proxy variables, see, e.g., Schlaak et al. (2021), Drautzburg and Wright (2023), Braun (2023), Keweloh et al. (2023), or Herwartz and Wang (2023). Identification through non-Gaussian errors is further backed by several studies documenting that the normal distribution does not provide a good approximation to many macroeconomic variables in OECD countries (Acemoglu et al., 2017; Cúrdia et al., 2014; Fagiolo et al., 2008).

The combination of statistical identification and proxy variables offers several advantages. We demonstrate that including a valid proxy increases the performance of the estimator in terms of bias, MSE, and the length of credible bands compared to an estimator based only on non-Gaussian shocks. In addition, the presence of non-Gaussian shocks offers insights into the exogeneity of a proxy, affording an opportunity to mitigate the bias introduced by endogenous proxies, a crucial departure from conventional methodologies using proxy variables.

Emphasizing the core intent of our proposed estimator is essential. Our aim extends beyond a mere test of proxy variable exogeneity. Instead, we endeavor to leverage prior knowledge of an exogenous proxy to enhance estimation precision. Yet, we remain flexible in disregarding the proxy should the data furnish evidence contradicting its exogeneity.

We use our non-Gaussian proxy weighting approach to estimate the effects of fiscal policy shocks and evaluate whether commonly used proxies fulfill the crucial exogeneity assumptions. We provide empirical evidence that the shocks identified using the fiscal proxy SVAR from Mertens and Ravn (2013) and using the non-fiscal proxy SVAR from Caldara and Kamps (2017) are non-Gaussian. In particular, all shocks show heavy tails and the tax and output shocks are left skewed. This data feature which is used for identification makes sense also from an economic point of view because tax reductions are generally larger (in absolute terms) than tax increases and GDP usually falls stronger during recessions than it rises during expansions.

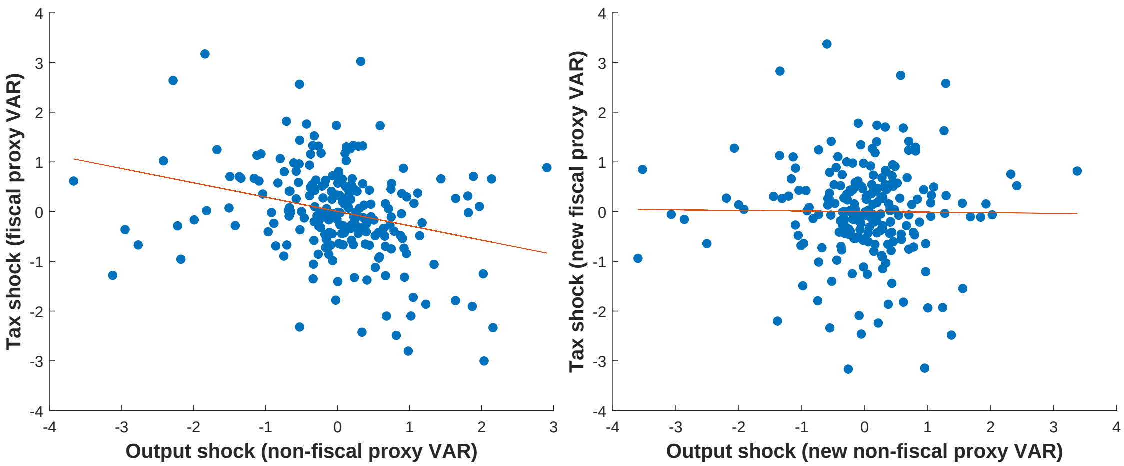
We find that the government spending multiplier is larger than the tax multiplier. In the impact period, the tax multiplier is close to zero and takes on values below unity for the entire forecast horizon of five years. The government spending multiplier however is above unity for the first year after the shock and then slowly converges back to its pre-shock level.

Thus, our findings imply that increasing government spending is a more effective tool to stimulate the economy than lowering taxes.

Evidently, our result of a large (small) government spending (tax) multiplier stands in some contrast to previous studies and in particular to the paper by Mertens and Ravn (2014) which estimate a tax multiplier around three. We show that the large differences between applying the Mertens and Ravn (2014) approach and our model is due to the tax proxy not being exogenous. Specifically, we provide evidence that the narrative tax measure is negatively correlated with structural output shocks. Intuitively, not accounting for this correlation leads to identified tax cuts that also include exogenous increases in output, and vice versa, which increases the size of the estimated tax multiplier. While Mertens and Ravn (2014) have to assume that the tax proxy is exogenous, our model shows that this assumption is not supported by the data, i.e. shrinking to an exogenous tax proxy leads to dependent structural shocks. We provide additional historical evidence indicating that the original tax proxy shows a tendency to indicate an exogenous tax increase during periods of economic recessions and an exogenous tax decrease during episodes of economic expansions. Relatedly, we also find some evidence that the TFP proxy used by Caldara and Kamps (2017) is negatively correlated with exogenous government spending shocks. Therefore, assuming an exogenous TFP proxy identifies output shocks which include spending shocks of opposite sign. As a consequence, the estimated spending multiplier is upward biased.

Finally, we use our estimation results to construct new proxies that account for confounding factors and argue that these proxies may fulfill the exogeneity assumption and can thus be used in a standard proxy VAR approach. The endogeneity feature of the original tax and TFP proxies is visualised in Figure 1. The left panel plots the estimated tax shocks following the Mertens and Ravn (2014) approach against the output shocks when applying the Caldara and Kamps (2017) strategy. If the proxy variables used in both papers are valid, one would expect no systematic correlation between the shocks. This is not the case: there is a negative relationship which indicates that at least one of the proxies is

Figure 1: Tax shocks vs. output shocks



Notes: The left side shows estimated tax shocks based on the fiscal tax proxy used by Mertens and Ravn (2014) vs. the output shocks based on the non-fiscal TFP proxy used by Caldara and Kamps (2017). The right side compares the shocks estimated using the new fiscal tax and new non-fiscal TFP proxies.

not exogenous thus questioning the validity of the reported estimates.[1] In contrast, the right panel of Figure 1 shows the tax shocks using the Mertens and Ravn (2014) approach with our new tax proxy against the output shocks using the Caldara and Kamps (2017) strategy with our new TFP proxy. The correlation vanishes implying that our new proxies should be preferred over the original proxies. Importantly, we show that while using the original (endogenous) proxies of the left column leads to substantial differences in the estimated fiscal multipliers when applying the traditional proxy VAR approach, relying on the exogenous proxies of the right column results in very similar estimates.

The remainder of the paper is organized as follows: Section 2 explains identification through proxy variables and non-Gaussianity. Specifically, we describe our novel proxy weighting approach and the combination of proxy variables with non-Gaussianity. Section 3 studies the performance of the proposed estimator using Monte Carlo simulations. In Section 4, we use the proposed model to analyze the effects of fiscal policy shocks. Section 5 concludes.

[1] The correlation between both shocks is not affected by the zero restrictions which again indicates that at least one of the identifying proxy exogeneity assumptions is invalid.

2 SVARs

A standard SVAR with n variables can be written as

$$\boldsymbol{y}_t = \boldsymbol{\nu} + \boldsymbol{A}_1 \boldsymbol{y}_{t-1} + ... + \boldsymbol{A}_p y_{t-p} + \boldsymbol{u}_t \tag{1}$$

$$\boldsymbol{u}_t = \boldsymbol{B}_0 \boldsymbol{\varepsilon}_t, \tag{2}$$

with parameter matrices $\boldsymbol{A}_1, ..., \boldsymbol{A}_p \in \mathbb{R}^{n\times n}$ which satisfy $\det(\boldsymbol{I} - \boldsymbol{A}_1 c - ... - \boldsymbol{A}_p c^p) \neq 0$ for $|c| \leq 1$, an intercept $\boldsymbol{\nu}$, an invertible matrix $\boldsymbol{B}_0 \in \mathbb{B} := \{\boldsymbol{B} \in \mathbb{R}^{n\times n} | \det(\boldsymbol{B}) \neq 0\}$, an n-dimensional vector of time series $\boldsymbol{y}_t = [\boldsymbol{y}_{1t}, ..., \boldsymbol{y}_{nt}]'$, an n-dimensional vector of reduced form shocks $\boldsymbol{u}_t = [u_{1t}, ..., u_{nt}]'$, and an n-dimensional vector of serially uncorrelated structural shocks $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}, ..., \varepsilon_{nt}]'$ with mean zero and unit variance.

We define $\boldsymbol{\pi} = \text{vec}([\boldsymbol{v}, \boldsymbol{A}_1', ..., \boldsymbol{A}_p']')$ and $e_{it}(\boldsymbol{B}, \boldsymbol{\pi})$ the ith component of $\boldsymbol{e}_t(\boldsymbol{B}, \boldsymbol{\pi}) = \boldsymbol{B}^{-1}(\boldsymbol{y}_t - \boldsymbol{\nu} - \boldsymbol{A}_1 \boldsymbol{y}_{t-1} - ... - \boldsymbol{A}_p \boldsymbol{y}_{t-p})$. Therefore, $\boldsymbol{e}_t(\boldsymbol{B}, \boldsymbol{\pi})$ represents the shocks given $\boldsymbol{\pi}$ and $\boldsymbol{B}$. If $\boldsymbol{\pi}$ and $\boldsymbol{B}$ are equal to the corresponding values of the data-generating process, $\boldsymbol{e}_t(\boldsymbol{B}, \boldsymbol{\pi})$ is equal to the structural shocks $\boldsymbol{\epsilon}_t$.

Without additional restrictions the model is not identified, i.e. any orthogonal matrix $\boldsymbol{Q}$ yields an observationally equivalent model $\tilde{\boldsymbol{B}} = \boldsymbol{B}\boldsymbol{Q}$. In the following, we discuss different restrictions imposed on the structural impact matrix $\boldsymbol{B}$ in order to uniquely pin down the impact effect of the structural shocks and hence to archive identification.

2.1 Proxy SVARs

This section provides a concise overview of the existing methods for incorporating proxy variables into SVAR models, namely the frequentist proxy approach based on moment conditions and the Bayesian augmented proxy approach. In addition, we introduce a new approach to incorporate proxy variables into a Bayesian SVAR using a weighting function. In the next sections we combine the proxy weighting approach with statistical identification through independent non-Gaussian errors. The combined approach allows for potentially endogenous proxy variables.

A proxy variable z_t for a target shock ϵ_{it} is valid if it satisfies the following assumption.

Assumption 1. *Let z_t be a proxy variable for the target shock ϵ_{it} which satisfies the following moment conditions:*

1. *Relevance:* $E[z_t \varepsilon_{it}] \neq 0$

2. *Exogeneity:* $E[z_t \varepsilon_{jt}] = 0$ *for* $j \neq i$

Therefore, a valid proxy variable is correlated with the target shock (relevance) and uncorrelated with all non-target shocks (exogeneity).

2.1.1 Existing proxy approaches

The frequentist proxy SVAR approach estimates the impact of the target shock ϵ_{it} based on moment conditions derived from the relevance and exogeneity moment conditions in Assumption 1. In particular, if z_t is relevant and exogenous, the impact b_{ij} of the target shock ϵ_{it} on some reduced form shock u_{jt} can be estimated by $\hat{b}_{ij} = \frac{\sum_{t=1}^{T}(z_t u_{jt})}{\sum_{t=1}^{T}(z_t u_{it})}$, see Stock and Watson (2012) and Mertens and Ravn (2013).

The augmented proxy SVAR approach, proposed by Caldara and Herbst (2019) and utilized in both the Bayesian proxy VAR literature (Arias et al. (2021), Braun and Brüggemann (2022), Bruns (2021), and Giacomini et al. (2022)) and occasionally in the frequentist literature (Angelini and Fanelli (2019)), involves augmenting the SVAR model with the proxy variable. The augmented proxy SVAR can be expressed as follows:

$$\begin{bmatrix} \boldsymbol{y}_t \\ z_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\nu} \\ \boldsymbol{\nu}_z \end{bmatrix} + \sum_{i=1}^{p} \begin{bmatrix} \boldsymbol{A}_i & 0 \\ \boldsymbol{\Gamma}_1 & \boldsymbol{\Gamma}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_{t-i} \\ z_{t-i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_0 & 0 \\ \boldsymbol{\Phi} & \boldsymbol{\Sigma}_\eta \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \eta_t \end{bmatrix}, \tag{3}$$

with a measurement error η_t uncorrelated with the structural form shocks $\boldsymbol{\varepsilon}_t$. To simplify, let $\boldsymbol{\nu}_z = 0$, $\Sigma_\eta = 1$, and $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_2 = 0$, which implies that the proxy variable is equal to a linear combination of the structural shocks and the measurement error, i.e. $z_t = \boldsymbol{\Phi}\boldsymbol{\varepsilon}_t + \eta_t$.

Imposing a valid proxy which satisfies Assumption 1 than implies zero restrictions on the $\mathbf{\Phi}$ matrix, such that the proxy variable is a linear combination of the target-shock and the measurement error, i.e. $z_t = \Phi_i \varepsilon_{it} + \eta_t$.

The augmented proxy SVAR approach, unlike the frequentist moment-based proxy approach, requires the specification of the data generating process for the proxy variable.[2] Specifically, the proxy variable is assumed to be a linear function of the structural shocks and the measurement error. However, it is important to note that many proxy variables do not follow a linear process like in our empirical work. If the proxy process is misspecified, it can lead to dependent shocks, which renders identification approaches based on independent shocks invalid.

The proxy linearity assumption raises particular concerns when applied to narrative proxy variables (like the tax proxy we use in our empirical work), which may follow a process like $z_t = \psi_t(\Phi_i \varepsilon_{it} + \eta_{it})$, where ψ_t is a Bernoulli random variable, compare Jentsch and Lunsford (2019), Bruns and Lütkepohl (2022), or Budnik and Rünstler (2022). Mischaracterizing this process with linearity leads to an estimated measurement error

$$\hat{\eta}_t = \begin{cases} (\Phi_i - \hat{\Phi}_i)\varepsilon_{it} + \eta_t & , \text{ if } \psi_t = 1 \\ -\hat{\Phi}_i \varepsilon_{it} & , \text{ else} \end{cases} . \tag{4}$$

A proper choice of $\hat{\Phi}_i$ leads to a measurement error $\hat{\eta}_t$ uncorrelated with ε_{it}, however, there exists no measurement error independent of the structural shock ε_{it}, which would be required to estimate the augmented proxy SVAR based on independent and non-Gaussian shocks.

[2] The augmented proxy SVAR approach coherently incorporates all sources of uncertainty in the estimation, see Caldara and Herbst (2019). Therefore, the proxy becomes informative about both the educed form and the structural parameters of the model.

2.1.2 A novel proxy weighting approach

We propose a Bayesian proxy approach that avoids the need to specify the functional form of the proxy variable. Specifically, we propose to use a weighting function based on the proxy exogeneity moment conditions to re-weight the likelihood of the SVAR. This approach allows us to overcome the drawbacks of the augmented proxy SVAR, as it does not rely on a specific functional form assumption for the proxy variable. Instead, it provides a flexible and robust framework for incorporating proxy variables into the Bayesian analysis.

In the absence of knowledge about the distribution of the structural shocks $\boldsymbol{\varepsilon}_t$ or the distribution and functional form of the proxy variable z_t, we can still approximate the distribution of the scaled sample mean of $z_t\boldsymbol{\varepsilon}_t$ using the central limit theorem. The approximation can be represented as:

$$\sqrt{T}(\frac{1}{T}\sum_{t=1}^{T} z_t\varepsilon_{jt} - \mu_j) \sim N(0, \sigma_j^2), \tag{5}$$

where $\mu_j = E[z_t\varepsilon_{jt}]$ and $\sigma_j^2 = Var(z_t\varepsilon_{jt})$. If we assume that the proxy variable is exogenous, specifically uncorrelated with the non-target shock ε_{jt}, then we have $\mu_j = 0$. Furthermore, if the proxy variable and the non-target shock are independent, we can deduce that $\sigma_j^2 = Var(z_t)$. Therefore, we can evaluate the likelihood of a given sample covariance of z_t and ε_t given the assumption of an exogenous proxy variable.

We can now construct a re-weighting function for the likelihood of proposals $\boldsymbol{B}$ and $\boldsymbol{\pi}$ based on Equation (5). Specifically, we impose the following probabilistic proxy exogeneity moment conditions:

$$\frac{1}{T}\sum_{t=1}^{T} z_t e_{jt}(\boldsymbol{B}, \boldsymbol{\pi}) \sim N\left(0, \frac{Var(z_t)}{T}\right), \tag{6}$$

which is motivated by the assumption of an exogenous proxy variable and the central limit theorem. For an exogenous proxy variable, define the exogenous proxy re-weighting function

$$r(\boldsymbol{z}|\boldsymbol{y}, \boldsymbol{B}, \boldsymbol{\pi}) = \prod_{j=1, j\neq i}^{n} f_j\left(\frac{1}{T}\sum_{t=1}^{T} z_t e_{jt}(\boldsymbol{B}, \boldsymbol{\pi})\right), \tag{7}$$

where f_j denotes the density of a normal distribution with mean zero and variance $Var(z_t)/T$. We can use the exogenous proxy re-weighting function to adjust the likelihood of the SVAR, $p(\boldsymbol{y}|\boldsymbol{B}, \boldsymbol{\pi})$, as follows:

$$p(\boldsymbol{y}|\boldsymbol{B}, \boldsymbol{\pi})r(\boldsymbol{z}|\boldsymbol{y}, \boldsymbol{B}, \boldsymbol{\pi}). \tag{8}$$

By re-weighting the likelihood using the exogenous proxy re-weighting function, we account for the probability of satisfying the proxy exogeneity restrictions. This means that the likelihood of values for $\boldsymbol{B}, \boldsymbol{\pi}$ that result in innovations $e_{jt}(\boldsymbol{B}, \boldsymbol{\pi})$ correlated with the proxy variable will be down-weighted.[3]

A Gaussian SVAR likelihood $p(\boldsymbol{y}|\boldsymbol{B}, \boldsymbol{\pi})$ is flat with respect to orthogonal rotations of the structural parameters in $\boldsymbol{B}_0$. However, re-weighting the likelihood using the exogenous proxy re-weighting function breaks the symmetry. That is, the likelihood of structural parameters which lead to an endogenous proxy variable get downweight. Consequently, the re-weighted likelihood exhibits a unique maximum. Asymptotically, this maximum corresponds to the column of $\boldsymbol{B}_0$. Therefore, by using the exogenous proxy re-weighting, we can identify the column in $\boldsymbol{B}$ that represents the exogenous relationship, as it yields the maximum likelihood estimate.

Importantly, if we assume a Gaussian likelihood we need to assume that $\boldsymbol{\mu} = 0$, i.e. the proxy variables are exogenous, to archive identification. Throughout the paper, we label the proxy weighting approach using the proxy exogeneity assumption $\boldsymbol{\mu} = 0$ and the assumption of Gaussian errors the *Gaussian proxy weighting approach*.

In the following, we show that combining our new proxy weighting approach with statistical identification allows to estimate $\boldsymbol{\mu}$ and thus detect and neglect endogenous proxy variables. In particular, we apply identification through non-Gaussian independent errors as described

[3]In this context, our approach bears conceptual similarity to the narrative sign restrictions proposed by Antolín-Díaz and Rubio-Ramírez (2018). Their method can be interpreted as a re-weighting of the likelihood based on whether the proposed shocks satisfy the narrative sign restrictions. Similarly, in our approach, we re-weight the likelihood based on whether the proposed shocks are uncorrelated with the proxy variable.

next.

2.2 Non-Gaussian SVARs

The purpose of this section is to to explain how the non-Gaussianity of independent structural shocks can be used to identify and estimate the simultaneous interaction in the SVAR. Figure 2 provides a visual representation of how the assumption of non-Gaussian and independent shocks can facilitate the identification of the SVAR. Consider a scenario with two independent and identically distributed structural shocks, denoted as $\boldsymbol{\varepsilon}_{1t}$ and $\boldsymbol{\varepsilon}_{2t}$, drawn from a uniform distribution. In the top-left panel of the Figure, we can observe these shocks. Importantly, knowing the value of $\boldsymbol{\varepsilon}_{1t}$ does not provide any information or predictive power for $\boldsymbol{\varepsilon}_{2t}$. In practice, the structural shocks are not directly observed but manifest as reduced form shocks $\boldsymbol{u}_t = \boldsymbol{B}_0\boldsymbol{\varepsilon}_t$, shown in the top-right panel. Assuming $\boldsymbol{B}_0$ is an orthogonal matrix, any rotation of the reduced form shocks, $e(\boldsymbol{B})_t = \boldsymbol{B}^{-1}u_t$, using an orthogonal matrix $\boldsymbol{B}$, will yield uncorrelated innovations $e(\boldsymbol{B})_t$. The bottom-left and bottom-right panels illustrate two different innovations, $e(\boldsymbol{B})_t$. While both sets of innovations are uncorrelated, the bottom-left panel clearly demonstrates their dependence. For instance, knowing that $e(\boldsymbol{B})_{1t}$ is equal to two, conveys the information that $e(\boldsymbol{B})_{2t}$ is likely to be negative and close to -1.5. In this case, the innovations are uncorrelated but not independent.[4] The only solution, up to sign and permutation, to disentangle the reduced form shocks into independent innovations is shown in the bottom-left panel. Here, the innovations align with the structural shocks, resulting in an identified system.

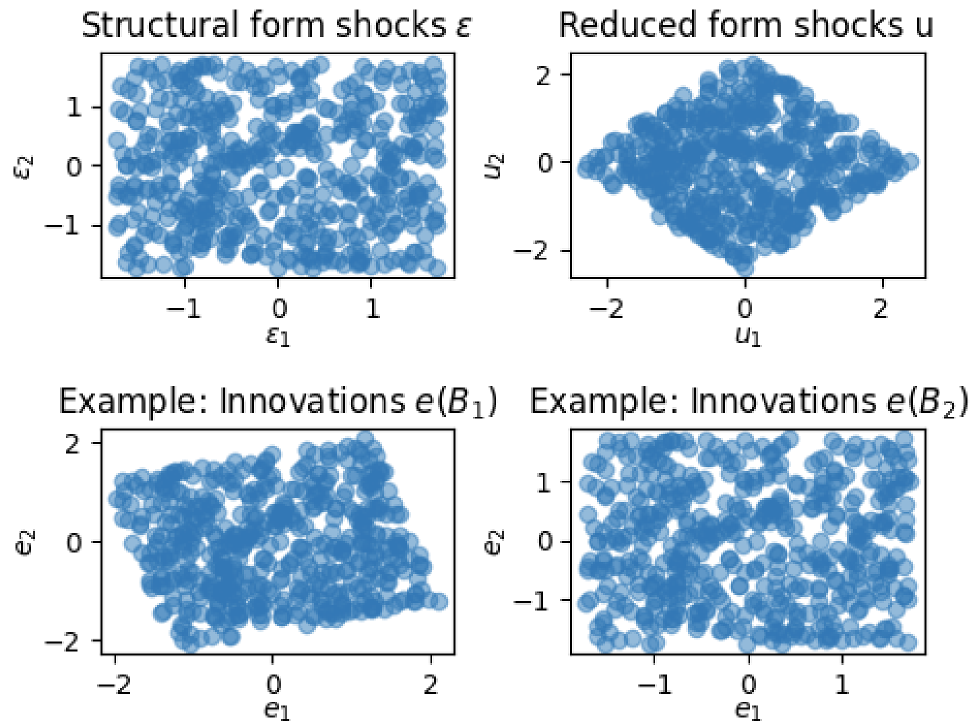
Technically, we impose the following assumptions used by Lanne et al. (2017):

Assumption 2. *1. The components of the structural shocks ε_t are a sequence of independent and identically distributed random vectors with zero mean and unit variance.*

2. The components of the structural shocks ε_t are mutually independent and at most one

[4]Consider the analogy of tax and government spending shocks. Does knowing the value of a tax shock convey information about the government spending shock? If not, merely assuming uncorrelated shocks is not sufficient to guarantee this property.

Figure 2: Identification with independent and non-Gaussian shocks



The top left plot shows a sample of independent and identically distributed structural shocks drawn from a uniform distribution. The top right plot shows the corresponding reduced form shocks equal to a rotation, $u_t = B_0 \varepsilon_t$, of the structural shocks. The bottom row shows two different sets of innovations obtained by using two different rotations $e(\boldsymbol{B}_1)$ and $e(\boldsymbol{B}_2)$. The bottom-left plot uses a rotation with $B_2 \neq B_0$. The bottom-right plot uses the rotation $B_2 = B_0$.

component has a Gaussian marginal distribution.

Lanne et al. (2017) show that these assumption are sufficient to identify the SVAR up to labeling of the shocks. Lanne and Luoto (2020), Anttonen et al. (2021), and Braun (2023) propose Bayesian non-Gaussian SVAR models based on these assumptions. Anttonen et al. (2021) assumes that each structural shock follows a generalized skewed t-distribution. We simplify the approach and assume that each shock follows a skewed t-distribution such that the density function of the ith shock is given by

$$f_i(\varepsilon_{it}; \lambda_i, q_i) = \frac{\Gamma(0.5 + q_i)}{v(\pi q_i)^{0.5}\Gamma(q_i)(\frac{|\varepsilon_{it}+m|^2}{q_i v^2(\lambda \mathrm{sign}(\varepsilon_{it}+m)+1)^2})^{0.5+q_i}}, \tag{9}$$

with $|\lambda_i| < 1$, $q_i > 2$ which implies that the fourth moment of the ith structural shock exists, and the normalization $m = \frac{2v\lambda q_i^{0.5}\Gamma(q_i-0.5)}{\pi^{0.5}\Gamma(q_i+.5)}$, $v = q_i^{-0.5}\left[(3\lambda^2+1)(\frac{1}{2q_i-2}) - \frac{4\lambda^2}{\pi}(\frac{\Gamma(q_i-0.5)}{\Gamma(q_i)})^2\right]^{-0.5}$ to mean zero and unit variance. The likelihood of a data set $\boldsymbol{y} = (\boldsymbol{y}_1', ..., \boldsymbol{y}_T')'$ given $\boldsymbol{y}_{-p+1}, ..., \boldsymbol{y}_0$ follows from Lanne et al. (2017) and is equal to

$$p(\boldsymbol{y}|\boldsymbol{\pi}, \boldsymbol{B}, \boldsymbol{\lambda}, \boldsymbol{q}) = |\det(\boldsymbol{B})|^{-T} \prod_{i=1}^{n} \prod_{t=1}^{T} f_i(\boldsymbol{e}_{it}(\boldsymbol{B}, \boldsymbol{\pi}); \lambda_i, q_i). \tag{10}$$

It is worth pointing out that by estimating $\boldsymbol{\lambda}$ and $\boldsymbol{q}$ our framework is flexible and the data can inform us over the degree of skewness and excess kurtosis, see Anttonen et al. (2021).

2.3 Combining proxy variables with non-Gaussianity

In this section, we propose a *non-Gaussian proxy weighting approach* that combines the proxy re-weighting approach from Section 2.1.2 with the non-Gaussian SVAR from Section 2.2. More precisely, we use the non-Gaussian likelihood from Equation (10) in Equation (8), which ensures identification under Assumption 2. Therefore, the proxy variable is not required for the identification of the SVAR. Consequently, in contrast to the existing proxy approaches, we are able to estimate the parameter $\boldsymbol{\mu}$, which determines the exogeneity of the proxy variables, directly from the data. Therefore, if the data does not support the validity of the proxy variable, the model can effectively ignore its information. However, if the proxy is exogenous and relevant, incorporating it leads to improved estimation accuracy and reduced estimation uncertainty in the non-Gaussian SVAR framework.

Therefore, we generalize the proxy moment condition in Equation (6) to

$$\frac{1}{T}\sum_{t=1}^{T} z_t e_{jt}(\boldsymbol{B}, \boldsymbol{\pi}) \sim N\left(\mu_j, \frac{Var(z_t)}{T}\right), \tag{11}$$

such that the density function f_j in the re-weighting function in Equation (7) is equal to the density of a normal distribution with mean μ_j and variance $Var(z_t)/T$. By doing so, we estimate the exogeneity of the proxy variable, specifically the expectation $E[z_t\varepsilon_{jt}]$. This approach allows us to adapt the re-weighting function based on the estimated values of

μ_j. Meaning, if the data provide evidence against exogeneity of the proxy variable, the estimated value μ_j will deviate from zero and no longer penalize proposals which lead to an exogenous proxy.

We propose a prior distribution for μ_j that reflects the belief in an exogenous proxy variable. The prior is designed to shrink the solution towards an exogenous proxy variable, i.e. we shrink μ_j to zero. We define the prior as follows:

$$\mu_j \sim N(0, \sigma^2_{\mu j}), \quad \sigma^2_{\mu j} \sim IG(a, b). \tag{12}$$

In our empirical work and in our simulations we set $a = b = 0$ which results in a flat prior, see Tipping (2001). Moreover, we use independent flat priors for all model parameters.[5] Hence, the joint posterior is proportional to

$$p(\boldsymbol{\pi}, \boldsymbol{B}, \boldsymbol{\lambda}, \boldsymbol{q}, \boldsymbol{\mu}, \boldsymbol{\Sigma}_{\boldsymbol{\mu}} | \boldsymbol{y}, \boldsymbol{z}) \propto p(\boldsymbol{y} | \boldsymbol{\pi}, \boldsymbol{B}, \boldsymbol{\lambda}, \boldsymbol{q}) r(\boldsymbol{z} | \boldsymbol{y}, \boldsymbol{B}, \boldsymbol{\pi}) p(\boldsymbol{\mu} | \boldsymbol{\Sigma}_{\boldsymbol{\mu}}), \tag{13}$$

with $p(\boldsymbol{\mu}|\boldsymbol{\Sigma}_{\boldsymbol{\mu}}) = N(\boldsymbol{\mu}; \boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\mu}})$ and $\boldsymbol{\Sigma}_{\boldsymbol{\mu}} = \text{diag}(\sigma^2_{\mu_1}, ..., \sigma^2_{\mu_n})$. The description of the Markov Chain Monte Carlo (MCMC) algorithm used to sample from the joint posterior distribution of the model is provided in the Appendix.

By estimating μ_j we allow for the possibility of both exogenous and endogenous proxies. Our model can utilize the information from valid proxy variables to improve estimation precision and lower estimation uncertainty of the non-Gaussian estimator, but also can alleviate biased estimation due to endogenous proxies. The prior variance $\sigma^2_{\mu j}$ is crucial as it controls how much we shrink μ_j towards zero (i.e. the case of an exogenous proxy). If we have strong beliefs in the validity of our proxy variable we would set $\sigma^2_{\mu j}$ to be small. However, such prior beliefs can be controversial and different proxy variables may lead to different economic conclusions, see our empirical work. In order to avoid fixing $\sigma^2_{\mu j}$ at an inappropriate value we instead will use the data to guide us and estimate $\sigma^2_{\mu j}$ in a

[5] Alternatively, it would be possible to use a Minnesota type prior or a more flexible global local prior on $\boldsymbol{\pi}$, see e.g., Huber and Feldkircher (2019), Cross et al. (2020) and Prüser (2023).

hierarchical fashion by using an inverse Gamma prior. Hence, by shrinking μ_j towards zero the model can benefit from valid proxies but if empirical warranted it can avoid such shrinkage and μ_j can take on values different from zero to allow for endogenous proxy variables.

The estimation of the covariance μ_j between the shock ε_{jt} and the proxy variable z_t, allowing for an endogenous proxy variable, becomes possible due to the fact that the proxy variable is not used for identification. Instead, the identification of the SVAR model relies on the assumption of independent and non-Gaussian shocks, as discussed in Section 2.2. However, the identification approach based on independent and non-Gaussian shocks only identifies the SVAR up to sign and permutations. This means that any sign-permutation of the shocks results in the same likelihood. Therefore, a MCMC algorithm may sample from different sign-permutations, such that posterior draws from the response of one variable to one shock does not come from a unique shock but rather from a combination of different shocks resulting in invalid inference, see Anttonen et al. (2021). The introduction of the proxy prior breaks this symmetry and provides a unique identification of the target shock associated with the proxy variable. Specifically, if the target shock ε_{it} exhibits the highest correlation, in absolute magnitude, with the proxy variable z_t, the sign-permutation that assigns the ith position to the shock ε_{it} will have the highest re-weighted likelihood.[6]

However, the previous method only labels the target shock of the proxy variable. Alternatively, various methods proposed in the literature can be applied to label the shocks and restrict the MCMC algorithm to these labeled shocks. Some of these alternative methods include using sign restrictions in Lanne and Luoto (2020), using knowledge of the distribution of the shocks in Gouriéroux et al. (2017), using the historical decomposition in Lewis (2023), using highest impact restrictions in Maxand (2020), or using a labeled first-step estimator in Keweloh (2023). In our simulations and applications, we utilize the labeling approach proposed by Keweloh (2023), which generalizes the labeling approach in Lanne

[6] Note that an endogenous proxy variable still fulfills this condition as long as the correlation of the proxy variable with the non-target shocks is smaller in absolute value than the correlation with the target shock.

et al. (2017). This method, employing a labeled first-step estimator $\tilde{\boldsymbol{B}}_0$ of $\boldsymbol{B}_0$ to restrict the posterior draws to unique sign-permutation representatives centered around the first-step estimator. More precisely, we reject each proposal $\boldsymbol{B}_{prop}$ in the MCMC algorithm which does not satisfy $|c_{kk}| > |c_{kl}|$ for all $k < l$ with c_{kl} elements of $\boldsymbol{C} := \tilde{\boldsymbol{B}}_0^{-1}\boldsymbol{B}_{prop}\boldsymbol{D}$, with a scaling matrix $\boldsymbol{D}$ such that each column has Euclidean norm one.

3 Monte Carlo Simulations

In this section, we demonstrate the ability of our non-Gaussian proxy weighting approach in handling both exogenous and endogenous proxy variables. In case of a valid proxy variable our model is able to use the information of the proxy which leads to a better performance compared to a purely non-Gaussian model. Furthermore, even when the proxy exhibits weak exogeneity, meaning it is only minimally affected by non-target shocks, we illustrate that our approach retains the ability to utilize the proxy to enhance efficiency, surpassing the performance of the non-Gaussian model alone. Moreover, unlike conventional proxy SVAR estimators, our approach is able to detect if the data provide evidence against the exogeneity of the proxy and thus, is able to to detect and neglect information from endogenous proxies.

We simulate a system containing a government spending shock $\varepsilon_{g,t}$, an output shock $\varepsilon_{y,t}$, and a tax shock $\varepsilon_{\tau,t}$ with

$$\begin{bmatrix} u_{g,t} \\ u_{y,t} \\ u_{\tau,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.15 & 1 & -0.5 \\ 0 & 1.5 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{g,t} \\ \varepsilon_{y,t} \\ \varepsilon_{\tau,t} \end{bmatrix}. \tag{14}$$

The structural shocks are independently and identically drawn from a Pearson distribution with mean zero, variance one, skewness 0.68 and excess kurtosis 2.33 and simulate 1000 data sets of length $T = 250$ and $T = 800$.

We construct a variable z_t as a proxy for the tax shock $\epsilon_{\tau,t}$. To demonstrate the flexibility

Table 1: Data-generating process of the proxy in different scenarios.

exogenous proxy	weakly endogenous proxy	endogenous proxy
$z_{\tau,t} = \varepsilon_{\tau,t} + \eta_t$	$z_{\tau,t} = \varepsilon_{\tau,t} - 0.05\varepsilon_{y,t} + \eta_t$	$z_{\tau,t} = \varepsilon_{\tau,t} - 0.37\varepsilon_{y,t} + \eta_t$

of our model we consider three different scenarios summarized in Table 1. In the first scenario the proxy is exogenous, in the second scenario, the proxy is weakly endogenous, and in the third scenario the proxy is endogenous. In all three simulations the proxy noise η_t is independently and identically drawn from the same distribution as the structural form shocks.

We consider four Bayesian estimation approaches. First, we estimate the SVAR using the standard augmented proxy SVAR approach, assuming Gaussian shocks and imposing exogeneity of the proxy. Our second approach involves estimating the model under the assumption of Gaussian shocks and employing the proxy weighting method, which enforces proxy exogeneity but lacks the flexibility to adapt this exogeneity assumption during estimation. The third estimation method involves estimating the SVAR using the non-Gaussian from Section 2.2 without the proxy variable. Finally, our fourth approach entails estimating the SVAR using our proposed non-Gaussian SVAR model in conjunction with the proxy weighting function described in Section 2.3 with

$$\frac{1}{T}\sum_{t=1}^{T} z_t e_{g,t}(\boldsymbol{B},\boldsymbol{\pi}) \sim N(\mu_g, 1/T) \text{ and } \frac{1}{T}\sum_{t=1}^{T} z_t e_{y,t}(\boldsymbol{B},\boldsymbol{\pi}) \sim N(\mu_y, 1/T) \tag{15}$$

with $\mu_g \sim N(0,\sigma^2_{\mu g})$ and $\sigma^2_{\mu g} \sim IG(0,0)$ and $\mu_y \sim N(0,\sigma^2_{\mu y})$ and $\sigma^2_{\mu y} \sim IG(0,0)$. Both non-Gaussian models are restricted to the set of unique sign-permutation representatives centered at the true impact matrix B_0 from the DGP in Equation (14) as described in Section 2.3.

Table 2 shows the average point estimates as well as the mean squared error (MSE) of the estimated impact of $\varepsilon_{\tau,t}$. The non-Gaussian model does not use the proxy variable and its performance is not affected by the different scenarios. Both Gaussian proxy approaches lead to very similar results and perform notably better than the non-Gaussian model if the proxy variable is exogenous, however, including an endogenous proxy leads to biased estimates. Combining statistical identification with proxy variables provides a balanced solution between these two extreme cases. If the proxy variable is exogenous, adding the proxy to the non-Gaussian estimator leads to an increase of the performance of the model, i.e. the MSE is twice as small compared to the non-Gaussian model. Therefore, the estimator is able to utilise the information of a valid proxy. However, in contrast to the Gaussian proxy estimators, our proposed combination approach is able to deal with endogenous proxy variables. If the proxy variable is only weakly endogenous our non-Gaussian proxy weighting approach can still exploit the information of the proxy to deliver improved estimation accuracy in comparison to the non-Gaussian estimator. Moreover, if the proxy variable is endogenous, our non-Gaussian proxy weighting approach is less biased compared to the pure proxy approaches, and with more data and more evidence against the validity of the proxy variable, the prior gets updated and the bias decreases.

Next we turn our attention to the finite sample properties of the 68% credible bands of the models. Table 3 shows the coverage rate (defined as the proportion in which the credible bands contain the true value) and the average length of the credible bands. The non-Gaussian model ignores the proxy variable, performs similar throughout the three specifications, and has correct coverage rates (the coverage rate is close to the probability chosen for the credible bands). If the prior belief of an exogenous proxy is correct, adding the proxy also leads to correct coverage and more informative credible bands in the sense that the bands are up to 50% smaller compared to the non-Gaussian model ignoring the proxy variable. Adding an endogenous proxy using to the non-Gaussian model via our weighting approach worsens the coverage rates. However, with an increasing sample size and more information against the prior belief of an exogenous proxy, the coverage rate improves and the difference of the average error bands length between the model with and without prior vanishes. Importantly, adding an weakly endogenous proxy also helps in a small sample size to lower the lengths of the credible bands without distorting the coverage much.

Table 2: Average point estimates and MSE for the impact of $\varepsilon_{\tau,t}$.

	exogenous proxy $z_{\tau,t} = \varepsilon_{\tau,t} + \eta_t$	weakly endogenous proxy $z_{\tau,t} = \varepsilon_{\tau,t} - 0.10\varepsilon_{y,t} + \eta_t$	endogenous proxy $z_{\tau,t} = \varepsilon_{\tau,t} - 0.37\varepsilon_{y,t} + \eta_t$
$T = 250$			
proxy (augmented)	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.009)}{-0.50} & \underset{(0.022)}{1.00}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.018)}{-0.59} & \underset{(0.046)}{0.86}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.106)}{-0.81} & \underset{(0.363)}{0.42}\end{bmatrix}'$
proxy (weighting)	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.009)}{-0.50} & \underset{(0.022)}{1.00}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.017)}{-0.59} & \underset{(0.046)}{0.85}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.106)}{-0.81} & \underset{(0.372)}{0.42}\end{bmatrix}'$
non-Gaussian	$\begin{bmatrix}\underset{(0.017)}{0.00} & \underset{(0.021)}{-0.49} & \underset{(0.051)}{0.95}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.018)}{0.00} & \underset{(0.019)}{-0.48} & \underset{(0.045)}{0.95}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.018)}{0.00} & \underset{(0.019)}{-0.48} & \underset{(0.046)}{0.95}\end{bmatrix}'$
non-Gaussian proxy weighting	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.009)}{-0.50} & \underset{(0.022)}{0.99}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.008)}{0.00} & \underset{(0.011)}{-0.54} & \underset{(0.028)}{0.93}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.009)}{0.00} & \underset{(0.026)}{-0.59} & \underset{(0.081)}{0.82}\end{bmatrix}'$
$T = 800$			
proxy (augmented)	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.003)}{-0.50} & \underset{(0.007)}{1.00}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.012)}{-0.60} & \underset{(0.032)}{0.84}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.103)}{-0.82} & \underset{(0.355)}{0.41}\end{bmatrix}'$
proxy (weighting)	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.003)}{-0.50} & \underset{(0.007)}{1.00}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.012)}{-0.60} & \underset{(0.031)}{0.85}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.102)}{-0.82} & \underset{(0.353)}{0.41}\end{bmatrix}'$
non-Gaussian	$\begin{bmatrix}\underset{(0.005)}{0.00} & \underset{(0.005)}{-0.49} & \underset{(0.012)}{0.99}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.005)}{0.00} & \underset{(0.005)}{-0.49} & \underset{(0.012)}{0.99}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.005)}{0.00} & \underset{(0.005)}{-0.49} & \underset{(0.012)}{0.99}\end{bmatrix}'$
non-Gaussian proxy weighting	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.003)}{-0.50} & \underset{(0.006)}{1.00}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.002)}{0.00} & \underset{(0.005)}{-0.53} & \underset{(0.012)}{0.94}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.003)}{0.00} & \underset{(0.006)}{-0.52} & \underset{(0.017)}{0.96}\end{bmatrix}'$

Note: The true impact of the shock $\varepsilon_{\tau,t}$ is $\begin{bmatrix}0 & -0.5 & 1\end{bmatrix}'$. The average and MSE of the Bayesian estimators are calculated based on the median of the posterior of B in each simulation.

Table 3: Coverage and average length of 68% credible bands of the estimated impact of $\varepsilon_{\tau,t}$.

	exogenous proxy $z_{\tau,t} = \varepsilon_{\tau,t} + \eta_t$	weakly endogenous proxy $z_{\tau,t} = \varepsilon_{\tau,t} - 0.10\varepsilon_{y,t} + \eta_t$	endogenous proxy $z_{\tau,t} = \varepsilon_{\tau,t} - 0.37\varepsilon_{y,t} + \eta_t$
$T = 250$			
proxy (augmented)	$\begin{bmatrix}\underset{(0.018)}{0.68} & \underset{(0.019)}{0.68} & \underset{(0.028)}{0.69}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.018)}{0.67} & \underset{(0.018)}{0.46} & \underset{(0.030)}{0.48}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.017)}{0.68} & \underset{(0.016)}{0.00} & \underset{(0.031)}{0.01}\end{bmatrix}'$
proxy (weighting)	$\begin{bmatrix}\underset{(0.018)}{0.68} & \underset{(0.019)}{0.67} & \underset{(0.029)}{0.70}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.018)}{0.68} & \underset{(0.018)}{0.47} & \underset{(0.030)}{0.48}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.017)}{0.56} & \underset{(0.015)}{0.06} & \underset{(0.030)}{0.07}\end{bmatrix}'$
non-Gaussian	$\begin{bmatrix}\underset{(0.026)}{0.73} & \underset{(0.027)}{0.68} & \underset{(0.041)}{0.69}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.026)}{0.70} & \underset{(0.027)}{0.70} & \underset{(0.040)}{0.71}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.026)}{0.69} & \underset{(0.027)}{0.69} & \underset{(0.040)}{0.70}\end{bmatrix}'$
non-Gaussian proxy weighting	$\begin{bmatrix}\underset{(0.017)}{0.70} & \underset{(0.018)}{0.68} & \underset{(0.028)}{0.70}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.017)}{0.70} & \underset{(0.018)}{0.64} & \underset{(0.029)}{0.64}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.018)}{0.71} & \underset{(0.024)}{0.52} & \underset{(0.040)}{0.53}\end{bmatrix}'$
$T = 800$			
proxy (augmented)	$\begin{bmatrix}\underset{(0.010)}{0.70} & \underset{(0.010)}{0.65} & \underset{(0.016)}{0.66}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.010)}{0.70} & \underset{(0.010)}{0.18} & \underset{(0.016)}{0.17}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.010)}{0.71} & \underset{(0.009)}{0.00} & \underset{(0.017)}{0.00}\end{bmatrix}'$
proxy (weighting)	$\begin{bmatrix}\underset{(0.010)}{0.70} & \underset{(0.010)}{0.65} & \underset{(0.016)}{0.66}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.010)}{0.70} & \underset{(0.010)}{0.19} & \underset{(0.016)}{0.17}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.018)}{0.67} & \underset{(0.016)}{0.01} & \underset{(0.031)}{0.01}\end{bmatrix}'$
non-Gaussian	$\begin{bmatrix}\underset{(0.013)}{0.66} & \underset{(0.013)}{0.66} & \underset{(0.020)}{0.66}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.013)}{0.67} & \underset{(0.013)}{0.66} & \underset{(0.020)}{0.66}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.013)}{0.67} & \underset{(0.013)}{0.66} & \underset{(0.020)}{0.66}\end{bmatrix}'$
non-Gaussian proxy weighting	$\begin{bmatrix}\underset{(0.009)}{0.68} & \underset{(0.010)}{0.68} & \underset{(0.015)}{0.66}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.009)}{0.68} & \underset{(0.010)}{0.54} & \underset{(0.017)}{0.54}\end{bmatrix}'$	$\begin{bmatrix}\underset{(0.010)}{0.70} & \underset{(0.014)}{0.64} & \underset{(0.021)}{0.63}\end{bmatrix}'$

In the Online Appendix we present results of additional Monte Carlos simulations. Importantly, we show that if the proxy variables does not follow a linear process (as in our empirical work) than we can not combine the augmented proxy approach (which uses a linear proxy specification) with statistically identification to detect exogenous proxy variables as shown in section 2.1.1. This is the reason why it is essential to use the proxy weighting approach with a non-Gaussian likelihood. Although both approach lead to the same results with a Gaussian likelihood.

To further demonstrate the flexibility of our approach we consider a range of different plausible scenarios an applied researcher may face. In many application the relevance of the proxy may be weak. In this case we show that pure proxy approaches lead to biased estimates. In contrast, our combined approach still works and can exploit the proxy information to improve on the pure statistical identification approach. In addition, alternative scenarios include a simulation with t-distributed shocks, with Gaussian shocks, a proxy with missing values, and a simulation with two proxy variables. Finally, we consider an alternative DGP based on the estimation results from Mertens and Ravn (2014). The results show that our model is highly flexible and can adapt to different scenarios.

4 Estimating fiscal multipliers

This section applies our proposed non-Gaussian proxy weighting approach to estimate the effects of exogenous changes in tax revenues and government spending. First, we describe the data and show that the time series feature a sizeable degree of non-Gaussianity. Thereafter our main findings are discussed. We find a larger government spending than tax multiplier and show that our proposed approach leads to different elasticities of tax revenues and government spending, respectively, compared to Mertens and Ravn (2014) and Caldara and Kamps (2017). Moreover, we provide evidence indicating that the fiscal and non-fiscal proxies used by Mertens and Ravn (2014) and Caldara and Kamps (2017), respectively, do not fulfill the crucial exogeneity assumption which biases the results of Gaussian proxy approaches. Finally, we construct new fiscal and non-fiscal proxies that are orthogonal to the structural shocks of the VAR model and thus can be used in a standard proxy VAR. The estimated multipliers using the new proxies using a Gaussian proxy approach are very similar to the ones of our baseline non-Gaussian proxy weighting approach.

4.1 Data and specification

To achieve comparability, we use the same trivariate VAR as adopted by Mertens and Ravn (2014).[7] The three endogenous variables are federal tax revenues τ_t, federal government consumption and investment expenditures g_t, and output y_t, all in log real per capita terms and for the sample 1950Q2 to 2006Q4. The data are downloaded from Karel Mertens' website. Additional details can be found in Mertens and Ravn (2014). The VAR has four lags of the endogenous variables and includes a constant, linear and quadratic trends, and a dummy for 1975Q2 all contained in $\boldsymbol{X}_t$. The SVAR is given by

$$\begin{bmatrix} \tau_t \\ g_t \\ y_t \end{bmatrix} = \boldsymbol{\gamma} \boldsymbol{X}_t + \sum_{i=1}^{4} \boldsymbol{A}_i \begin{bmatrix} \tau_{t-i} \\ g_{t-i} \\ y_{t-i} \end{bmatrix} + \begin{bmatrix} u_{\tau,t} \\ u_{g,t} \\ u_{y,t} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} u_{\tau,t} \\ u_{g,t} \\ u_{y,t} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{g,t} \\ \varepsilon_{y,t} \end{bmatrix}, \tag{16}$$

with structural tax shocks $\varepsilon_{\tau,t}$, structural government spending shocks $\varepsilon_{g,t}$, and output shocks $\varepsilon_{y,t}$. For comparison, we also map the estimated simultaneous interaction into the notation used by Mertens and Ravn (2014)

$$u_{\tau,t} = \Theta_G \sigma_G \varepsilon_{g,t} + \Theta_Y u_{y,t} + \sigma_\tau \varepsilon_{\tau,t} \tag{17}$$

$$u_{g,t} = \gamma_\tau \sigma_\tau \varepsilon_{\tau,t} + \gamma_Y u_{y,t} + \sigma_G \varepsilon_{g,t} \tag{18}$$

$$u_{y,t} = \eta_\tau u_{\tau,t} + \eta_G u_{g,t} + \sigma_Y \varepsilon_{y,t}. \tag{19}$$

[7] Caldara and Kamps (2017) consider a slightly larger VAR consisting of five endogenous variables as baseline model but also report results for the three variables specification we rely on. We show below that our main findings are robust to extending the baseline trivariate VAR by inflation and the interest rate as done by Caldara and Kamps (2017).

Mertens and Ravn (2014) use a tax proxy $z_{\tau,t}$ and an additional zero restriction imposing that government spending does not respond contemporaneously to changes in economic activity. The tax proxy relies on a series of possibly unanticipated tax shocks, a subset of the Romer and Romer (2010) tax shocks identified by studying narrative records of tax policy decisions.[8] As argued, the tax proxy measures changes in the tax system that are primarily not related to the state of the economy and thereby it should offer a valid proxy for tax shocks. Therefore, Mertens and Ravn (2014) impose the following exogeneity assumptions

$$E[z_{\tau,t}\varepsilon_{g,t}] = 0 \quad \text{and} \quad E[z_{\tau,t}\varepsilon_{y,t}] = 0. \tag{20}$$

Instead of using a fiscal proxy, Caldara and Kamps (2017) rely on the non-fiscal Fernald (2012) TFP measure as a proxy $z_{y,t}$ for output shocks and additionally assume that government spending does not respond contemporaneously to structural tax shocks. The Fernald (2012) technology series measures total factor productivity adjusted for changes in factor utilization. Fernald (2012) carefully eliminates these sources of endogenous movements such that his resulting purified TFP series can be understood as solely reflecting exogenous technology variations which motivates the two exogeneity assumptions

$$E[z_{y,t}\varepsilon_{\tau,t}] = 0 \quad \text{and} \quad E[z_{y,t}\varepsilon_{g,t}] = 0 \tag{21}$$

used by Caldara and Kamps (2017). In what follows, we rely on the original TFP measure used by Caldara and Kamps (2017) which is provided by the replication codes available on the website of the Review of Economic Studies.

[8] To achieve exogeneity of the proxy the authors exploit the narrative information in official historical documents in two ways. First, they verify that the policy documents do not discuss a desire to respond to current or prospective economic conditions and return growth to normal. Second, within the set of policy changes not motivated by the near-term economic outlook, they focus on tax changes motivated either by a desire to reduce the budget deficit or by raising long-run growth. Moreover, Mertens and Ravn (2014) use only those tax shocks to instrument exogenous policy changes for which potential anticipation effects are arguably unlikely. More precisely, they omit all tax liability changes that were implemented more than 90 days (one quarter) after becoming law.

Both approaches start with carefully motivated identifying assumptions, yet both approaches lead to different conclusions regarding the effects of tax and spending shocks. In particular, using the fiscal proxy leads to a large tax multiplier while the non-fiscal proxy leads to a large spending multiplier. This difference indicates that at least one of the identifying proxy exogeneity assumptions is invalid.[9] However, both approaches rely on the exogeneity assumptions to identify the SVAR and hence, cannot detect or update invalid proxy exogeneity assumptions. Our proposed model fills this gap since it allows to evaluate the empirical support for the exogeneity assumptions and thus helps in understanding diverging findings regarding the size of fiscal multipliers between the fiscal and non-fiscal approach.

In particular, we estimate the SVAR in Equation (16) using the non-Gaussian proxy weighting approach without any zero restrictions on the response of government spending. We normalize both proxies to mean zero and unit variance and use the following four proxy weighting moment conditions

$$\frac{1}{T}\sum_{t=1}^{T} z_{\tau,t} e_{g,t}(\boldsymbol{B},\boldsymbol{\pi}) \sim N(\mu_{\tau g}, 1/T), \qquad \frac{1}{T}\sum_{t=1}^{T} z_{\tau,t} e_{y,t}(\boldsymbol{B},\boldsymbol{\pi}) \sim N(\mu_{\tau y}, 1/T) \tag{22}$$

$$\frac{1}{T}\sum_{t=1}^{T} z_{y,t} e_{\tau,t}(\boldsymbol{B},\boldsymbol{\pi}) \sim N(\mu_{y\tau}, 1/T), \qquad \frac{1}{T}\sum_{t=1}^{T} z_{y,t} e_{g,t}(\boldsymbol{B},\boldsymbol{\pi}) \sim N(\mu_{yg}, 1/T) \tag{23}$$

with $\mu_j \sim N(0, \sigma^2_{\mu j})$ and $\sigma^2_{\mu j} \sim IG(0,0)$ for $j \in \{\tau g, \tau y, y\tau, yg\}$. Therefore, we start with the prior that the fiscal and the non-fiscal proxies are both valid and shrink towards the exogenous proxy solution. However, the data can update the prior and deviate from a given exogeneity assumption. Moreover, we use the proxy variables to label the shocks. Specifically, the shock exhibiting the highest correlation in absolute terms with the TFP proxy is labeled as the output shock. Among the two remaining shocks, the one displaying the strongest correlation in absolute terms with the tax proxy is identified as the tax shock,

[9] Figure 1 shows the estimated tax shocks using the tax proxy in comparison to the estimated output shocks using the TFP proxy. The correlation between both shocks is not affected by the zero restrictions and indicates that at least one of the identifying proxy exogeneity assumptions is invalid.

while the remaining shock is attributed as the government spending shock.[10]

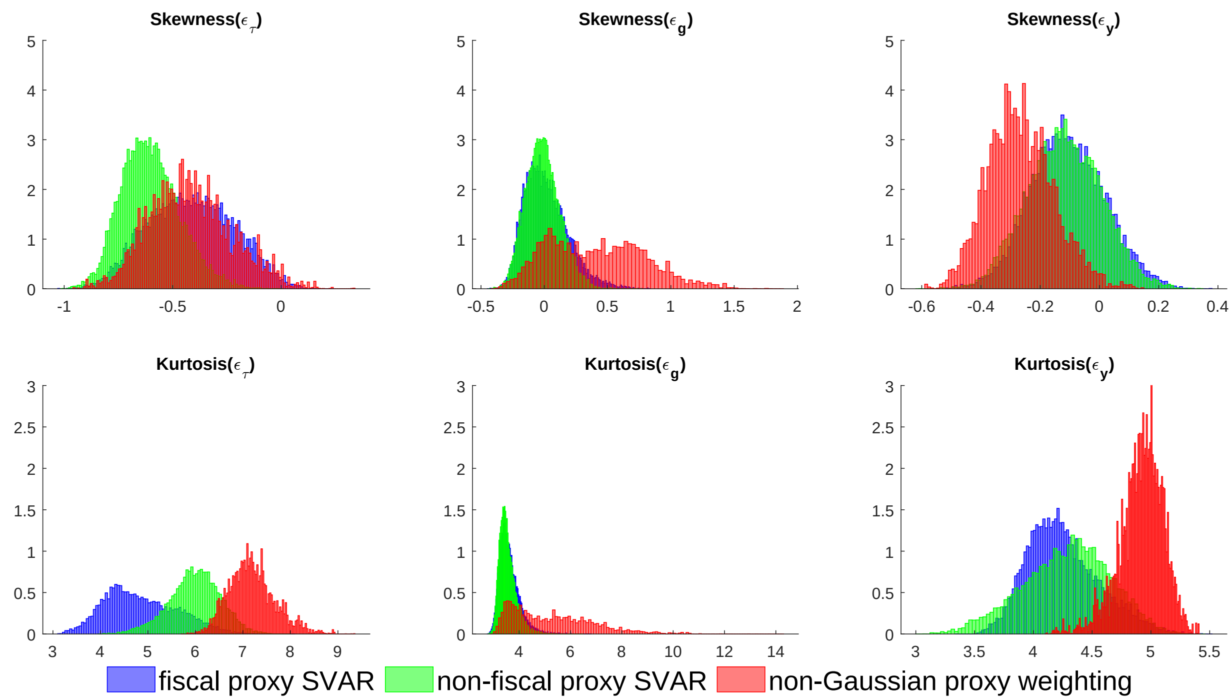
For comparison, we also estimate the Mertens and Ravn (2014) fiscal proxy SVAR with the Gaussian proxy weighting approach, the government spending restriction $b_{23} = 0$, and the tax proxy weighting moment conditions in Equation (22) with $\mu_{\tau g} = \mu_{\tau y} = 0$ and the Caldara and Kamps (2017) non-fiscal proxy SVAR with the Gaussian proxy-weighting approach, the government spending restriction $b_{13} = 0$, and the TFP proxy weighting moment conditions in Equation (23) with $\mu_{y\tau} = \mu_{yg} = 0$. Therefore, both Gaussian models impose the corresponding proxy prior without the ability to update the proxy prior. Note that the two Gaussian proxy weighting models yield impulse responses akin to those acquired through the conventional moment-based frequentist proxy estimator, as outlined in the Appendix.

A necessary requirement for updating the proxy exogeneity by the data is that we work with non-Gaussian structural shocks. Figure 3 shows the posterior of the skewness and kurtosis for the estimated structural shocks in the non-Gaussian proxy weighting SVAR and the two Gaussian proxy weighting SVARs. All three models show a sizeable degree of non-Gaussianity in the estimated structural shocks. In particular, the skewness of the tax and output shock is centered around non-zero values and the kurtosis shows positive values above three. Across all models, the tax and output shocks are left skewed, which is economically reasonable because tax cuts tend to be larger than tax hikes and output falls stronger during recessions than it rises during expansions. In addition, there is some evidence that the government spending shock is right skewed, indicating that spending stimuli are larger than spending consolidations.

Figure 4 plots results on the relevance of the tax and TFP proxies in our non-Gaussian proxy weighting model. The left column shows the posterior distribution of the correlation between the tax proxy and the structural tax shock and the right column presents the distribution of the correlation between the TFP proxy and the structural output shock. Both distributions are centered around a positive mean with no support of values close

[10] An alternative labeling approach relying on a first step estimator is shown in the Appendix and leads to similar results.

Figure 3: Evidence for non-Gaussianity



The figure shows the posterior distributions of the skewness as well as the kurtosis of the structural shocks. We show results for our non-Gaussian proxy weighting SVAR, the non-fiscal proxy SVAR proposed by Caldara and Kamps (2017) as well as the fiscal proxy SVAR from Mertens and Ravn (2014).

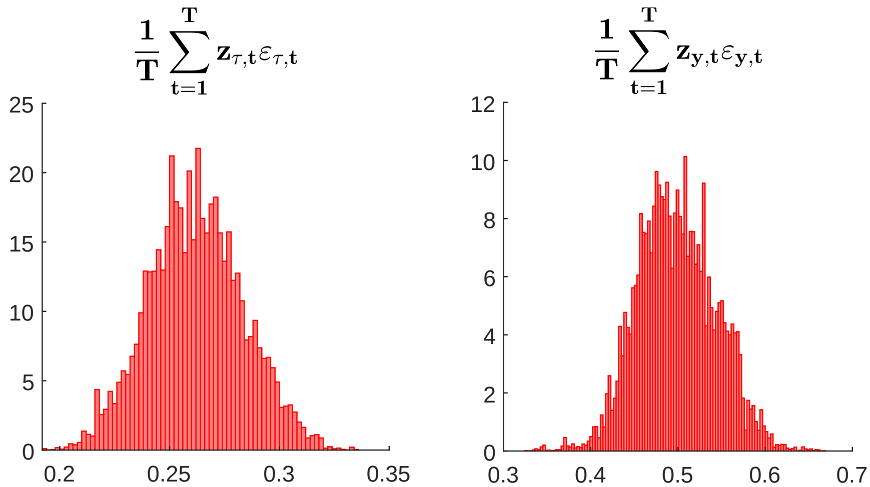
to zero. Thus, we find that both instruments are strong in the sense that they fulfill the relevance condition which is very much in line with the evidence provided by Mertens and Ravn (2014) and Caldara and Kamps (2017).[11]

4.2 Baseline results

Figure 5 presents the estimated tax and spending multipliers, where the first column reports the tax multiplier, the second column shows the spending multiplier, and the third column presents the estimated difference between the tax and spending multiplier. Similar to Mertens and Ravn (2014), we calculate tax multipliers by dividing the output response of a tax revenue shock of minus one percent by the average ratio of federal tax revenues to GDP in the sample of 17.5%. Government spending multipliers are calculated by dividing

[11]Note that, we plot the correlation between the instrument and the structural shock, whereas Mertens and Ravn (2014) and Caldara and Kamps (2017) have to focus on the correlation between the instrument and the reduced form shock.

Figure 4: Posterior of proxy relevancy moment conditions

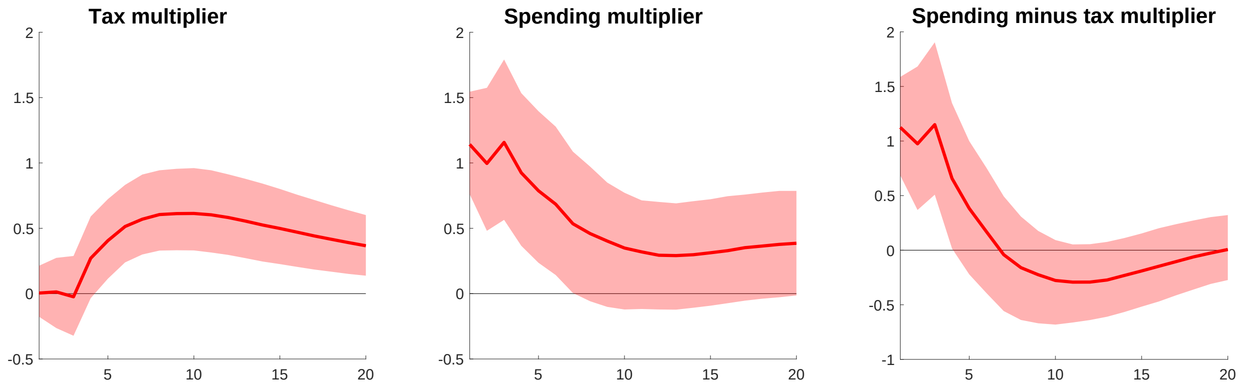


The figure shows that both the tax proxy and the TFP are related to their target shocks.

the output response of a public spending shock of one percent by the average ratio of federal spending to GDP in the sample of 9.1%. Equivalently, the numbers reflect the present response to a tax cut (government spending increase) that lowers (increases) tax revenues (government spending) by one percentage point of GDP.

The tax multiplier as reported in the first column of Figure 5 is estimated to be close to zero for the impact period. Only around one year after the shock, the multiplier increases. The multiplier peaks at a value of 0.71 two years after the shock materialized. Thereafter, the response slowly converges back to its pre-shock level. The estimated government spending multiplier presented in the second column shows some stark differences. For the impact period, the spending multiplier is estimated to be different from zero taking on a value above but close to unity. The peak response is reached three quarters after the shock with a value of 1.16 and the credible bands around the estimated spending multiplier do not contain the value of zero for more than two years following the shock. The third column showing the difference between the government spending and tax multiplier clearly depicts the divergent dynamics of both multipliers. For the first year after the shock, the government spending multiplier clearly exceeds the tax multiplier. Over the medium term, the difference becomes negative implying that the tax multiplier is larger than the government spending multiplier, although the point estimate for the difference is small.

Figure 5: Estimated output multipliers for the non-Gaussian proxy weighting SVAR

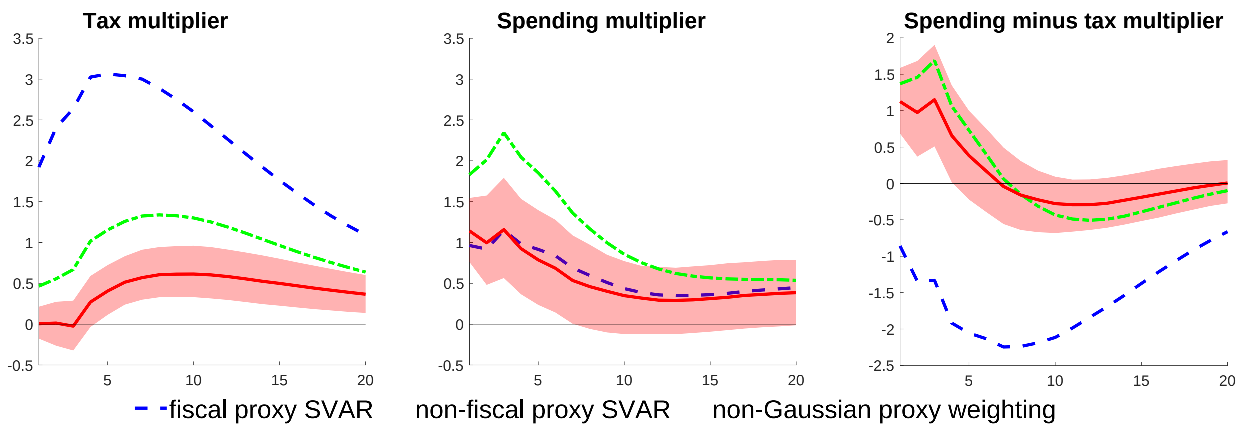


The figure shows the posterior median as well as 68% credible bands for the impulse responses of output to a tax shock and a government spending shock. The right subfigure shows the posterior median as well as 68% credible bands for the difference of the two output responses.

Thus, our baseline non-Gaussian proxy weighting model suggests that positive shocks to government spending have a larger stimulating impact on economic activity than exogenous tax cuts.

Figure 6 compares our baseline estimates of the non-Gaussian model to the fiscal proxy SVAR from Mertens and Ravn (2014) and to the non-fiscal proxy SVAR from Caldara and Kamps (2017). Notably, our estimates show strong differences to the fiscal proxy SVAR. The fiscal proxy SVAR leads to a much larger tax multiplier compared to our baseline estimates. In particular, the fiscal proxy SVAR delivers an on-impact tax multiplier close to two and the multiplier further increases in the subsequent periods reaching a peak value close to three. Given the much larger tax multiplier, the fiscal proxy SVAR implies that exogenous tax cuts are a more powerful tool to stimulate the economy compared to exogenous increases in government spending, which is the opposite to our baseline results. Additionally, the non-fiscal proxy SVAR leads to a larger government spending multiplier compared to our baseline estimates. In particular, the spending multiplier peaks at a value above two, whereas our baseline estimate shows a maximum value above but close to unity. Similar to our non-Gaussian proxy weighting model, applying the the non-fiscal proxy SVAR also results in a positive difference between the government spending and tax multiplier.

Figure 6: Comparison of estimated output multipliers between the different models



The figure compares the output responses between our non-Gaussian proxy weighting SVAR with 68% credible bands to the median responses in the non-fiscal proxy SVAR proposed by Caldara and Kamps (2017) as well as the fiscal proxy SVAR from Mertens and Ravn (2014).

Figure 7 reports the posterior of the cyclical elasticities of tax revenues (Θ_y) and government spending (γ_y) respectively. As discussed in Caldara and Kamps (2017), these two parameters crucially determine the size of the estimated multipliers. The tax multiplier increases in the size of the elasticity of tax revenues. In contrast, the smaller the elasticity of government spending, the larger the spending multiplier. The intuition for this relation can be summarized as follows. There exists a positive correlation between government spending and output in the data, which any identification approach decomposes into a fraction explained by government spending shocks and a fraction explained by the remaining shocks of the SVAR. The timing assumption imposed by Mertens and Ravn (2014) implies that government spending does not respond contemporaneously to any other shock. Therefore, all the positive contemporaneous relations in the data must be explained by the government spending shock. This leads to a spending multiplier of about one. If the systematic response of government spending increases, the remaining shocks explain a larger part of the positive correlation and the spending multiplier decreases. In contrast, if the systematic response decreases, that is, turns negative, the multiplier increases. A similar reasoning applies to the identification of tax shocks.

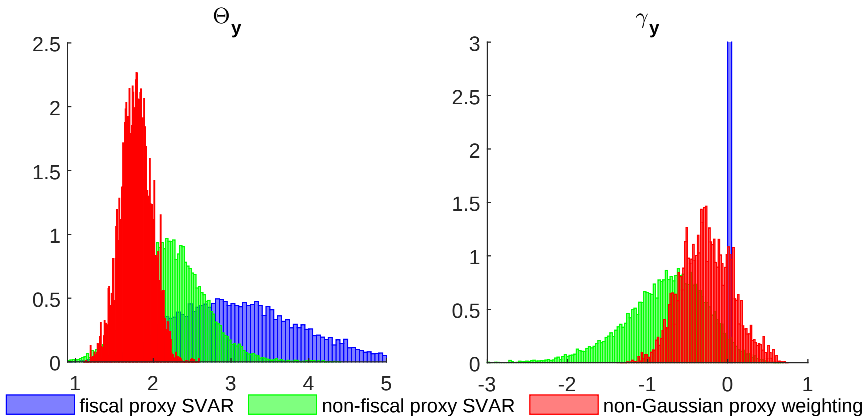
As shown in Figure 7, the different identification approaches provide different estimates for the elasticities of tax revenues and government spending. The fiscal proxy SVAR based on Mertens and Ravn (2014) leads to a tax revenue elasticity centered around three. The estimated elasticity is considerably smaller for the non-fiscal proxy SVAR and our non-Gaussian proxy weighting model. While the non-fiscal proxy approach results in an elasticity centered above two, our proposed non-Gaussian model implies an elasticity centered below two.[12] Given the positive relationship between the tax elasticity and the tax multiplier, this explains the large differences between the estimated tax multipliers across identification strategies. With a larger tax elasticity, the fiscal proxy SVAR produces the largest tax multiplier, whereas our data driven strategy leads to the smallest tax elasticity and tax multiplier. The picture is similar for the spending elasticity and the government spending multiplier. By adopting a zero restriction on the spending elasticity, the fiscal proxy SVAR results in the largest spending elasticity across identification strategies. In contrast, the non-fiscal proxy SVAR produces the smallest spending elasticity and thus the largest spending multiplier. Our non-Gaussian proxy weighting model implies a value in between the estimates of the two other approaches and thus the estimated government spending multiplier is larger (smaller) than the one obtained by the fiscal proxy (non-fiscal proxy) approach.

To summarize, our non-Gaussian proxy weighting model produces a tax multiplier that is smaller than the one found when relying the on the fiscal proxy SVAR. Furthermore, we find a considerably smaller government spending multiplier compared to the non-fiscal proxy SVAR. We will show below that the differences in the estimated multipliers across models is due to the fact that the proxies used in the fiscal and non-fiscal SVAR do not fulfil the crucial exogeneity restrictions. When relying on newly constructed exogenous proxies, the fiscal and non-fiscal SVAR lead to estimated multipliers that are very similar to the ones of our proposed non-Gaussian proxy weighting SVAR.

Our main finding that the government spending multiplier is larger than the tax multiplier

[12]The value of the estimated elasticity of our non-Gaussian model is close to the one calculated by Follette and Lutz (2010) based on institutional details of tax revenues. In particular, Follette and Lutz (2010) estimate the elasticity of tax revenues with respect to output for the federal government, and obtain a value of 1.6 for the period 1986–2008 and 1.4 for 1960–85.

Figure 7: Posterior of tax revenue elasticity Θ_y and government spending elasticity γ_y



The figure compares the Posterior of tax revenue elasticity Θ_y and government spending elasticity γ_y between our proxy shrinkage VAR with the non-fiscal proxy SVAR proposed by Caldara and Kamps (2017) as well as the fiscal proxy SVAR from Mertens and Ravn (2014).

is actually highly robust to modifications of the baseline empirical specification. We report a battery of robustness checks in the appendix. We show that our results are robust to including additional endogenous variables in the VAR, controlling for fiscal foresight, and splitting the sample. Moreover, the results are not significantly affected when separately estimating two models that use only one of the proxies compared to the baseline model that uses both proxies in the estimation.

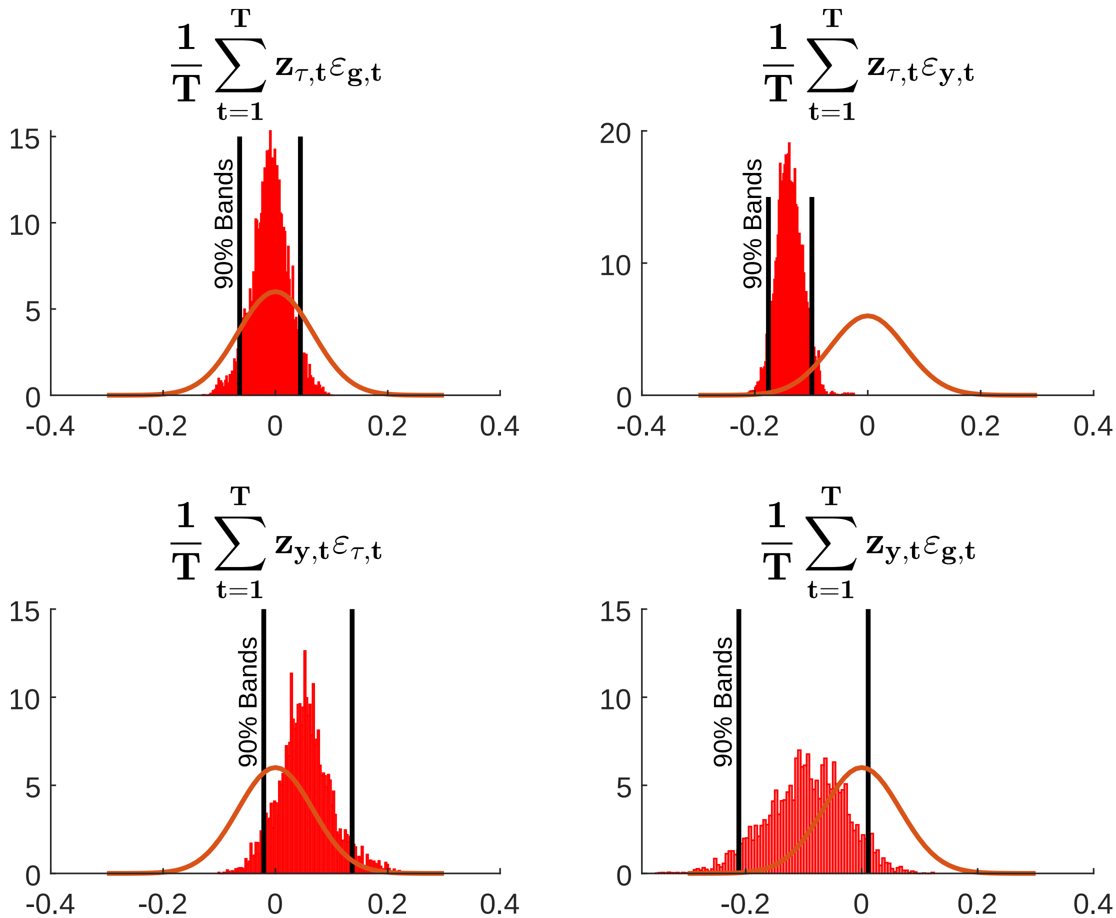
4.3 Understanding the differences

The key advantage of our non-Gaussian proxy weighting approach is that we can update the proxy exogeneity priors. That is we shrink towards exogenous proxies, however, if the data provides evidence against a given exogeneity assumption, our model can update the prior and stop to shrink towards exogenous proxies. Contrary, when applying the Gaussian fiscal proxy SVAR from Mertens and Ravn (2014) or the Gaussian non-fiscal proxy SVAR from Caldara and Kamps (2017), it has to be assumed that the respective proxy is exogenous and this prior cannot be updated. In the following, we show that the data provides evidence against the exogeneity of both proxies which further helps in understanding the different multiplier estimates across identification strategies.

Figure 8 provides evidence on the exogeneity assumptions obtained from our non-Gaussian model. The two graphs in the first row show the posterior distributions of the correlation between the tax proxy and the structural government spending and output shock, respectively. The second row presents the posterior distributions of the correlation between the TFP proxy and the structural tax and government spending shock, respectively. With exogenous proxy variable, meaning $\mu_j = 0$ in Equation (22), the proxy prior shrinks towards shocks with no systematic correlation with the proxy variables as indicated by the solid lines in Figure 8. However, this is not the case. Specifically, the data provides evidence against the exogeneity assumptions and the estimator stops to shrink towards the exogenous proxy solution. For example, the tax proxy has a clear negative correlation with the output shock. Put differently, positive (negative) output shocks coincide with negative (positive) values for the tax instrument. Intuitively, not accounting for this correlation leads to identified tax cuts that also include exogenous increases in output, and vice versa, which increases the size of the estimated tax multiplier. Concerning the TFP proxy, we find some evidence that the instrument is negatively correlated with exogenous government spending shocks. Therefore, the TFP proxy identifies positive (negative) output shocks that also include negative (positive) government spending shocks which reduces the fraction of GDP movements explained by the identified output shocks. As a result, the estimated government spending elasticity is reduced which, as already discussed above, increases the size of the estimated government spending multiplier.

In summary, our findings reveal that the fiscal proxy used by Mertens and Ravn (2014) and the non-fiscal proxy used by Caldara and Kamps (2017) do not fulfill the crucial exogeneity assumption which leads to biased estimates. In particular, the negative correlation between the tax proxy and the output shock induces an upward bias in the estimated tax multiplier by Mertens and Ravn (2014). In addition, the negative correlation between the TFP proxy and the government spending shock leads to an upward bias in the estimated government spending multiplier by Caldara and Kamps (2017). Our proposed non-Gaussian proxy weighting model accounts for these correlations between instruments and structural shocks and therefore leads to a smaller tax multiplier compared to Mertens and Ravn (2014) and a smaller government spending multiplier compared to Caldara and Kamps (2017).

Figure 8: Prior vs. posterior of exogeneity moment conditions



The figure shows the prior (solid line) for exogenous proxy variables, i.e. with $\mu_j = 0$, and the posterior of the exogeneity moments for the tax proxy as well as the TFP proxy.

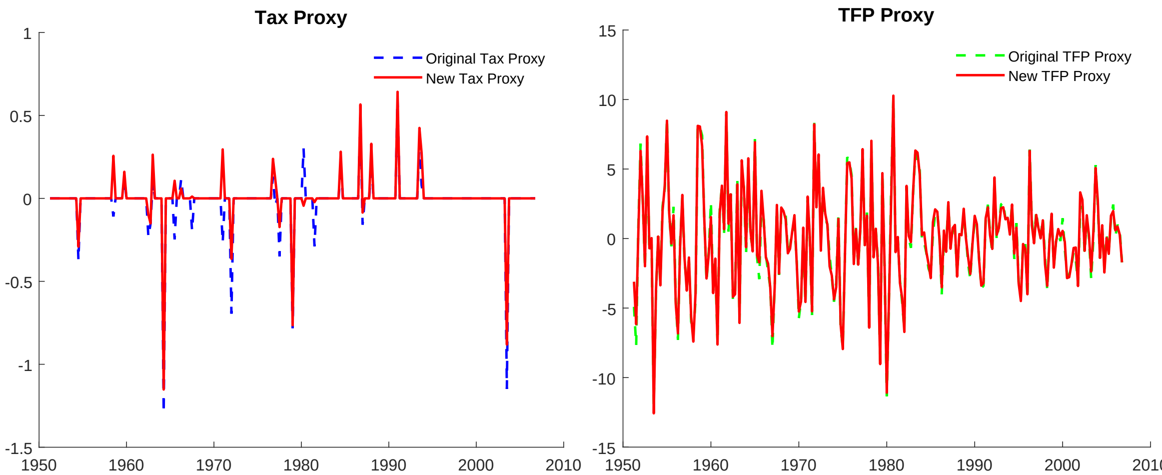
To further highlight the implied differences across identification strategies, in the Appendix we show the times series for the estimated structural shocks. While the estimated shocks share a similar pattern for most periods, there are some discrepancies worth mentioning. In general, we find that the fiscal proxy VAR approach shows the tendency to interpret positive output shocks as negative tax shocks, and vice versa. As a consequence, by taking this shortcoming of the fiscal proxy into account, our non-Gaussian proxy weighting approach leads to a much smaller tax multiplier compared to the standard fiscal proxy identification strategy. We explain in detail why we think that the identified shock of our proposed non-Gaussian proxy weighting model fits better to the historical narrative.

4.4 Constructing new proxies

Given our result from the previous section that there exists evidence of a clear correlation between both the fiscal and the non-fiscal proxy, respectively, and the structural shocks, we can go one step further and construct new proxy measures that are orthogonal to the disturbances of the model. Thus these new proxies arguably fulfill the exogeneity assumption and therefore can be used in a standard proxy variable VAR approach. To get new proxy measures we proceed as follows. For the narrative tax proxy, we regress the proxy on a constant and the median structural government spending and output shocks obtained from applying the non-Gaussian proxy weighting approach. Similarly, for the TFP measure, we regress the proxy on a constant and the median structural tax and government spending shocks. The estimated residuals of these regressions capture movements in the original proxy measures that are not related to other disturbances of the model.

Figure 9 presents our new tax proxy measure and compares it with the original series. For most episodes both measures move in the same direction and show only marginal differences. However, for some key dates there are sizeable discrepancies. The original tax proxy shows a tendency to indicate an exogenous tax increase during periods of economic recessions and an exogenous tax decrease during episodes of economic expansions. For example, in 1971Q1 the original measure shows a relatively strong tax cut, although the actual change to the tax code included only modest adjustments to depreciation rules. In general this period better fits the narrative of an expansionary output shock. In addition, with the US economy entering a recession in the early 1980s, the original measure shows two pronounced tax changes, a tax increase in 1980Q2 and a tax cut in 1981Q3. However, the macroeconomic narrative seems to be more consistent with a series of contractionary output shocks. Our new measure which accounts for confounding innovations in the tax proxy measure, indicates a tax increase in the early 1970s economic expansion and almost no change in the tax code during the early 1980s recession.

Figure 9: New proxy variables

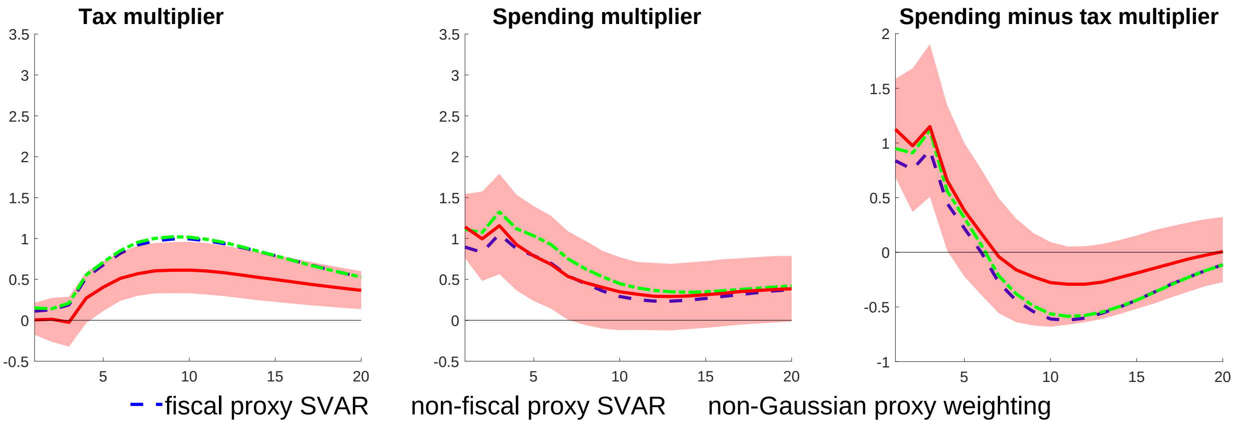


The new tax proxy is residual of the regression $z_{\tau,t} = \beta_0 + \beta_1 \varepsilon_{g,t} + \beta_2 \varepsilon_{y,t} + u_t$, meaning the variation of the tax proxy unexplained by the government spending and output shock.The new TFP proxy is residual of the regression $z_{TFP,t} = \beta_0 + \beta_1 \varepsilon_{\tau,t} + \beta_2 \varepsilon_{g,t} + u_t$, meaning the variation of the TFP proxy unexplained by the tax and government spending shock.

We also calculate a new TFP proxy as shown in Figure 9. The differences between the original and the new TFP proxy which takes potential endogeneity concerns into account are relatively small, both measures are highly correlated. However, differences can indeed be observed for the periods like 1965Q3, 1984Q1, 1984Q2, 2002Q1. In the Appendix, we discuss in detail why we think our new TFP proxy measure better aligns with the historical narrative.

Given that our new proxy measures removes endogenous variation in the original tax and TFP series, we can use them as instruments in a standard proxy VAR setting that rests on the idea of an available instrument fulfilling the exogeneity assumption. Figure 10 presents the results of the fiscal and non-fiscal model, respectively, when relying on the newly constructed proxies and compares the obtained estimates to our non-Gaussian proxy weighting model. Notably, the differences in estimated multipliers across models become much smaller. As such, the fiscal proxy model now leads to an estimated tax multiplier that is much smaller compared to the case when using the original (potentially endogenous) tax proxy as shown in Figure 6. Both the fiscal and non-fiscal model induce estimated tax multipliers that mainly lie within the credible bands of our non-Gaussian model. The same

Figure 10: Comparison of estimated output multipliers between the different models with new proxies



The figure compares the output responses between our non-Gaussian proxy weighting VAR with 68% credible bands to the median responses in the non-fiscal proxy SVAR proposed by Caldara and Kamps (2017) as well as the fiscal proxy SVAR from Mertens and Ravn (2014). But we replace the old proxy variables with the new proxy variables.

applies to the estimated government spending multiplier which takes on values close to unity across all models. Importantly, when removing endogenous variations in the instruments, all models lead to the conclusion that the government spending multiplier is larger than the tax multiplier. Thus our evidence suggests that endogeneity in the tax and TFP proxies are responsible for the large differences in estimated multipliers across estimation strategies. Because the fiscal and non-fiscal approach have to assume that the proxies are indeed exogenous, this finding can only be obtained by a strategy that evaluates the exogeneity of the proxies like our proposed non-Gaussian proxy weighting model.

5 Conclusion

This paper discusses the challenge of measuring the effects of fiscal policy and the recent use of the proxy VAR approach as a tool for identifying fiscal policy shocks. We propose a new non-Gaussian proxy weighting approach that combines non-Gaussian identification with proxy variables. The method shrinks towards exogenous proxy variables but allows for updating the prior and stopping the shrinkage if empirically warranted. We use our model to provide evidence that the contradicting results of Mertens and Ravn (2014) and Caldara and Kamps (2017) may be due to the use of invalid instruments. Furthermore, we find that increasing government spending is more effective in stimulating the economy than lowering taxes. Finally, we utilize our estimation results to construct new proxy variables. These new proxies result in very similar fiscal multipliers estimates and can be used in traditional proxy SVARs.

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