BAYESIAN MULTIPLE-INDICATOR MIXED-FREQUENCY MODEL WITH MOVING AVERAGE STOCHASTIC VOLATILITY *

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Abstract

We suggest a Bayesian mixed-frequency multiple-indicator model with moving-average stochastic volatility that nests 1) U-MIDAS model of Foroni et al. (2015), 2) U-MIDAS model with MA-component of Foroni et al. (2019); and 3) multiple-indicator model with stochastic volatility of Carriero et al. (2015). The general models as well as its restricted versions can be efficiently estimated using the precision-based algorithm of Chan and Jeliazkov (2009) and Chan (2013). We re-examine the evidence presented in Foroni et al. (2019) on the usefulness of including a moving-average component in the mixed-frequency forecasting models. Our results are less optimistic on the additional value of the MA-component based on out-of-sample model evaluation using either point or density forecasts.

Keywords: US GDP, nowcasts, real-time, mixed-frequency data, Bayesian estimation

*The views are solely of the authors and under no circumstances represent those of the Bank of Latvia. All computations were performed in R (R Core Team, 2012).

1 Introduction

Starting with the seminal contribution of Ghysels et al. (2006) there was a rapid development of models that are able to cope with data sampled at heterogeneous frequencies. In these models the frequency mismatch between modelled variables is generally solved by skip-sampling high-frequency variables to low frequency (e.g. monthly series is converted into three quarterly ones with the first/second/third quarterly series containing the first/second/third months of each quarter; cf. Foroni et al. (2015)). As a result, the number of explanatory regressors is a multiple of the frequency scaling parameter (in the previous example 3). Following Ghysels et al. (2006) the most popular approach of the mixed-data sampling (MIDAS) models to mitigate the problem of parameter proliferation are distributed lag polynomials that link low- and skip-sampled high-frequency variables. In such cases, however, the model parsimony comes at a cost that one has to rely on non-linear iterative methods for estimation of parameters in MIDAS regressions. The use of non-linear rather than ordinary least squares estimator typically makes it substantially more difficult to fit a model with more than one explanatory high-frequency variable. As the result, MIDAS regressions in which regressors generated by skip-sampling of a *single* high-frequency indicator are very common in empirical studies (Claudio et al., 2020).

Fortunately, Foroni et al. (2015) pointed out that when the sampling frequency difference is small one can dispense with distributed lag polynomials and estimate an unrestricted version of MIDAS regressions (U-MIDAS), in which a dependent low-frequency variables is directly regressed on one or several skip-sampled high-frequency variables using OLS estimator. However, because of potential overfitting problems, one has to be careful not to include too many regressors in this framework.

Carriero et al. (2015) go one step further and propose a Bayesian multiple-indicator mixed-frequency model as a direct extension of Foroni et al. (2015). In the former model the potential caveat of data overfitting is directly addressed by imposed shrinkage on the parameter values by means of Minnesota prior. Since the model of Carriero et al. (2015) is estimated by Markov Chain Monte Carlo method its performance can be assessed not only by point but also by density forecast. The accuracy of density forecasts can be further improved by augmenting the model by stochastic volatility component, which is straightforward in the Bayesian setting of Carriero et al. (2015).

It is well known that a moving-average (MA) component can be introduced in aggregated data as a result of temporal aggregation: Specifically, in recent contribution Foroni et al. (2019) pointed out that models dealing with data sampled at differing frequencies is not an exception. Based both on the Monte Carlo simulation results and evidence from MIDAS regressions estimated with actual economic data, they conclude that the incorporation of moving-average components into otherwise standard (U-)MIDAS regression yields more accurate out-of-sample (point-)forecasts. This improvement in accuracy for MA-augmented (U-)MIDAS models, however, comes at the cost of a more complex iterative estimation procedure. Furthermore, it creates a challenge on how to combine moving-average component suggested in Foroni et al. (2019) with the stochastic volatility component of Carriero et al. (2015) within a mixed-frequency model.

The solution to this challenge is provided in Chan (2013). Chan (2013) proposes a model with moving average stochastic volatility. Estimation of model parameters is carried out by means of the Bayesian approach that capitalises on recent advances in precision-based algorithms and banded sparse matrices (Fahrmeir and Kaufmann, 1991). The model is formalized in a state space form, but instead of relying on Kalman filter iterations, parameters are efficiently estimated by the direct approach of Chan and Jeliazkov (2009) based on a precision-based algorithm. McCausland et al. (2011) systemtically investigate performance of the Kalman filter routines and precision-based algorithms and conclude that the gains of the latter approach can be substantial when dealing with high-dimensional settings and in cases when state vectors are repetitively drawn for fixed parameter values.

We suggest a model that bridges these separate developments on the specification and estimation of mixed-frequency models for nowcasting purposes (Banbura et al., 2011). Borrowing ideas from Foroni et al. (2019), Foroni et al. (2015), Carriero et al. (2015) and Chan (2013) we suggest an unifying approach that combines characteristic features of each of these contributions in a single model. This makes it practical for daily use at institutions that routinely engaged in monitoring and forecasting economic conditions.

We make the following contributions to the nowcasting literature with mixed-frequency data. First, we provide additional evidence on the usefulness of the MA component for point forecast accuracy using a different data set. Our data set was previously analysed in Carriero et al. (2015) and Siliverstovs (2020) and it comprises real-time vintages of US GDP and twelve national economic and financial indicators, sampled at the monthly frequency. We investigate the influence of the MA component on the out-of-sample point forecast accuracy of both single- and multiple-indicator U-MIDAS regressions. An additional distinctive feature of our analysis is that the significance of the MA component can be assessed from the Bayesian point of view, e.g. by computing Dickey-Savage density ratios for models with and without MA component. Second, we extend the results of Foroni et al. (2019) on the usefulness of the MA-component in mixed-frequency data models by expanding the focus from point forecasts to entire density forecasts. To this end, it is of particular interest whether the MA-component on its own may result in better calibrated densities or only in combination with the stochastic volatility component. Third, Chan (2013) illustrates the performance of the moving average stochastic volatility models using univariate regressions fitted to monthly inflation data. Despite the flexibility that the model framework offers, for example, to include additional explanatory variables, Chan (2013) did not pursue research with additional predictors. Nonetheless, we shall estimate models that include both single and multiple regressors. Moreover, we follow Carriero et al. (2015) and introduce Minnesota priors for parameters in the conditional mean in a mixed-frequency adapted model of Chan (2013). Fourth, given the findings of Chauvet and Potter (2013) and Siliverstovs (2020) concerning the importance to distinguish relative forecasting performance during recessionary and expansionary business cycle phases we will verify whether the usefulness of the MA component also varies with the state of the business cycle both for point and density forecasts. Finally, we provide evidence on the usefulness of the

Bayesian approach for density forecasting in mixed-frequency models, which can be viewed as a viable alternative to the bootstrap based predictive density simulation in this class of models as suggested in Aastveit et al. (2017).

The remainder of this paper is organised as follows. Section 2 contains description of the model proposed in Chan (2013) that is adopted to mixed-frequency data. The dataset is described in Section 3. In Section 4 we evaluate the empirical results based both on the in-sample fit of the models and out-of-sample forecasting accuracy in terms of point and density forecasts. The final section concludes.

2 Model

2.1 Specification

Building upon the state space framework, the most general specification of the model specified at the quarterly frequency is given as follows,

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t^y, \\ \varepsilon_t^y &= u_t + \psi u_{t-1}, \quad u_t \sim \mathcal{N}(0, \exp(h_t)), \qquad\qquad u_0 = 0 \\ h_t &= h_{t-1} + \varepsilon_t^h, \qquad\qquad v_t \sim \mathcal{N}(0, \omega_h^2), \quad h_0 \sim \mathcal{N}(a_0, b_0). \end{aligned} \tag{1}$$

Depending on the particular form of the conditional mean, the following models emerge:

- a historical mean model (HMM) for
$$\mu_t = \phi_0$$
- an AR(2) model for
$$\mu_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-1}$$
- a single-indicator U-MIDAS model for
$$\mu_t = \phi_0 + \phi_1 y_{t-1} + \beta_1 x_t^{(1)} + \beta_2 x_t^{(2)} + \beta_3 x_t^{(3)},$$
where regressors $x_t^{(3)}, x_t^{(2)}, x_t^{(1)}$ denote skip-sampled values of the monthly indicator x_t with the superscript (m) indicating values retained in the m-month of quarter t.
- a multiple-indicator U-MIDAS model
$$\mu_t = \phi_0 + \phi_1 y_{t-1} + \beta_1 x_{1,t}^{(1)} + \beta_2 x_{1,t}^{(2)} + \beta_3 x_{1,t}^{(3)} + \beta_4 x_{2,t}^{(1)} + \beta_5 x_{2,t}^{(2)} + \beta_6 x_{2,t}^{(3)} + \ldots$$
The model in Equation (1) also encompasses previously discussed models. For example, by setting the MA(1) parameter $\psi = 0$ and $\omega_h^2 = 0$ we obtain the U-MIDAS model of Foroni et al. (2015) with homoskedastic and uncorrelated innovations or, equivalently, the model of Carriero et al. (2015) without the stochastic volatility component. For $\psi \neq 0$ and $\omega_h^2 = 0$,

instead, we obtain the moving-average U-MIDAS model of Foroni et al. (2019). By setting $\psi = 0$ and $\omega_h^2 \neq 0$ we end up with the stochastic volatility U-MIDAS model of Carriero et al. (2015).

The model in Equation (1) can be compactly written in the matrix form. Denoting $\mathbf{y} = (y_1, ..., y_T)'$:

$$\mathbf{y} = \mu + \mathbf{H}_\psi \mathbf{u}$$
$$\mathbf{H}\mathbf{h} = \varepsilon$$

where $\mu = (\mu_1, ..., \mu_T)'$ and $\mathbf{u} = (u_1, ..., u_T)' \sim N(0, \mathbf{S_y}), \mathbf{S_y} = diag(e^{h_1}, ..., e^{h_T})$, $\varepsilon \sim N(0, \omega_h^2 I_T)$. The matrix $\mathbf{H}_\psi$ is a lower diagonal matrix with one on the main diagonal and the parameter ψ on the first lower diagonal. The matrix $\mathbf{H}$ is the first difference matrix with the one on the main diagonal and -1 on its first lower diagonal.

2.2 Priors

The following priors are imposed:

- for ARX-MA model type we use the uniform prior $\psi \sim \mathcal{U}(-1, 1)$ and for ARX-MA-SVRW model type we use the truncated normal prior $\psi \sim \mathcal{N}(0, V_\psi)1(|\psi| < 1)$ with $V_\psi = 1$. Both priors restrict the parameter ψ values to the invertibility region;
- $\omega_h^2 \sim IG(a_h, b_h)$ is an inverse Gamma distribution with scale $a_h = 3$ and rate $b_h = 0.5(a_h - 1)$ parameters;
- for models without stochastic volatility we have $\mathbf{u} = (u_1, ..., u_T)' \sim \mathcal{N}(0, \sigma^2 I_T)$ with $\sigma_y^2 \sim IG(a_y, b_y)$ with $a_y = 3$ and $b_y = 5(a_y - 1)$;
- the initial observation $h_0 \sim \mathcal{N}(a_0, b_0)$ with $a_0 = 0$ and $b_0 = 5$;

For parameters in the conditional mean μ we specify the following priors. For the benchmark HMM and AR2 models as well as for single-indicator U-MIDAS models we set diffuse normal prior $\gamma \sim N(0, 100 I_\gamma)$ with γ vector dimensions appropriately specified as given below:

- HMM: $\gamma = (\phi_0)$
- AR2: $\gamma = (\phi_0, \phi_1, \phi_2)'$
- SIM-U-MIDAS: $\gamma = (\phi_0, \phi_1, \beta_1, \beta_2, \beta_3)'$.

For parameters in the conditional mean μ of the multiple-indicator U-MIDAS models we follow Carriero et al. (2015) and specify the Minnesota prior adapted to the mixed frequency case. The Minnesota prior specified for the vector $\gamma = (\phi_0, \phi_1, \beta_1, \beta_2, \beta_3, ...)'$ is the joint distribution with zero mean and diagonal covariance matrix. The entries of the diagonal covariance matrix are defined as follows:

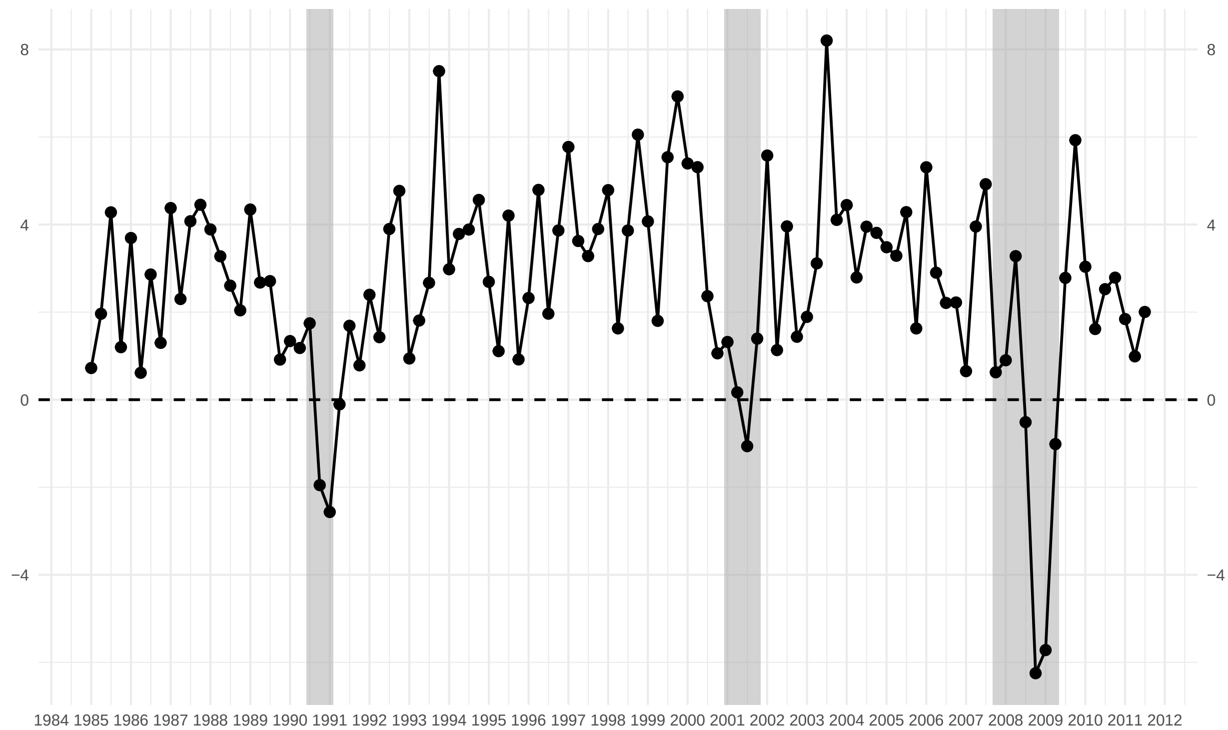

Figure 1: GDP growth, second release

- intercept parameter, $\sigma^2_{\phi_0} = 100 * \sigma^2_y$;
- parameter for the autoregressive lag, $\sigma^2_{\phi_l} = \lambda_1^2/(l^{\lambda_3})^2$ with $l = 1$;
- parameters for the explanatory variables, $\sigma^2_{\beta_i} = \sigma^2_y/\sigma^2_{x_i} * (\lambda_1\lambda_2/l^{\lambda_3})^2$,

where σ^2_y and $\sigma^2_{x_i}$ are variances of residuals from AR(4) models fitted for the variables y_t and $x^{(m)}_{i,t}$. The vector of the hyperparameter values are set the same as in Carriero et al. (2015): $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, and $\lambda_3 = 1$. Recall that the hyperparameter λ_1 is responsible for the overall degree of shrinkage; λ_2 determines degree of shrinkage of the parameters β_i relative to the shrinkage on the parameter ϕ_1; and λ_3 sets the shrinkage rate for the quarterly lags of the regressors.

3 Data

The data set used in the analysis is the same as used in Carriero et al. (2015) and Siliverstovs (2020). It comprises real-time vintages of quarterly GDP releases as well as releases of twelve economic and financial indicators available at the monthly frequency. The targeted time series is shown in Figure 1. The period used for out-of-sample forecast evaluation starts in 1985Q1 and ends in 2011Q3. Each point in the plot indicates second estimate of annualised quarterly GDP growth that was historically released. The shaded areas indicate NBER-dated recessions in the sample.

Table 1: Monthly indicators

Indicator	Description	Transformation	Availability
ISM	ISM index (overall) for manufacturing	level	1-2-3
EMPLOY	Payroll employment	log-change	1-2-3
SUPDEL	ISM index for supplier delivery times	level	1-2-3
ORDERS	ISM index for orders	level	1-2-3
HOURS	Average weekly hours of production workers	log-change	1-2-3
SP500	S&P500 index	log-change	1-2-3
TBILL	3-month Treasury bill rate	level	1-2-3
TBOND	10-year Treasury bond yield	level	1-2-3
CLAIMS	New claims for unemployment insurance	level	1-2
RSALES	Real retail sales	log-change	1-2
IP	Industrial production	log-change	1-2
STARTS	Housing starts	log-level	1-2

Notes:
Entries in column Availability indicate for which months skip-sampled values of the monthly indicators are available for estimation at the chosen forecast horizon.

The set of the auxiliary indicators represents various aspects of the US economy: soft economic indicators in the form of business tendency surveys (ISM, SUPDEL, ORDERS), several hard economic indicators capturing labour market (EMPLOY, CLAIMS) and production conditions (IP, HOURS) as well as consumption and housing (RSALES, STARTS) sides of the economy. Sentiments in the financial markets are captured by the following variables SP500, TBILL, and TBOND, see Table 1. In the previous two studies four forecast origins were used: three in the beginning of either first, second, third months of the targeted quarter and the fourth forecast origins corresponding to backcasting exercise is in the beginning of the first month (during its first week) of the following quarter. In the current study we produce backcasts since as shown in Carriero et al. (2015) and Siliverstovs (2020) for this forecast origina maximum gains in forecasting accuracy in comparison to univariate benchmark models are realised. Observe that the monthly indicators are characterised by different release timing: some of them are available during the first week of a month, whereas others are available only during the second week. As shown in column *Availability* of Table 1, because of the differences in timing releases for the first group of indicators with earlier release timing we have all three skip-sampled monthly values whereas for the second group of indicators we have only values for the first two months of the targeted quarter. All monthly indicators have the same publication lag of one months, i.e. a monthly release during current month contains the last value for the previous month.

4 Results

The effect of inclusion of moving-average component is evaluated using estimation results of the ψ parameter in sample as well as evaluation of accuracy of point and density forecasts out of sample.

As discussed above there are three groups of time series models. The first group entails univariate benchmark models, historical mean and autoregressive models (HMM and AR2). The second group is mixed frequency models with a single indicator listed in Table 1 used as expalanatory variable (single-indicator models, SIM). The third group of models consists of two multiple-indicator models employed in Carriero et al. (2015). The large model (LRG) that includes all the twelve monthly indicators and its smaller version (SML) based on five monthly indicators (ISM, EMPLOY, IP, RSALES, STARTS). The number of regressors in each of these two multiple-indicator models (MIM) can be directly assessed from column *Availability* in Table 1. Hence the total number of regressors in LRG and SML models is equal to $8*3+4*2+1+1 = 34$ and $2*2+2*3+1+1 = 14$, respectively, taking into account one lag of the dependent variable y_{t-1} and an intercept.

4.1 In-sample estimation

The results of in-sample estimation of the three groups of models are obtained on the full sample of data spanning 1970Q1-2011Q2 period using the data vintage that was available in the beginning of October 2011. We use these estimation results in order to make a backcast of US GDP for 2011Q3 - a final quarter in our out-of-sample forecast evaluation sample. The summary of the posterior distribution of the MA(1) parameter ψ for each model is presented in Figure 2 in the form of estimated median and 90% credible interval. In the left and rigth panels of the figure the estimates of the quantiles of the porsterior distribution of the parameter ψ are reported for ARX-MA and ARX-MA-SVRW model specifications, respectively. The estimates of ψ in general are very similar in both model specifications in all but one case -, namely, a single-indicator model with CLAIMS regressor variable.

For the benchmark models (top two models) we have a mixed result. For the AR2 model the 90% credible interval is very large and it included zero, whereas for the historical mean model the main mass of the posterior distribution is located on the positive half axis of the real line. Clearly, for the HMM the MA-component makes up for the omitted autoregressive dynamics that is present in the benchmark AR2 model.

For the SIMs the evidence is rather mixed. For models without stochastic volatility, there are some explanatory variables for which the main mass of the posterior distribution is located in the negative part (RSALES, SP500, HOURS, TBILL, IP). For model with the SV-component we find more models (RSALES, SP500, CLAIMS, HOURS, STARTS, SUPDEL, TBILL, TBOND) for which the 90% credible interval is mainly located in the negative part. For both model specification without and with the SV-component the main mass of the posterior distribution for ψ is located in the positive part. For ISM- and ORDERS-models the ψ posterior distribution is located around zero value for both specification types.

For the two MIMs the posterior distributions of ψ also are different. For the smaller model with five indicators the median of the distribution is centered around zero, whereas for the larger model with twelve indicators the whole distribution is shifted to the negative region with the median value reported about -0.187 and -0.153 for specification types without and with stochastic volatility

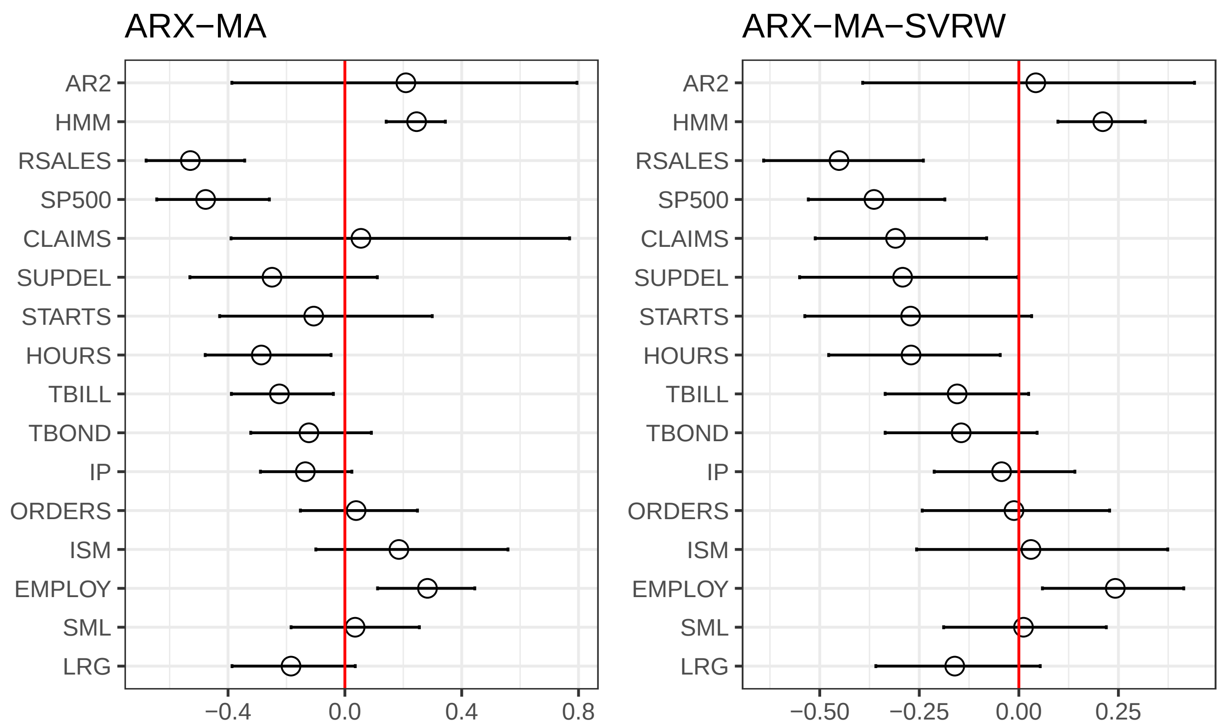

Figure 2: MA(1) parameter (median and 90% credible interval), vintage October 2011

component. It is interesting to observe that for the model with richer dynamics the evidence in favour of moving average term is stronger.

It is further instructive to examine how the posterior distribution of ψ evolves over time when estimated recursively using real-time GDP vintages. For the sake of brevity, the time series plots of the 5th, 50th, and 95th percentiles are reported for two selected SIMs (EMPLOY and RSALES) and for the two MIMs (SML and LRG) without stochastiv volatility, see Figure 3. The recursive plots confirm the findings reported for the full estimation sample. The 90% credible intervals for EMPLOY- and RSALES-models stay consistently above, respectively, below the zero line. The 90% credible interval for the ψ parameter in the SML-model is centered at zero value and for the LRG-model it lies in the negative territory with its upper percentile near the zero borderline.

4.2 Out-of-sample forecast accuracy

The results of out-of-sample (OOS) assessment of forecast accuracy are collected in Table 2 for point forecasts and in Tables 3 and 4 for density forecasts. In line with the literature we evaluate accuracy of point forecasts in terms of the Root Mean Squared Forecast Error and goodness of density forecasts in terms of mean logarithmic score (MLOGS) and mean continuous ranked probability score (MCRPS).

The results for models without moving-average term reported for the full forecast evaluation sample (1985Q1-2011Q3) are comparable with those in Carriero et al. (2015) and Siliverstovs (2020)

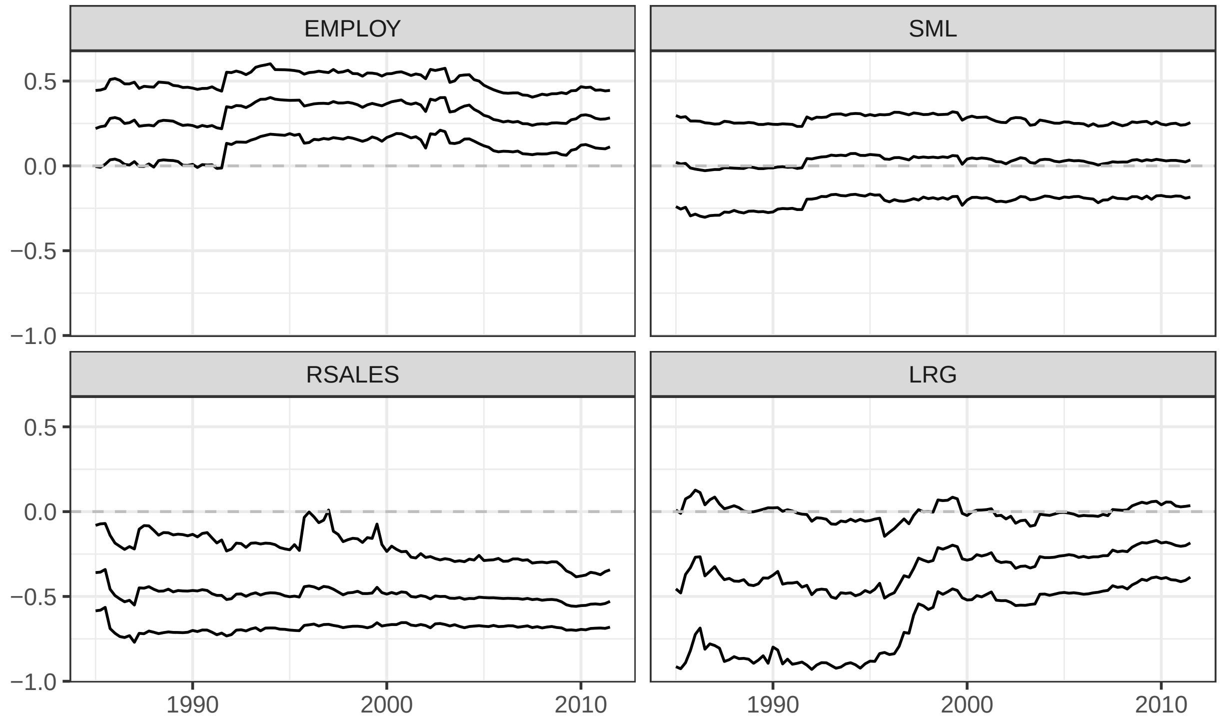

Figure 3: MA(1) parameter in ARX-MA model (median and 90% credible interval), recursively estimated using real-time vintages

and for expansion and recession sub-samples in Siliverstovs (2020). This is encouraging, especially, for models with stochastic volatility for which another estimation approach was applied. These results indicate that data pooling in one model as done in Carriero et al. (2015) is beneficial when compared to the forecasting performance of the benchmark as well as single-indicator mixed-frequency models but these benefits are mainly due to few observations during the recessionary periods (especially during the Great Recession) when the benchmark models (and to the lesser extent SIMs) perform substantially worse than during expansionary periods.

This section is sub-divided into two parts. In Section @ref(sec:effe_ma) we investigate the effect on OOS forecast acccuracy of adding moving average component to model without and with stochastic volatility. In Section @ref(sec:effe_lv) we investigate the effect on OOS forecast accuracy of adding stochastic volatility component to model without and with MA-term. This comparison helps us identify model's feature that has the most profound effect on OOS forecast accuracy. Is it MA- or SV-component or synergy of both?

The information in the tables is summarised in the graphical form in Figures 4-6 and 7-9 for the effect of MA- and SV-components, respectively. Each plot is sub-divided into six sub-plots. These sub-plots are grouped by evaluation sample: full sample(top panel), expansion sample (middle panel), and recession sample (bottom panel). The left panels in Figures 4-6 display the effect of MA-term in models without stochastic volatility (ARX vs ARX-MA), whereas the right panels display the effect of MA-term in models with stochastic volatility (ARX-SVRW vs ARX-

MA-SVRW). Using the same organisational layout, in Figures 7-9 the effect of SV-component is analised for models without MA-term (ARX vs ARX-SVRW) and with MA-term (ARX-MA vs ARX-MA-SVRW).

4.2.1 Effect of MA-component

Figure 4 contains cross-plots of RMSFE for models without stochastic volatility (ARX vs ARX-MA) in the left column and for models with stochastic volatility (ARX-SVRW vs ARX-MA-SVRW) in the right column. The horizontal and vertical dashed lines indicate RMSFE for the benchmark AR2 model. Models which lie either above or further to the right to these dashed lines have higher RMSFE than the corresponding version of the benchmark model.

The red line is a 45-degree line. The position of the models relative to this line is informative whether model's version with MA-term produces different RMSFE from that produced by the model without MA-term. For those models which have similar RMSFE values, i.e. adding the MA-term has no or negligible effect on point forecast accuracy, the corresponding dots lie close to this 45-degree line.

These cross-plots are also informative about models' relative forecasting performance. The best forecasting models with lowest RMSFE are located in the bottom-left corner whereas the worst performing models are located in the opposite top-right corner. For example, the two multiple-indicator models (SML and LRG) are among the best performing models.

When judged for the full sample there are four models (SML, LRG, ISM, CLAIMS) with the highest forecasting accuracy. However, incorporation of MA-term did not bring any noticable improvements in point forecast accuracy for these models. On the contrary, in LRG-model without stochastic volatility the RMSFE is larger for the MA-augmented version. In fact, for the boom sample the performance of the multiple-indicator model LRG-MA is worse than that of any benchmark models that are augmented with the MA-term (AR-MA and HMM-MA), see the middle-left plot at Figure 4. It is interesting to observe that in model version with stochastic volatiliy (LRG-MA-SVRW) this negative influence of the MA-component is neutralised, see the middle-right plot. For the bust sub-sample, there are no discernible effects of the MA-term for the top-performing models. Among the models for which augmentation with the MA-component resulted in the improved point forecast accuracy one can mention SIMs with SP500, RSALES, EMPLOY variables.

Figures 5 and 6 compare MLOGS and MCRPS measure of density forecast accuracy respectively. The two multiple-indicator models are also the best-performing models in terms of these metrics. In general, we find very little evidence supporting the usefulness of augmenting mixed-frequency models with MA-terms, especially, for those models that produce most accurate density forecasts. This conclusion is robust when the effect of adding MA-terms is evaluated by means of MLOGS and MCRPS and whether the underlying models have SV-component or not. Nevertheless, some SIMs such as EMPLOY, RSALES, SP500 benefited from adding the MA-term.

4.2.2 Effect of SV-component

The evaluation results of the effect of enlarging models with the stochastic volatility component on point and density forecast accuracy are presented in Figures 7 and Figures 8 and 9, respectively. In terms of point forecasts, the multiple-indicator with MA-component (LRG-MA) model seems to strongly benefit from the SV-term during expansion period (see middle-right plot). Also for a number of SIMs (HOURS, STARTS, RSALES, ORDERS) augmentation with the SV-term results in improved OOS point forecast accuracy during expansionary phase, but for the recessionary phase the effect is rather negative for these models. For the MIMs there no discernible effect of the SV-term during recessions, as both models perform very similarly in terms of RMSFE.

In the previous studies where the same data set was analysed it was already pointed out on the indispensible effect of stochastic volatility on density forecast accuracy (Carriero et al., 2015; Siliverstovs, 2020). In this section we provide an additional evidence for models with moving average component. Essentially, the results shown in Figures 8 and 9 support the earlier conclusions. This is especially evident during the expansionary business cycle phase, see plots in the middle panel of these two figures. All dots in these two figures lie below the 45-degree line!!! This clearly indicates the benefits of augmenting forecasting models of US GDP growth with the stochastic volatility component when one is interested in accurate density forecasts. Since the expansionary phase takes the lion share in our forecast evaluation sample, this naturally translates when one evaluates forecasting performance for the full sample, see the upper panel in Figures 8 and 9. For the recessionary phase the evidence is not so clear-cut, see the lower panel of the figure. For the most accurate models (SML and LRG) practically there is no difference in forecasting accuracy of models without and with stochastic volatility. The single-indicator ISM-model seems to benefit whereas for such SIMs as EMPLOY-, SP500-, and RSALES-models the forecasting performance deteriorates during recessions.

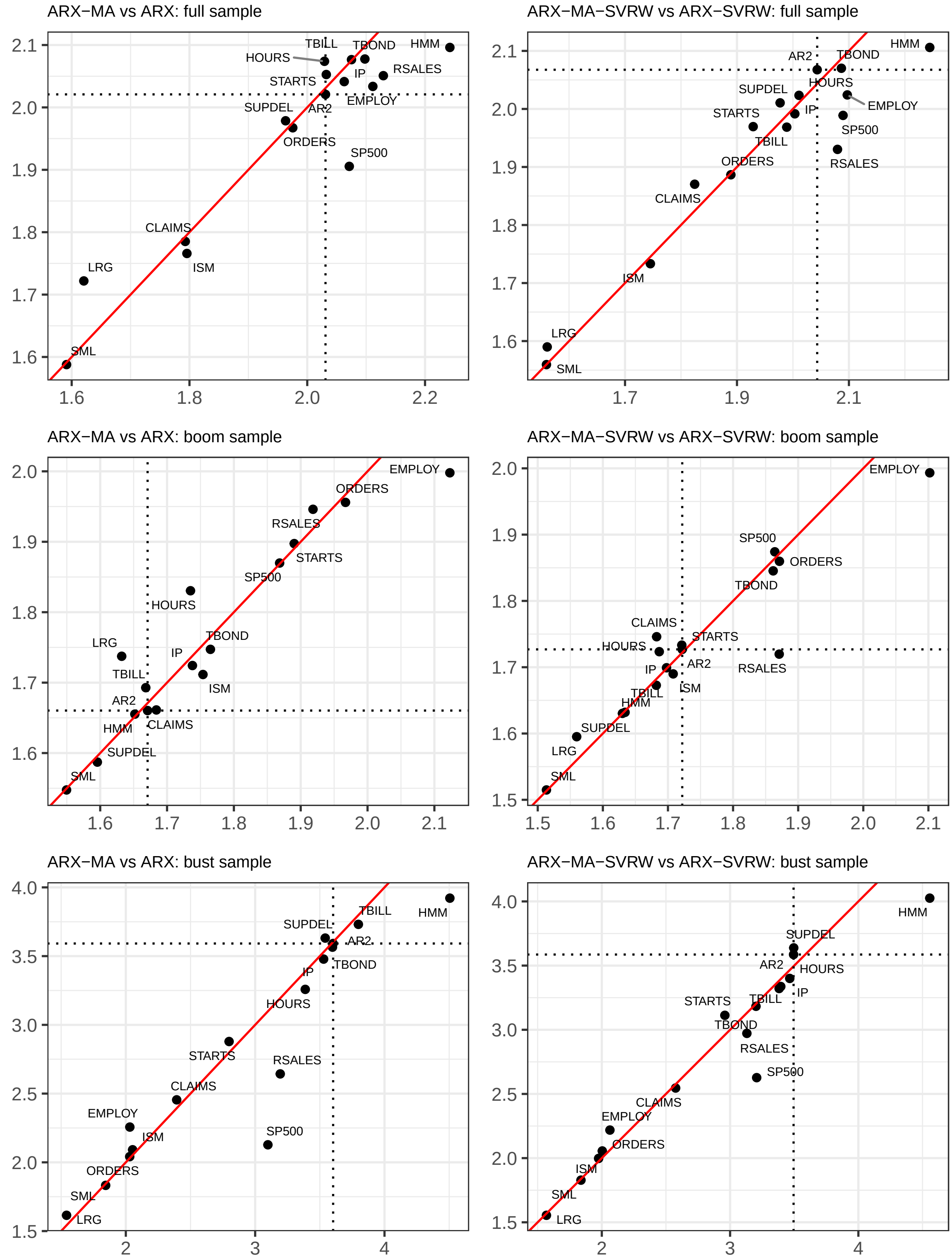

Figure 4: RMSFE for models without and with moving-average component

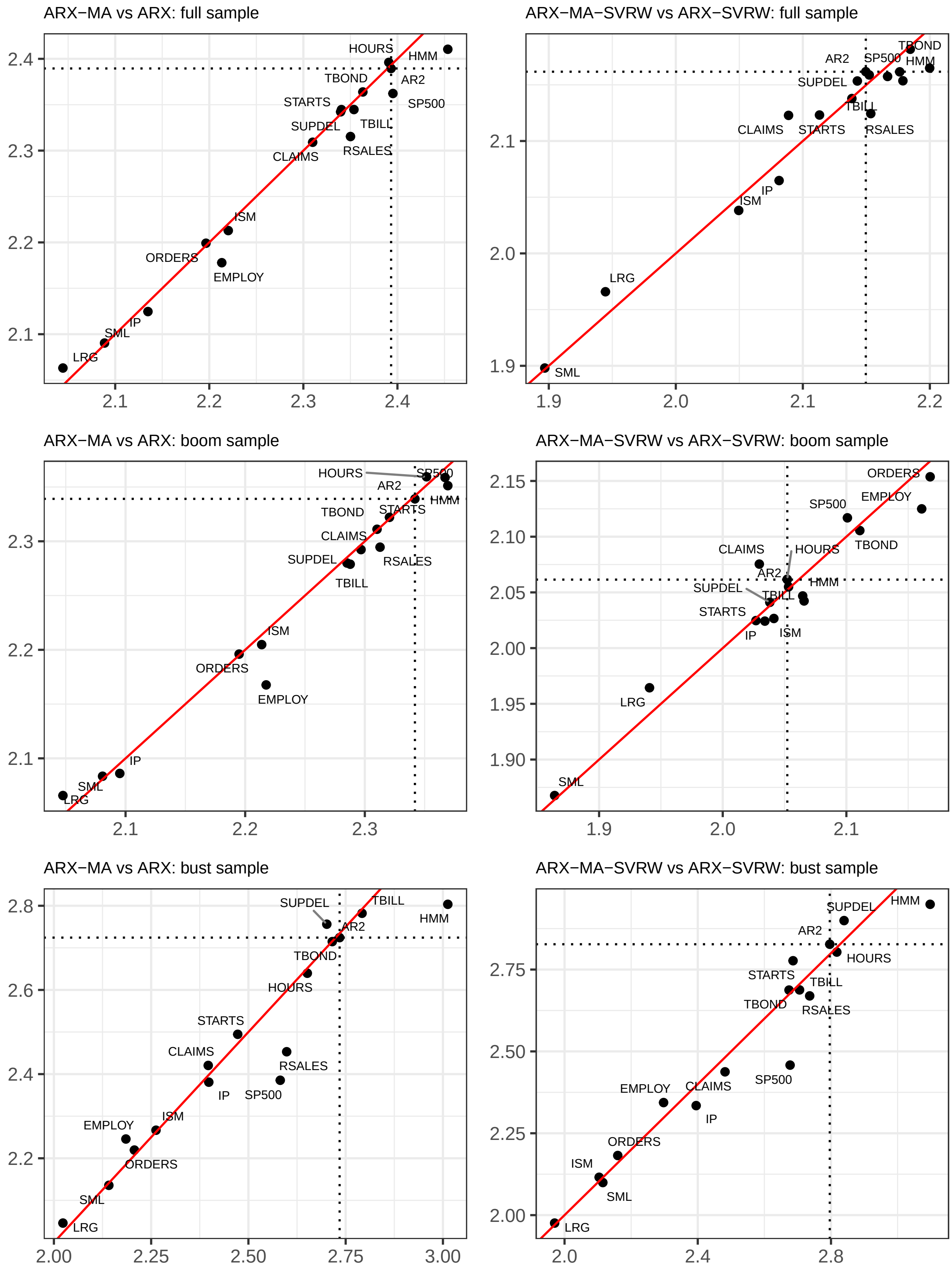

Figure 5: MLOGS for models without and with moving-average component

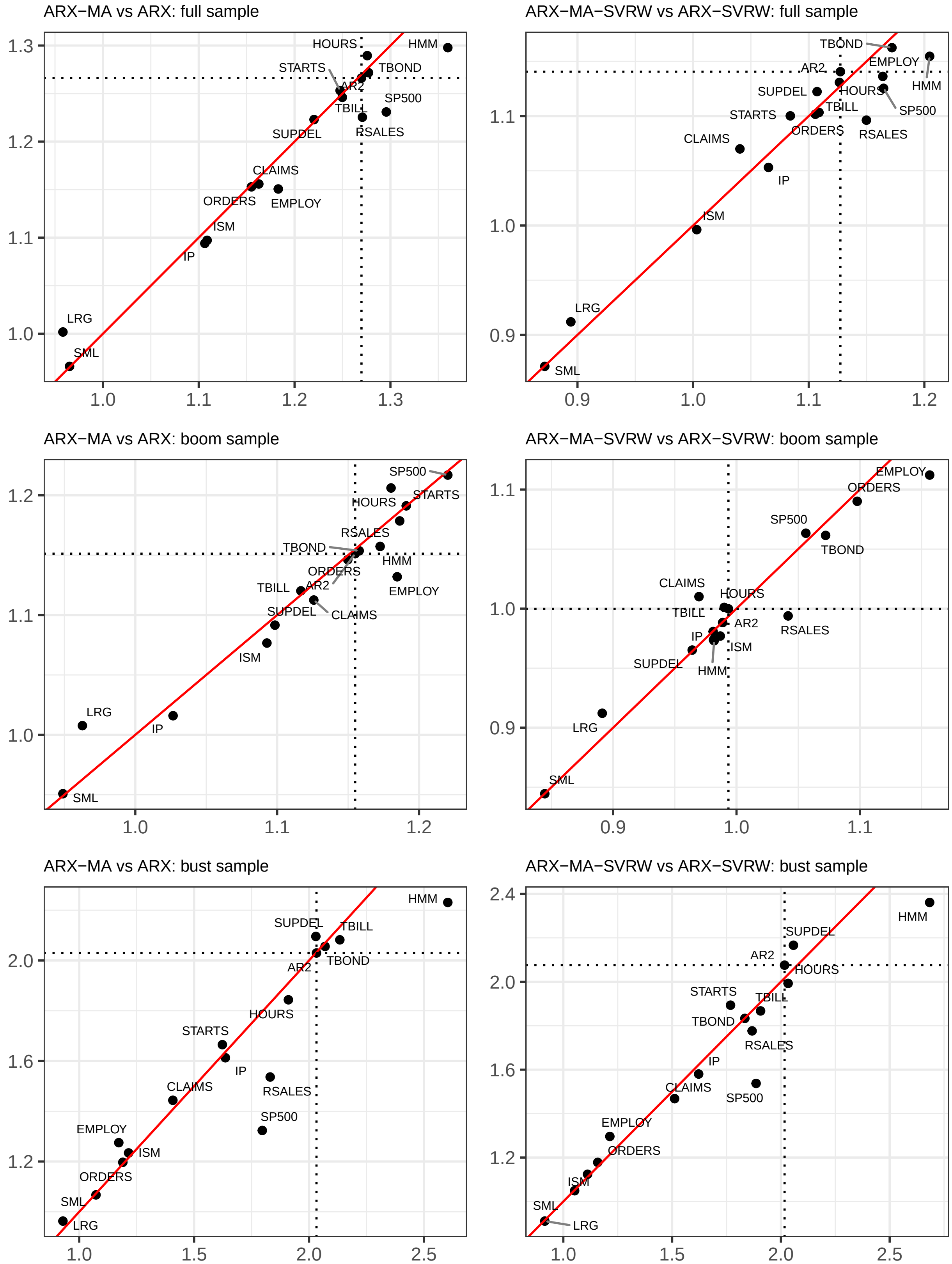

Figure 6: MCRPS for models without and with moving-average component

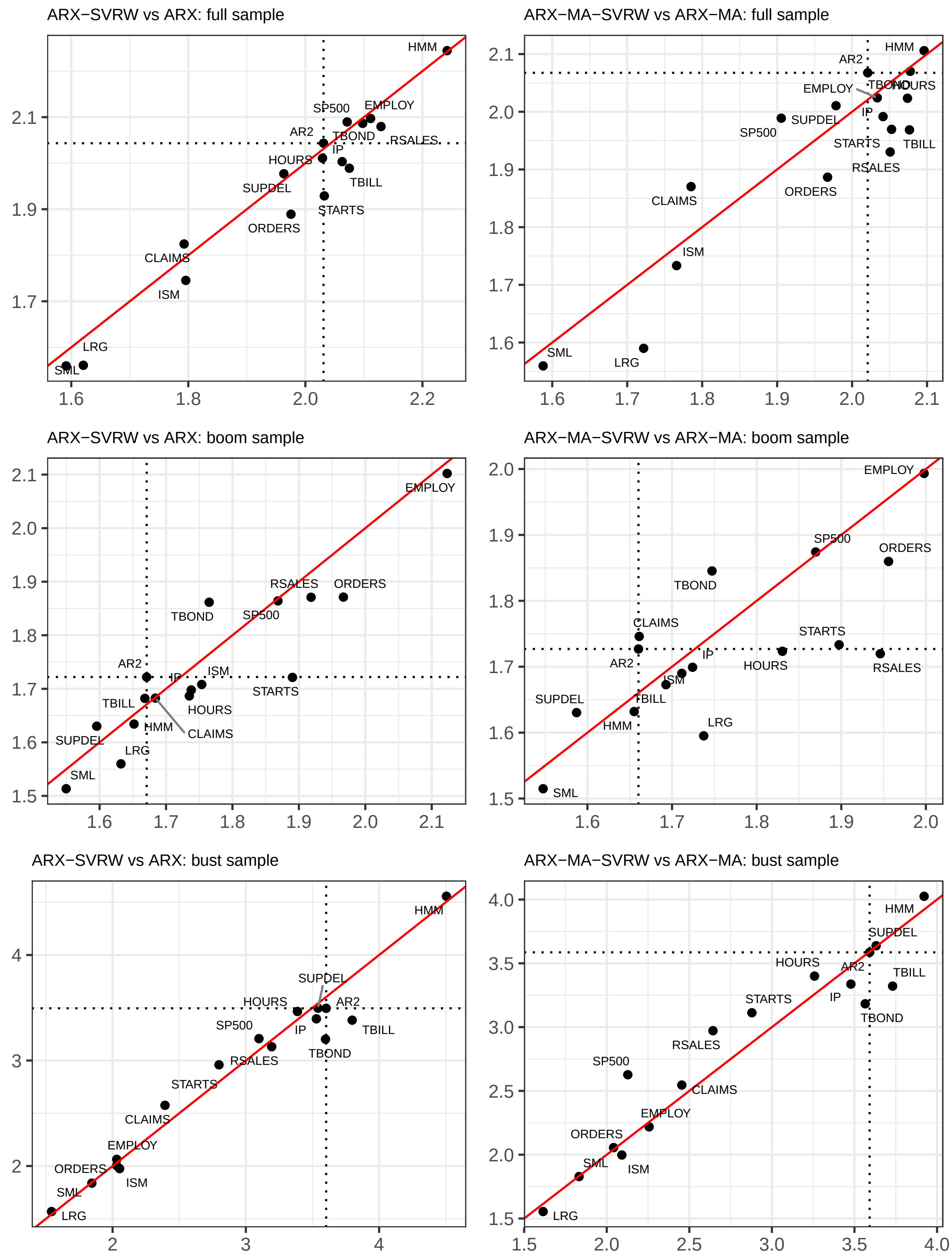

Figure 7: RMSFE for models without and with stochastic-volatility component

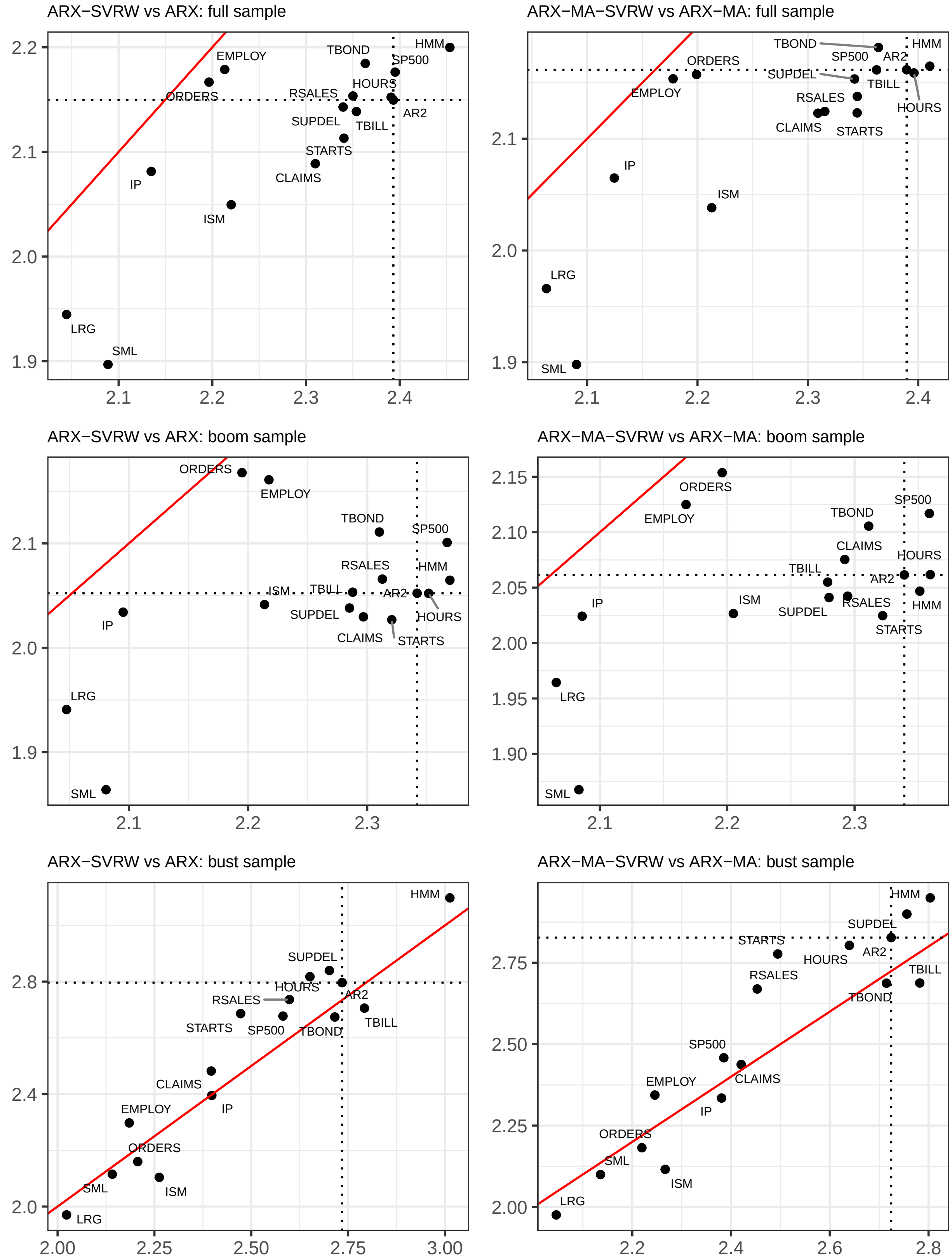

Figure 8: MLOGS for models without and with stochastic-volatility component

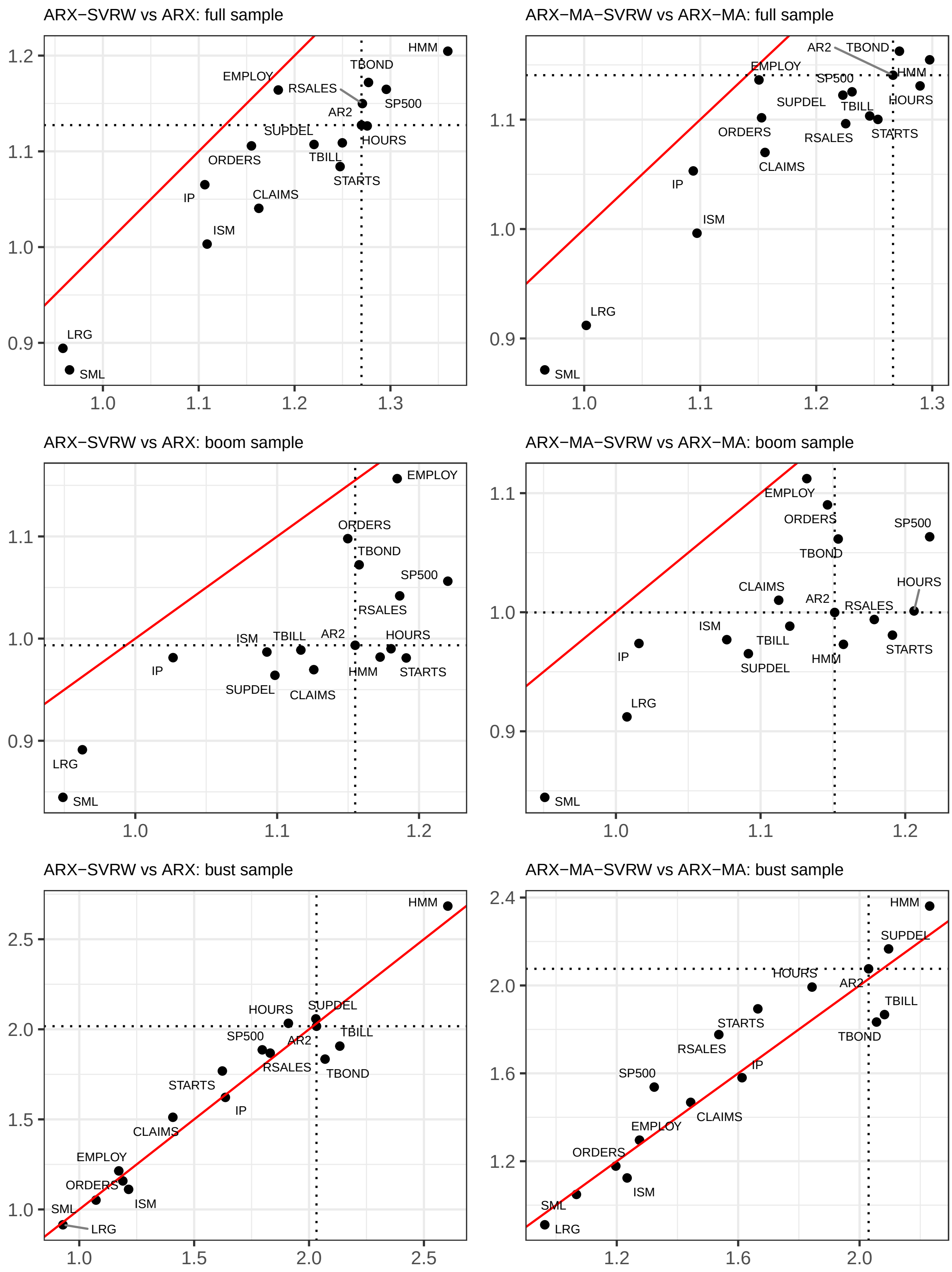

Figure 9: MCRPS for models without and with stochastic-volatility component

5 Conclusions

In this paper we suggest an eclectic model that is based on the fusion of several types of models already suggested in the forecasting literature. More specifically, our specification of the mixed-frequency multiple-indicator model with moving-average stochastic volatility nests 1) U-MIDAS model of Foroni et al. (2015) based either on a single or multiple high-frequency explanatory variables, 2) U-MIDAS model with MA-component of Foroni et al. (2019); and 3) multiple-indicator model with stochastic volatility of Carriero et al. (2015). We show how the most general model specification as well as its restricted versions can be estimated by application of the Bayesian approach using the precision-based algorithm of Chan (2013). Our model also serves as a viable alternative to the bootstrap-based density forecasting in mixed-frequency models as suggested in Aastveit et al. (2017).

In a recent study Foroni et al. (2019) point out that in mixed-frequency models it is natural to expect moving-average component in the regression residuals. Foroni et al. (2019) argue that by integrating explicitly the moving-average term into the model specification tends to improve point forecast accuracy in this class of models. Nevertheless the analysis of Foroni et al. (2019) seems to be unnecessary restricted as they focus on a) single-indicator mixed-frequency models with b) homoskedastic innovations and their analysis is centered on c) point forecasts.

Contributing to the forecasting literature with mixed-frequency models our general specification of the model allows us to re-assess marginal improvements from the moving-average component a) by employing models both with single- and multiple auxiliary high-frequency indicators, b) by examining models both with homo- and heteroskedastic innovations, and c) by focusing not only on point- but also density forecast accuracy. An additional difference of our approach from that in Foroni et al. (2019) is that instead of relying on the method of Non-linear Least Squares we apply Bayesian approach for model estimation and generation of out-of-sample forecast densities.

In our analysis we employ real-time vintages of the dependent variable (US GDP growth) and twelve auxiliary economic and financial indicators collected by Carriero et al. (2015). By focusing on this data set allows us to compare our estimation results with those of Carriero et al. (2015) and Siliverstovs (2020) obtained using different estimator of stochastic volatility.

We find a rather week evidence of the usefulness of incorporating the moving-average component in the mixed-frequency forecasting regressions. We find no systematic evidence that the inclusion of the MA-term significantly boosts forecast accuracy. In fact, for the most accurate models the difference in the forecasting performance between models without and with MA-term is negligible. This holds both for point and density forecasts and irrespective whether stochastic volatility component is absent or present in the model.

Our analysis further confirms findings of Chauvet and Potter (2013) and Siliverstovs (2020) on differences in the forecasting performance during expansions and recessions. Furthermore, our findings strongly suggest to incorporate stochastic volatility in the models when assessing the goodness of a forecasting model of US GDP growth in terms of density forecasts.

Table 2: RMSFE

MODEL	Full sample				Boom sample				Bust sample			
	ARX	ARX MA	ARX SVRW	ARX MA SVRW	ARX	ARX MA	ARX SVRW	ARX MA SVRW	ARX	ARX MA	ARX SVRW	ARX MA SVRW
Benchmark												
AR2	2.03	2.02	2.04	**2.07**	1.67	1.66	1.72	1.73	3.60	3.59	3.49	3.59
HMM	2.24	2.10	2.24	**2.11**	1.65	1.66	1.63	1.63	4.51	3.92	4.56	4.03
SIM												
TBOND	2.10	2.08	2.09	**2.07**	1.76	1.75	1.86	1.85	3.60	3.57	3.20	3.18
EMPLOY	2.11	2.03	2.10	**2.02**	2.12	2.00	2.10	1.99	2.03	2.26	2.06	2.22
HOURS	2.03	2.07	2.01	**2.02**	1.74	1.83	1.69	1.72	3.39	3.26	3.46	3.40
SUPDEL	1.96	1.98	1.98	**2.01**	1.60	1.59	1.63	1.63	3.54	3.63	3.50	3.64
IP	2.06	2.04	2.00	**1.99**	1.74	1.72	1.70	1.70	3.53	3.48	3.40	3.34
SP500	2.07	1.91	2.09	**1.99**	1.87	1.87	1.86	1.87	3.10	2.13	3.21	2.63
STARTS	2.03	2.05	1.93	**1.97**	1.89	1.90	1.72	1.73	2.80	2.88	2.96	3.11
TBILL	2.08	2.08	1.99	**1.97**	1.67	1.69	1.68	1.67	3.80	3.73	3.38	3.32
RSALES	2.13	2.05	2.08	**1.93**	1.92	1.95	1.87	1.72	3.19	2.64	3.13	2.97
ORDERS	1.98	1.97	1.89	**1.89**	1.97	1.96	1.87	1.86	2.03	2.04	2.00	2.06
CLAIMS	1.79	1.79	1.82	**1.87**	1.68	1.66	1.68	1.75	2.39	2.45	2.58	2.55
ISM	1.80	1.77	1.75	**1.73**	1.75	1.71	1.71	1.69	2.05	2.09	1.98	2.00
MIM												
SML	1.59	1.59	1.56	**1.56**	1.55	1.55	1.51	1.51	1.84	1.83	1.84	1.83
LRG	1.62	1.72	1.56	**1.59**	1.63	1.74	1.56	1.60	1.54	1.61	1.57	1.55

Table 3: MLOGS

MODEL	Full sample ARX	Full sample ARX MA	Full sample ARX SVRW	Full sample ARX MA SVRW	Boom sample ARX	Boom sample ARX MA	Boom sample ARX SVRW	Boom sample ARX MA SVRW	Bust sample ARX	Bust sample ARX MA	Bust sample ARX SVRW	Bust sample ARX MA SVRW
Benchmark												
AR2	2.39	2.39	2.15	**2.16**	2.34	2.34	2.05	2.06	2.73	2.72	2.80	2.83
HMM	2.45	2.41	2.20	**2.16**	2.37	2.35	2.06	2.05	3.01	2.80	3.10	2.95
SIM												
TBOND	2.36	2.36	2.18	**2.18**	2.31	2.31	2.11	2.11	2.72	2.71	2.67	2.69
SP500	2.40	2.36	2.18	**2.16**	2.37	2.36	2.10	2.12	2.58	2.39	2.68	2.46
HOURS	2.39	2.40	2.15	**2.16**	2.35	2.36	2.05	2.06	2.65	2.64	2.82	2.80
ORDERS	2.20	2.20	2.17	**2.16**	2.19	2.20	2.17	2.15	2.21	2.22	2.16	2.18
EMPLOY	2.21	2.18	2.18	**2.15**	2.22	2.17	2.16	2.12	2.19	2.25	2.30	2.34
SUPDEL	2.34	2.34	2.14	**2.15**	2.29	2.28	2.04	2.04	2.70	2.76	2.84	2.90
TBILL	2.35	2.34	2.14	**2.14**	2.29	2.28	2.05	2.05	2.79	2.78	2.71	2.69
RSALES	2.35	2.32	2.15	**2.12**	2.31	2.29	2.07	2.04	2.60	2.45	2.74	2.67
STARTS	2.34	2.34	2.11	**2.12**	2.32	2.32	2.03	2.02	2.47	2.49	2.69	2.78
CLAIMS	2.31	2.31	2.09	**2.12**	2.30	2.29	2.03	2.08	2.40	2.42	2.48	2.44
IP	2.13	2.12	2.08	**2.06**	2.10	2.09	2.03	2.02	2.40	2.38	2.40	2.33
ISM	2.22	2.21	2.05	**2.04**	2.21	2.20	2.04	2.03	2.26	2.27	2.10	2.12
MIM												
SML	2.09	2.09	1.90	**1.90**	2.08	2.08	1.86	1.87	2.14	2.14	2.11	2.10
LRG	2.04	2.06	1.94	**1.97**	2.05	2.07	1.94	1.96	2.02	2.05	1.97	1.98

Table 4: MCRPS

MODEL	Full sample				Boom sample				Bust sample			
	ARX	ARX MA	ARX SVRW	ARX MA SVRW	ARX	ARX MA	ARX SVRW	ARX MA SVRW	ARX	ARX MA	ARX SVRW	ARX MA SVRW
Benchmark												
AR2	1.27	1.27	1.13	**1.14**	1.15	1.15	0.99	1.00	2.03	2.03	2.02	2.08
HMM	1.36	1.30	1.20	**1.15**	1.17	1.16	0.98	0.97	2.60	2.23	2.68	2.36
SIM												
TBOND	1.28	1.27	1.17	**1.16**	1.16	1.15	1.07	1.06	2.07	2.06	1.83	1.83
EMPLOY	1.18	1.15	1.16	**1.14**	1.18	1.13	1.16	1.11	1.17	1.27	1.21	1.30
HOURS	1.28	1.29	1.13	**1.13**	1.18	1.21	0.99	1.00	1.91	1.84	2.03	1.99
SP500	1.30	1.23	1.16	**1.13**	1.22	1.22	1.06	1.06	1.80	1.32	1.89	1.54
SUPDEL	1.22	1.22	1.11	**1.12**	1.10	1.09	0.96	0.97	2.03	2.10	2.06	2.17
TBILL	1.25	1.25	1.11	**1.10**	1.12	1.12	0.99	0.99	2.13	2.08	1.91	1.87
ORDERS	1.15	1.15	1.11	**1.10**	1.15	1.15	1.10	1.09	1.19	1.20	1.16	1.18
STARTS	1.25	1.25	1.08	**1.10**	1.19	1.19	0.98	0.98	1.62	1.66	1.77	1.89
RSALES	1.27	1.23	1.15	**1.10**	1.19	1.18	1.04	0.99	1.83	1.54	1.87	1.78
CLAIMS	1.16	1.16	1.04	**1.07**	1.13	1.11	0.97	1.01	1.41	1.44	1.51	1.47
IP	1.11	1.09	1.07	**1.05**	1.03	1.02	0.98	0.97	1.64	1.61	1.62	1.58
ISM	1.11	1.10	1.00	**1.00**	1.09	1.08	0.99	0.98	1.21	1.23	1.11	1.12
MIM												
SML	0.97	0.97	0.87	**0.87**	0.95	0.95	0.84	0.84	1.07	1.07	1.05	1.05
LRG	0.96	1.00	0.89	**0.91**	0.96	1.01	0.89	0.91	0.93	0.96	0.91	0.91

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