Domestic policy or global influences? - the natural rate of interest, house prices and global spill-overs*

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Abstract

In Sweden, the Riksbank's policy of negative interest rates amid an economic upturn and a booming housing market has received a lot of criticism. The Riksbank, on the other hand, claims that it is not primarily policy but global factors that have had a negative influence on domestic interest rates. We use a version of the Laubach and Williams model to estimate the Swedish natural interest rate and find evidence of such global spill-overs. We also find that separating the influence of the natural interest rate from monetary policy can be advantageous when analyzing house prices.

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1 Introduction

In many advanced economies inflation has been low for an extended period of time while interest rates have reached new record lows. Currently, negative interest rates have been introduced in several advanced economies. In the Unites States (US), the Federal Reserve has started to gradually raise interest rates. However, both market pricing and FOMC members' forecasts indicate that interest rates over the longer term will settle at a lower level than before the financial crisis. This reflects a widely held belief that there has been a shift downward globally in the natural rate of interest in recent years (see, e.g., Williams, 2016; Holston et al., 2017; Rachel and Smith, 2015; Christensen and Rudebusch, 2017). Although it is a crucial concept for gauging the expansiveness of monetary policy, and thus serves as an important input to macroeconomic forecasting and policy analysis, formal estimates of the natural rate are missing for many smaller countries, including Sweden.

The Riksbank has stated that global trends stemming from abroad have had a large influence on domestic interest rates, and that the decline in global rates can be traced to structural factors (Sveriges Riksbank, 2017b). Meanwhile, in the domestic policy debate negative interest rates have often been criticized, in particular since growth has been high and house prices have increased at a faster rate than disposable income, which has raised concerns about a possible housing bubble.

In this paper we estimate a model which makes it possible to disentangle the effect of global structural factors, i.e. the natural interest rate, and the expansiveness of monetary policy (the interest rate gap). Even if we accept that global factors are important for the general international interest rate levels, Holston et al. (2017) have shown that even over longer horizons about half of the variation in the natural interest rate is due to domestic factors in other small open economies like the United Kingdom (UK) and Canada. This implies that foreign proxies do not capture an important component of the domestic natural rate. In order to judge the expansiveness of domestic monetary policy country-specific estimates are thus necessary. We contribute to the literature by producing an estimate of the natural rate in Sweden, following in spirit the methods outlined in Laubach and Williams (2003) and, more specifically, the Bayesian methods extended by Berger and Kempa (2014).

Furthermore, we apply our estimated natural interest rate in two additional studies. First, following Holston et al. (2017), we study the spill-overs from larger trading partners to Sweden, using an error-correction model framework. We find evidence of global spill-overs to the Swedish natural rate. Second, we study real house price developments in Sweden using a model similar to Claussen (2013). Using our estimate of the natural interest rate and the monetary policy gap in place of the real interest rate, we find that separating the two helps to improve the analysis. Our findings have implications for both policy and forecasting in Sweden, and contribute to the wider discussion regarding the global dimension and spill-overs in interest rates.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the estimation results. Section 4 studies the global spill-overs, and section 5 analyzes the influence of the natural interest rate in a model of fundamental house prices. Section 6 concludes. The outline of the estimation is placed in the Appendix.

2 The model

The concept of the natural interest rate in our model is similar to the Wicksellian (1936) concept of the natural rate as the real interest rate that is consistent with stable inflation and output. It is thus a more longer-run concept than for instance the natural interest rate in DSGE-models, which is the interest rate that would prevail in a world with flexible prices (see, e.g., Woodford, 2003).

Our semi-structural model assumes a relationship between potential growth and the natural rate of interest, consistent with basic economic theory, but also allows for lasting deviations between the two. To see this, we part from the household intertemporal utility maximization, where using a standard CES utility function, the solution yields the following (log-linear) relationship between the interest rate and steady state growth:

$$r^* = \frac{1}{\sigma} g + \rho, \tag{1}$$

where σ is the intertemporal elasticity of substitution, g is the growth rate, and ρ is the rate of time preference. As in the seminal paper by Laubach and Williams (2003), we link a time-varying version of Equation (1) to the observed economy, and then apply the Kalman filter to data on real gross domestic product (GDP), inflation, and the short-term interest rate to estimate jointly the natural rate of interest, the natural rate of output and trend growth. We use a small semi-structural open economy model, following closely the Bayesian methods proposed by Berger and Kempa (2014).

We define the real interest rate as the nominal rate (i_t) minus expected inflation one year ahead (π^e_{t+4}), $r_t = i_t - \pi^e_{t+4}$, where, as suggested by Laubach and Williams (2003), inflation expectations are proxied by the forecast for the four-quarter-ahead percentage change in inflation, $\pi^e_{t+4} = \hat{\pi}_{t+4|t}$, where $\hat{\pi}_{t+4|t}$ is a dynamic forecast from an univariate AR(3) process with a rolling estimation window of 40 quarters. In line with Berger and Kempa (2014), we add the exchange rate to the Laubach and Williams-model in order to capture the important effects of the exchange rate on inflation and output in a small open economy such as Sweden. In contrast to Berger and Kempa (2014), however, we model the three gap relationships that are present in the model endogenously as a VAR-system, allowing each gap to react to the others. Furthermore, we assume that potential growth is stationary, rather than difference-stationary, since the former seems to be a more plausible assumption.

The real GDP (y_t), the real interest rate (r_t), and the real effective exchange rate (q_t) can each be expressed as the sum of two unobserved components: an equilibrium level, denoted by an asterisk, and a gap, denoted by a tilde,

$$y_t = y_t^* + \tilde{y}_t, \tag{2}$$
$$r_t = r_t^* + \tilde{r}_t, \tag{3}$$
$$q_t = q_t^* + \tilde{q}_t, \tag{4}$$

where y_t^* is the potential GDP, r_t^* is the natural real rate of interest, and q_t^* is the natural real effective exchange rate. Based on the relation (1), and in line with Laubach and Williams (2003), we assume that the natural rate depends on potential growth (which is unobserved and will be estimated simultaneously with the natural interest rate), and a remaining unobserved component that follows a random walk,

$$r_t^* = cg_t + z_t, \tag{5}$$
$$z_t = z_{t-1} + \varepsilon_t^z. \tag{6}$$

The component z_t can be thought of as everything that affects the natural rate that is not related to growth, such as an increased desire to save, changes in fiscal policy, changes in the demand for safe assets, etcetera; see, e.g., Rachel and Smith (2015), Bean et al. (2015), and Armelius et al. (2014), for overviews.

The trend growth is related to potential GDP through the equations

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}, \tag{7}$$
$$g_t = (1 - \varphi_2)\varphi_1 + \varphi_2 g_{t-1} + \varepsilon_t^g. \tag{8}$$

That is, the trend growth rate is assumed to follow a stationary AR(1) process with unconditional mean $\mathrm{E}(g_t) = \varphi_1$, implying that potential GDP is an $I(1)$ process.

In the absence of a strong view regarding the level of the long run exchange rate, the natural effective exchange rate is assumed to follow a random walk,

$$q_t^* = q_{t-1}^* + \varepsilon_t^q. \tag{9}$$

The GDP gap, the real interest rate gap, and the real effective exchange rate gap are explained by a first-order VAR system,

$$\tilde{x}_t = \Psi \tilde{x}_{t-1} + \tilde{\varepsilon}_t, \tag{10}$$

where $\tilde{x}_t = (\tilde{y}_t, \tilde{r}_t, \tilde{q}_t)'$ is a time series vector of gaps, Ψ is a 3×3 parameter matrix, and $\tilde{\varepsilon}_t = (\varepsilon_t^{\tilde{y}}, \varepsilon_t^{\tilde{r}}, \varepsilon_t^{\tilde{q}})'$ is a time series vector of specific errors. The VAR system is consistent with economic theory in that deviations from fundamental values in all the macroeconomic variables are allowed to influence the other variables. For instance, the first equation in the system,

$$\tilde{y}_t = \psi_{11}\tilde{y}_{t-1} + \psi_{12}\tilde{r}_{t-1} + \psi_{13}\tilde{q}_{t-1} + \varepsilon_t^{\tilde{y}}, \tag{11}$$

is a reduced-form of an aggregate demand equation, an "IS-curve", where the output gap is determined by its own lag, and lags from, respectively, the real interest rate gap and the exchange rate gap. When the actual interest rate is above the natural rate, monetary policy is contractionary, which will have a negative impact on the output gap. Similarly, the second and third equations in the system determine, respectively, how the real interest rate gap reacts to cyclical variations in output and the exchange rate, and how the real exchange rate gap reacts to variations in the output gap and the interest rate gap. The natural interest rate is thus the rate that will prevail when the output gap is closed, and the exchange rate is not over- or undervalued, in the absence of other shocks.

Finally, we have an aggregate supply equation, a "Phillips curve", given by

$$\pi_t = \delta_1 + \delta_2\pi_{t-1} + \delta_3 q_{t-1}^n + \delta_4\tilde{y}_t + \varepsilon_t^{\pi}, \tag{12}$$

where π_t is inflation at time t, that is dependent on its own lag, changes in the *nominal* exchange rate q_t^n and the output gap $\tilde{y}_t$. Here, the nominal exchange rate should capture changes in international prices and the contribution of imports. Note that, in the VAR-system (10) we use the real exchange rate gap, since the output gap and the interest rate gap are expressed in real terms.

Equations (2) to (12) are estimated using Bayesian methods based on the Kalman smoother. A Bayesian approach is suitable for dealing with the identification issues that are associated with the unobserved component model at hand. It is well-known that identification can be difficult when the unobserved natural rate in turn depends on other unobserved components; see, for instance, Laubach and Williams (2003), Mesonnier and Renne (2007), and Berger and Kempa (2014). To make use of the Kalman smoother, the equations are cast in state space form. Equations (2)-(4) and Equation (12) have left-hand side variables that are observed, and are therefore modeled as signal (measurement) equations. Equations (5)-(10) have left-hand side variables that are unobserved, and are therefore modeled as state (transition) equations.[1] The details are outlined in Appendix A.

[1]All modelling was done in EViews 9.5, using the EViews programming language. The codes can be sent upon request.

3 Estimation results

We use data on the log of real quarterly GDP and annualized log-difference of seasonal adjusted quarterly core inflation (CPIF).[2] The policy rate is the nominal repo rate, and the exchange rate is the nominal and real exchange rate KIX published by the Riksbank.[3] A key variable is the real interest rate, defined as the repo rate with expected inflation subtracted (see aforementioned). Figure 1 displays the data and modeled inflation expectations. The model is estimated over the sample 1996Q1-2016Q4.

Following Berger and Kempa (2014), we use independent Gaussian prior distributions for the non-variance parameters, and independent gamma prior distributions for the variance

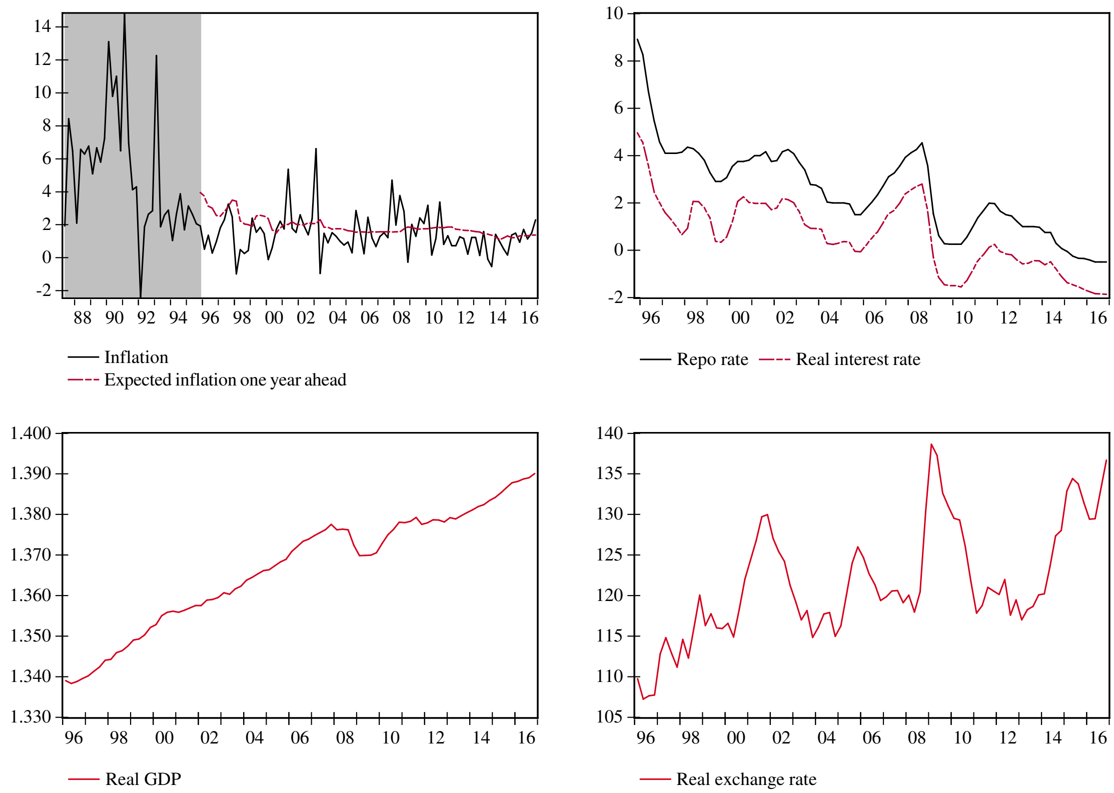

Figure 1. Data used and modeled inflation expectations.

[2]CPIF is inflation adjusted for interest rate changes; it is the measure that the Riksbank is officially targeting since September 2017, but served as the implicit target prior to that (see Sveriges Riksbank, 2017a).

[3]The first two series are from Statistics Sweden, and can be downloaded from www.scb.se. The latter two series are from the Riksbank, and can be downloaded from www.riksbank.se. A description of KIX can be found in Erlandsson and Markowski (2006).

Table 1. Prior and posterior parameter distributions.

	Prior distribution		Posterior distribution	
Parameter	Mean	90% interval	Mean	90 % interval
Natural rate of interest				
c	4	[2.34, 5.65]	0.388	[0.305, 0.472]
σ_z^2	0.25	[0.11, 0.43]	0.059	[0.048, 0.072]
Potential output and growth				
$\sigma_{y^*}^2$	0.50	[0.06, 1.28]	0.090	[0.032, 0.218]
φ_1	0.80	[0.64, 0.96]	0.689	[0.650, 0.726]
φ_2	0.57	[0.41, 0.73]	0.565	[0.523, 0.609]
σ_g^2	0.25	[0.11, 0.43]	0.148	[0.116, 0.187]
Natural exchange rate				
σ_q^2	0.25	[0.11, 0.43]	0.226	[0.168, 0.303]
Output gap				
ψ_{11}	0.50	[0.09, 0.91]	0.985	[0.941, 1.028]
ψ_{12}	0.00	[-0.41, 0.41]	-0.332	[-0.400, -0.263]
ψ_{13}	0.00	[-0.41, 0.41]	0.002	[-0.005, 0.009]
$\sigma_{\tilde{y}}^2$	0.50	[0.06, 1.28]	0.312	[0.248, 0.405]
Real interest rate gap				
ψ_{21}	0.00	[-0.41, 0.41]	0.264	[0.233, 0.294]
ψ_{22}	0.50	[0.09, 0.91]	0.630	[0.589, 0.669]
ψ_{23}	0.00	[-0.41, 0.41]	-0.024	[-0.029, -0.019]
$\sigma_{\tilde{r}}^2$	0.50	[0.06, 1.28]	0.005	[0.001, 0.026]
Real exchange rate gap				
ψ_{31}	0.00	[-0.41, 0.41]	-0.048	[-0.135, 0.038]
ψ_{32}	0.00	[-0.41, 0.41]	0.202	[0.105, 0.298]
ψ_{33}	0.50	[0.09, 0.91]	0.941	[0.920, 0.962]
$\sigma_{\tilde{q}}^2$	1.00	[0.13, 2.57]	5.426	[5.087, 5.798]
Phillips curve				
δ_1	1.00	[0.18, 1.82]	1.334	[1.222, 1.442]
δ_2	0.50	[0.09, 0.91]	0.125	[0.075, 0.174]
δ_3	0.25	[-0.16, 0.66]	0.040	[0.013, 0.065]
δ_4	0.25	[-0.16, 0.66]	0.163	[0.114, 0.209]
σ_{π}^2	2.00	[0.68, 3.88]	1.439	[1.332, 1.555]

parameters. Table 1 shows some specific details of the prior and posterior distributions. Some remarks are at place. We set the prior mean of the potential growth coefficient c in Equation (5) to 4, in line with the idea that the natural real rate of interest should be closely related to annual potential growth. In the growth rate equation (8), we set the prior mean of φ_1 (the AR unconditional mean) to 0.57, in line with an annual steady state growth rate of around 2.3 percent, and the prior mean of φ_2 to 0.8, in line with the belief of a somewhat persistent stationary potential growth. The prior means of the parameters in the gap VAR-system (10) are set to 0.5 for own lags, and 0 for the other lags. This equals a prior belief of independent stationary AR(1) processes - an agnostic view of the nature of the interaction between the gaps.

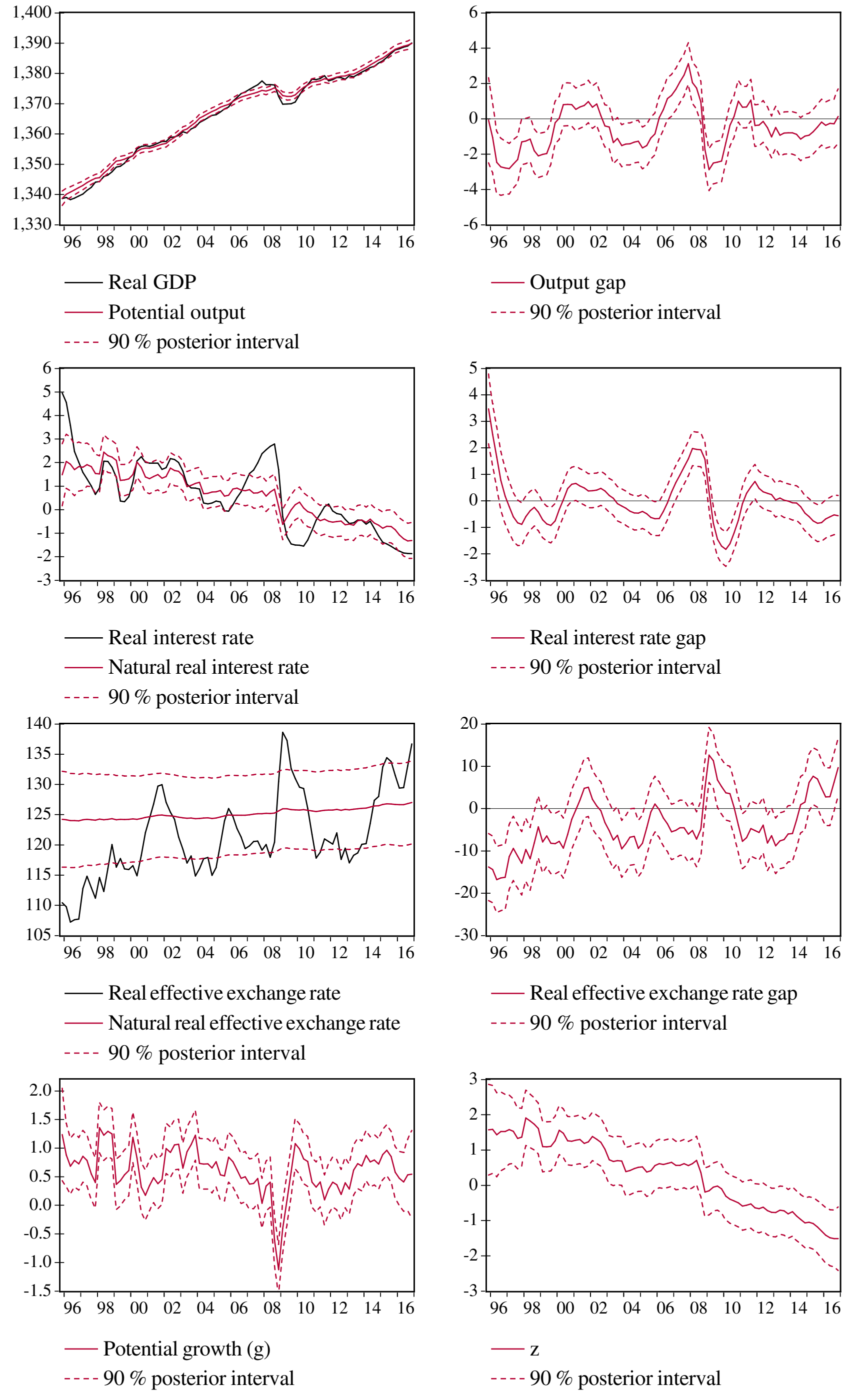

Figure 2. Estimated gaps and trends with posterior intervals.

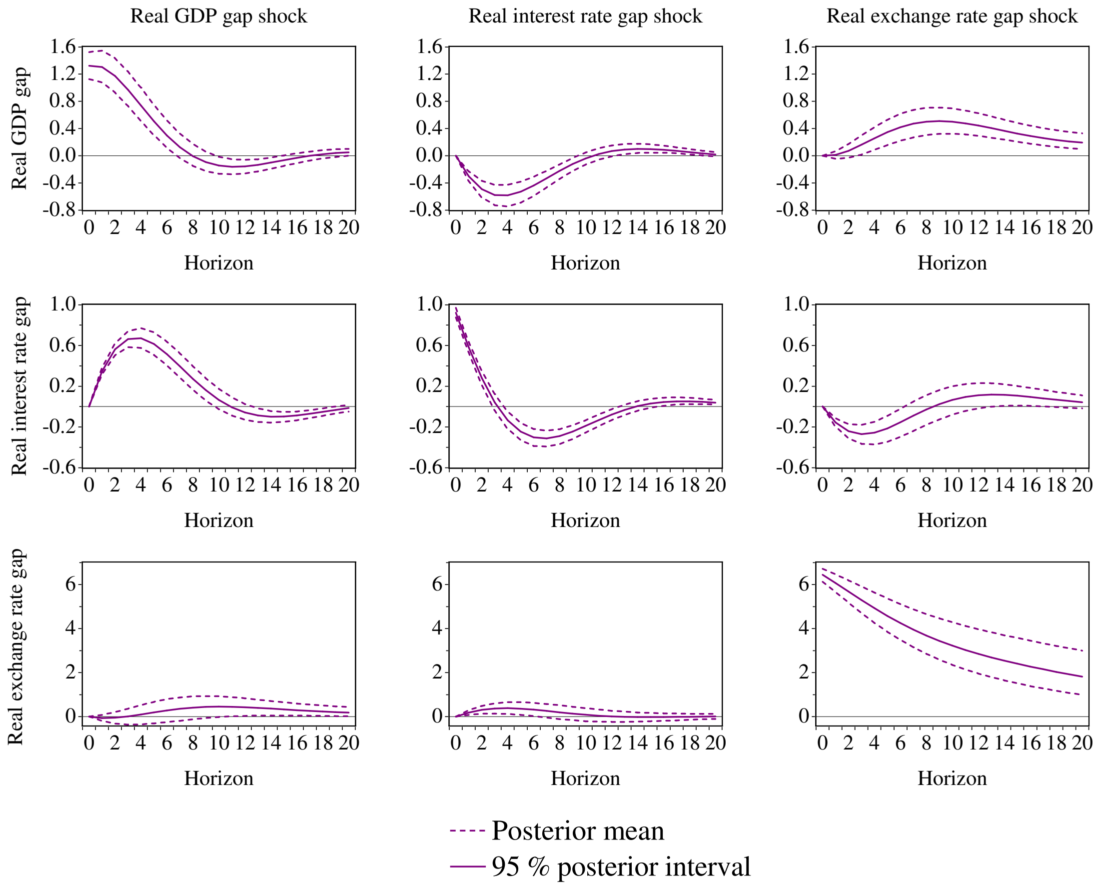

Figure 3. Gap impulse responses.

Figure 2 shows the estimated gaps and trends together with the respective 90 % posterior intervals. As can be seen, the intervals are quite wide for most estimates. Similar uncertainties are typical findings in these types of models, and were one of the major conclusions in Laubach and Williams (2003). As in many previous studies (see, e.g., Rachel and Smith, 2015; Laubach and Williams, 2016, and references therein), there is a clear downward sloping trend in the natural rate of interest since the beginning of the sample period. Moreover, in line with Holston et al. (2017), there is no sign of a recent pick-up in the natural interest rate in our estimate, which has been below zero since the third quarter of 2010. It is also clear that a large fraction of the decline in the natural interest interest rate originates from the unobserved component z_t. Potential growth, on the other hand, has been more stable. Meanwhile, as in, e.g., Hamilton et al. (2015), the connection between potential growth and the natural interest rate appears weak, with a posterior mean for the growth parameter c estimated to 0.388.

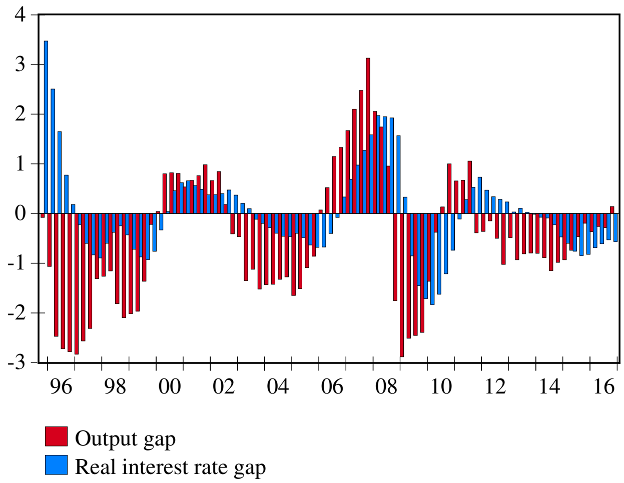

Figure 4. Output and interest rate gaps.

For a small open economy like Sweden it would be natural to expect that the unobserved component consists mainly of influences from abroad. Economic developments in Sweden have been much more favorable than in for instance the Euro Area (EA), the major trading partner of Sweden. We explore this idea in Section 4.

We also produce impulse responses from the gap VAR-system (10) in terms of effects from one standard deviation positive shock in the respective gaps (see Figure 3). A positive shock to the output gap leads to a contractive monetary policy (positive real interest rate gap), which contributes to a faster return to balanced resource utilization. A positive shock in the real interest rate gap leads to a negative development of the output gap, which within four quarters leads to a negative real interest rate gap to get the economy back to equilibrium. The effects of these shocks on the real exchange rate gap are relatively small. A positive shock to the real exchange rate gap is associated with an expansionary monetary policy and a positive output gap. Shocks to the exchange rate gap appear quite persistent.

Figure 4 illustrates the dynamics of the model. When an output gap opens up monetary policy responds with a lag, creating an interest rate gap. This, in turn, eventually brings output back to the equilibrium level, thus closing the output gap.

3.1 Comparison to a model with constant GDP growth

Based on Economic theory, our model allows the natural rate of interest to vary over time in response to shifts in preferences and the growth rate of output. Naturally, it is of interest to study the robustness of our results to changes in these underlying assumptions. Figure 5

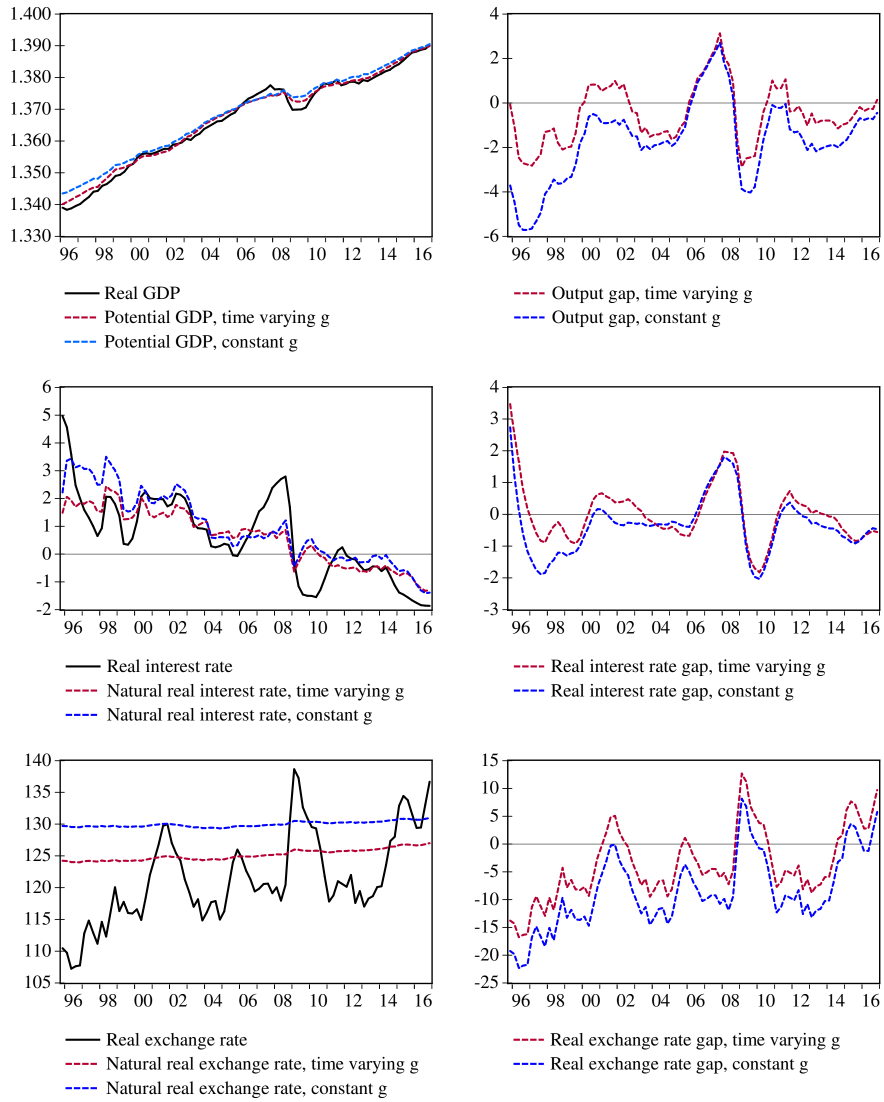

Figure 5. Comparison of time-varying and constant GDP growth.

shows a comparison between the time-varying growth model and a model where growth is assumed to be constant, estimated by the model as a parameter. The latter model is formed by replacing g_t with the parameter φ_1 in Equations (5) and (7), and then removing Equation (8) all together. Note that, while g_t is constant, potential output is still allowed to vary due to the error term in Equation (7). As can be seen in Figure 5, imposing constant potential GDP growth affects both the output gap and the exchange rate gap, which are subject to downward level shifts. These shifts do not, however, affect the estimate of the natural interest rate much, with the exception of the first three years in the sample. There is still a clear downward sloping trend in the natural rate, and the current estimate is still clearly below zero. Thus, the allowance of time-varying growth does not seem to be of major importance for our results. Berger and Kempa (2014) came to the same conclusion for Canadian data.

4 International spill-overs

In this section, we study international spill-overs using cointegration analysis. More precisely, we study if our estimate of the Swedish natural real rate of interest cointegrates with the natural interest rates for some important trading partners to Sweden, namely the US, the UK, and the EA, as estimated by Holston et al. (2017).[4] These natural interest rates, as well as our estimated natural interest rate for Sweden, are displayed in Figure 6.

The interest rates are themselves estimated. Hence, some care should be taken when interpreting the results. However, as in Holston et al. (2017), we view the analysis as a simple way to describe a possible non-stationary comovement between the interest rates. For all natural interest rates, augmented Dickey-Fuller (ADF) tests (Dickey and Fuller, 1979) cannot reject the null hypotheses of a unit root in (see Table 2). Hence, in a descriptive manner, the question of cointegration becomes interesting. Because we primarily expect the other economies' interest rates to affect Swedish interest rates, we use a single-equation error-

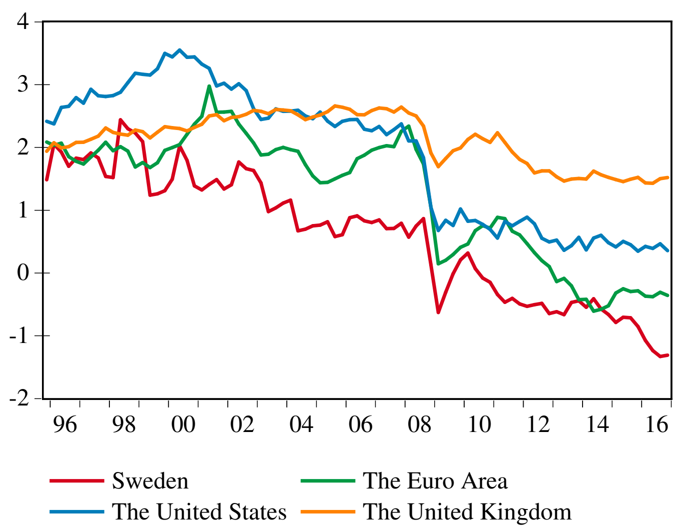

Figure 6. Estimated natural interest rates.

[4]The estimated natural interest rates by Holston et al. (2017) were downloaded from John Williams' San Francisco Fed website: http://www.frbsf.org/economic-research/economists/john-williams.

correction framework, estimated by the two-step Engle and Granger (1987) approach. That is, we first estimate the following equation by OLS:

$$r^*_{t,SWE} = \beta_0 + \beta_{US} r^*_{t,US} + \beta_{EA} r^*_{t,EA} + \beta_{UK} r^*_{t,UK} + \varepsilon^c_t, \tag{13}$$

where β_j are parameters, $r^*_{t,SWE}$ is our estimate of the natural real interest rate in Sweden, $r^*_{t,US}$, $r^*_{t,EA}$ and $r^*_{t,UK}$ are the estimates of the natural real interest rates in the US, the EA and the UK, respectively, from Holston et al. (2017), and ε^c_t is the error term. Second, we test for a unit root in the residual series $\hat{\varepsilon}^c_t$ using an ADF test. As shown in Table 2, the null hypothesis of a unit root is rejected, suggesting cointegration between the natural interest rates.

Based on the residual series and estimated parameters, we can construct a cointegrating equation normalized with respect to the Swedish natural interest rate,

$$\hat{\varepsilon}^c_t = r^*_{t,SWE} - \hat{\beta}_0 - \hat{\beta}_{US} r^*_{t,US} - \hat{\beta}_{EA} r^*_{t,EA} - \hat{\beta}_{UK} r^*_{t,UK}. \tag{14}$$

Figure 7 shows the fitted values and residuals (i.e. the cointegrating equation) from the estimation of the level regression (13). According to the estimation, the natural rate in Sweden has been below the fitted values in recent years. This implies that the Swedish natural real interest rate is somewhat lower than what can be expected by just looking at the international interest rate environment. However, the longer-term decline in the natural interest rate for the last decades seems to be largely driven by international factors, in line with what the Riksbank has been arguing (see, e.g., Sveriges Riksbank, 2017b). Our analysis thus gives some formal support for that type of statement.

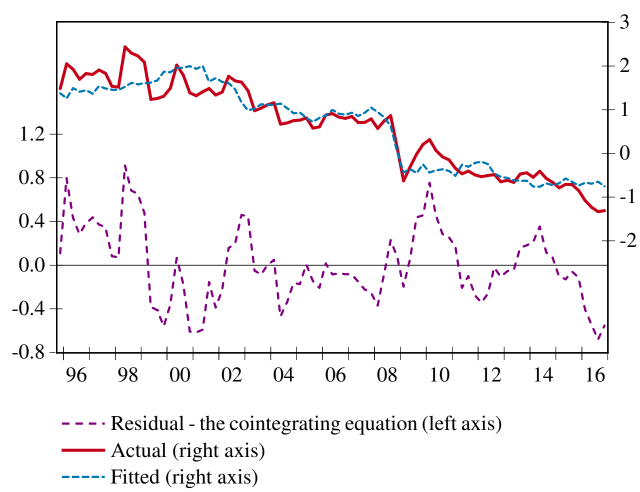

Figure 7. Actual versus fitted values from first-step regression.

Table 2. Estimation results, cointegration analysis.

	$r^*_{t,SWE}$	$r^*_{t,US}$	$r^*_{t,EA}$	$r^*_{t,UK}$	$\hat{\varepsilon}^c_t$
ADF, p-values	0.98	0.96	0.80	0.76	0.02
Level	$\hat{\beta}_0$	$\hat{\beta}_{US}$	$\hat{\beta}_{EA}$	$\hat{\beta}_{UK}$	
	-0.081	**0.691**	**0.377**	**-0.516**	
	(0.362)	(0.095)	(0.134)	(0.202)	
Error correction	$\hat{\alpha}_0$	$\hat{\alpha}_\varepsilon$	$\hat{\alpha}_{US}$	$\hat{\alpha}_{EA}$	$\hat{\alpha}_{UK}$
A		**-0.216**			
		(0.078)			
B	-0.032	**-0.214**			
	(0.027)	(0.078)			
C		**-0.219**	**0.456**		
		(0.075)	(0.162)		
D		**-0.210**		0.084	
		(0.078)		(0.142)	
E		**-0.233**			0.399
		(0.078)			(0.288)
F	-0.024	**-0.241**	**0.459**	-0.175	0.290
	(0.026)	(0.078)	(0.183)	(0.172)	(0.343)

Note: Standard errors in parentheses. Bold numbers are significant at the 5 percent level. For ADF tests, p-values are in accordance to MacKinnon (1996).

To further assess the nature of the Swedish natural interest rate's relationship to other estimated interest rates, we construct error correction models on the form

$$\Delta r^*_{t,SWE} = \alpha_0 + \alpha_{US}\Delta r^*_{t-1,US} + \alpha_{EA}\Delta r^*_{t-1,EA} + \alpha_{UK}\Delta r^*_{t-1,UK} + u_t, \qquad (15)$$

where α_j are parameters to be estimated, and u_t is an error term. Table 2 shows six different error correction model (ECM) specifications from Equation (15), denoted A to F. The first ECM includes only $\hat{\varepsilon}^c_t$, the cointegrating equation (14). The second ECM adds the constant α_0, and so on. Note that the error correction term coefficient α_ε is significant, and estimated to be less than -0.2 for all specifications. As such, more than one fifth of the disequilibrium from the international rates is expected to be regained each quarter. Additionally, specifications C and F show a significant short term lead from the US natural rate to the Swedish natural rate.

5 The influence of the natural rate on real house prices

Swedish house prices have received a lot of attention in both the media and in policy circles in Sweden (see, for instance, Dermani et al., 2016). The main issue is whether there is overvaluation, or if the continuous and significant rise can be explained by fundamental economic circumstances. In this section, we extend the analysis on Swedish house prices in Claussen (2013), originally part of a larger study by Sveriges Riksbank (2011).

Claussen (2013) uses cointegration analysis to find a long-run relationship between house prices and a few macroeconomic variables that should be able to explain their fundamental value: real disposable income per capita, household financial net worth, and the real mortgage rate. We use similar variables, but replace the real interest rate with our estimated natural real interest rate. According to theory, the latter should reflect more long-term fundamental forces and it is thus conceivable that it could have a bigger influence on long-run house prices than the policy rate, since a house purchase typically is a very long-run investment for a household. On the other hand, if the decline in the natural interest rate reflects some form of pessimism regarding the future, which would be associated with falling asset prices, it is possible that it could have the opposite effect. We also add the estimated real interest rate gap, which can be interpreted as the monetary policy stance. In the debate, monetary policy is often blamed for keeping interest rates low, and thus contributing to the increase in housing prices. However, in absence of a measure of the natural rate, it is impossible to distinguish if it is policy that is contributing to rising prices or not, since any measure of real interest rates will capture both the natural rate and the policy stance.

Whereas Claussen (2013) uses dynamic OLS to estimate the long-run equilibrium relationship, we apply a vector ECM (VECM),

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \cdots + \Gamma_{p-1} \Delta x_{t-p+1} + v_t, \tag{16}$$

where x_t is a $n \times 1$ vector consisting of the relevant time series, $\Pi, \Gamma_1, ..., \Gamma_{p-1}$ are $n \times n$ parameter matrices, and v_t is an error term. It is well-established (see Johansen, 1995) that $I(1)$ cointegration is a restriction on the matrix Π, which under reduced rank r $(0 < r < n)$

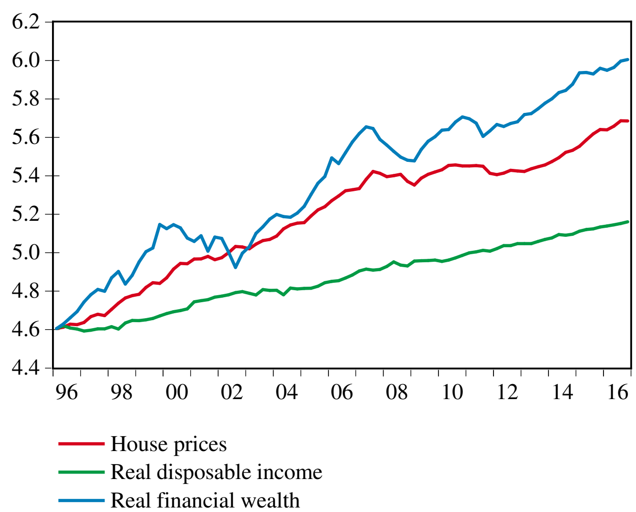

Figure 8. Log of indexed data in house price models, excluding interest rates.

Table 3. VECMs of fundamental house prices.

	House prices	Interest rate	Natural interest rate	Interest rate gap	Disposable income	Financial wealth	Constant
ADF, p-values	0.48	0.22	0.98	< 0.01	0.98	0.68	
VECM1: $adj.R^2 = 0.46$							
Long-run parameters	1	0.01			**6.13**	**-3.69**	32.18
		(0.08)			(1.93)	(0.85)	
Adjustment parameters	0.008	-0.102			-0.015	0.038	
	(0.007)	(0.170)			(0.134)	(0.021)	
VECM2: $adj.R^2 = 0.50$							
Long-run parameters	1		**-0.25**	0	0.91	**-1.90**	7.98
			(0.12)		(0.95)	(0.38)	
Adjustment parameters	0.013		0.145	0.084	-0.025	**0.124**	0.038
	(0.017)		(0.255)	(0.200)	(0.014)	(0.049)	

Note: Standard errors in parentheses. Bold numbers are significant at the 5 percent level.

can be decomposed into $\Pi = \alpha\beta'$, where α $(n \times r)$ is the adjustment matrix, and β $(n \times r)$ is the cointegrating matrix. The r cointegrating (long-run) equations are given by $\beta' x_t$. We estimate two models. The first model (VECM1) includes variables according to Claussen (2013), that is, the vector x_t consists of the log of real house prices, the log of real disposable income per capita, the log of household financial net worth, and the real interest rate (i.e., $n = 4$).[5] The second model (VECM2) replaces the real rate with our estimated natural interest rate, and adds the real interest rate gap (i.e., $n = 5$). The latter is, however, not allowed to enter the cointegrating equation, by imposing the restriction that the associated elements in β are set to zero. The real interest rate and the natural real interest rate were shown in Figure 2. The remaining data are indexed and shown in Figure 8 (in logs), for the same sample period that we used to estimate the natural rate, that is, 1996Q1-2016Q4. The sample period does not include any significant downturn in the housing market. What's more, during the period, house prices in Sweden have grown faster than real disposable income, which is part of the reason for the concern over a possible debt-financed bubble.

Table 3 displays individual ADF unit root tests. At the five percent level, the test rejects the null hypothesis of a unit root in the natural real interest rate gap (i.e., the gap is stationary, as implied by our model setup in Section 2), but fails to reject the null hypothesis for the other series. Thus, we can move on to test for cointegration. We apply the trace test by Johansen (1991) that allows for a constant in the cointegrating relationship. Because the trace test is sensitive, in terms of size distortions, to under-parametrization, but not to over-

[5] As in Claussen (2013), real house prices are measured by the Statistics Sweden's real-estate price index, deflated by CPIF. The real disposable income is from the national accounts, Statistics Sweden. While Claussen uses real mortgage rates, we simplify by using the real repo rate (see Section 3).

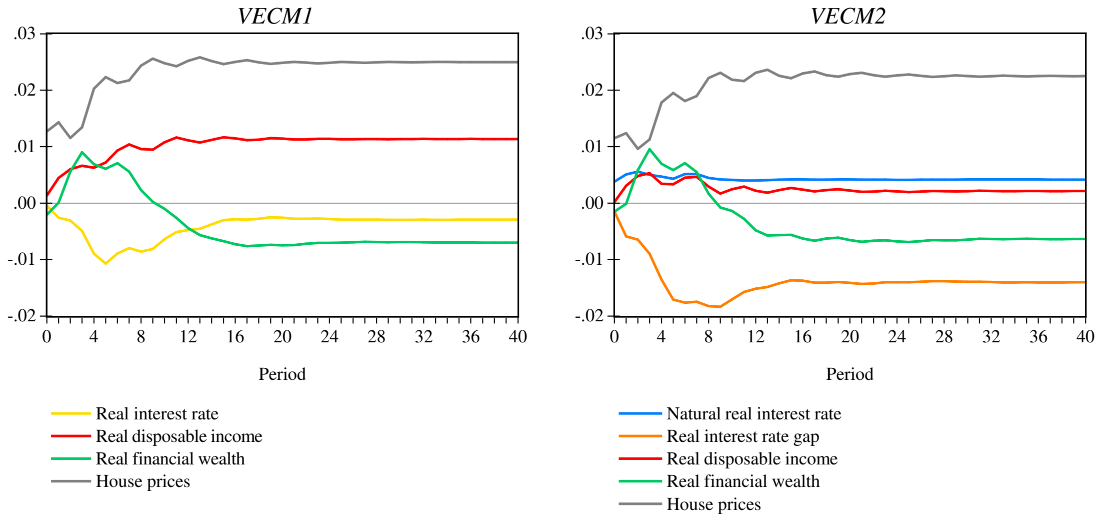

Figure 9. Responses in house prices to one standard deviation innovations.

parametrization (see Cheung and Lai, 1993), we set the lag number in (16) to a rather large number, $p = 8$; the test suggests one cointegrating relationship for both VECM1 and VECM2 (i.e., $r = 1$). The models are, however, both estimated using the smaller lag number $p = 5$, as suggested by information criteria. The most central output - the cointegrating (long-run) vectors and adjustment vectors - are shown in Table 4; VECM2 has a slightly higher fit in terms of adjusted R^2 than VECM1. Note that, without imposing identifying restrictions, the estimated coefficients do not need to be interpretable in an economic sense. Moreover, since all the series in the long-run relationship of the model are endogenously affecting each other, the coefficients tend to change with respect to the lag length. The separate effect of each series is therefore easier to disentangle in the form of impulse responses, which are shown in Figure 9. Based on the impulse responses, the real interest rate has a small negative effect on house prices, with a maximum impact of 0.1 percent after 4 quarters. When interest rates are separated into the natural rate and the interest rate gap, the effect of monetary policy almost doubles, while the effect of the natural rate is very close to zero.

We also compute Cholesky variance decompositions for (log of) house prices with respect to shocks to the other time series (see Figure 10). The orderings in the decompositions are the same as the orderings in the respective legends. It is noticeable that the model including the natural interest rate and the real interest rate gap is able to explain about 40 percent of the variation in house prices, which is about double of what is explained in the model where only the real interest rate is included.[6] The higher adjusted R^2 in VECM2 suggests that this

[6]This result is robust to the choice of ordering of the series.

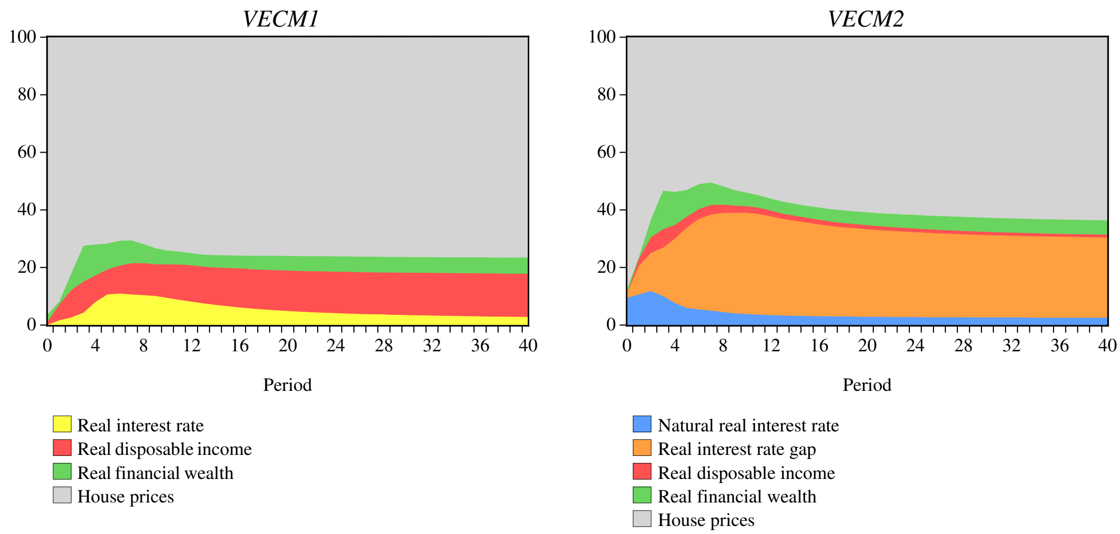

Figure 10. Variance decompositions for house prices.

is not only due to the increased number of variables. Hence, separating the natural part of the interest rates from the monetary policy stance could be informative.

Figure 11 shows the estimated cointegrating relationships from the models VECM1 and VECM2. According to the models, the house prices were not far from the estimated equilibriums by the end of 2016.

All in all, our analysis indicates that the fall in the natural interest rate is not a main reason for increasing house prices in Sweden. Moreover, our findings can also be viewed as a demonstration of the advantages of having a measure of the natural rate available, since they indicate that separating the natural rate from the policy stance can influence the results in these types of models.

6 Conclusions

We have produced an estimate of the natural rate of interest in Sweden, using a small-scale macroeconomic model. We found that the natural rate is currently negative in Sweden, and that it has been on a declining trend for the past two decades. Most of the decline in the Swedish natural rate can in our model be traced to unobserved components that are unrelated to growth of potential GDP. The Riksbank has in its' communication stated that the decline in the Swedish natural interest rate has its origin in global factors. We assess this hypothesis by testing for a cointegrating relationship between our estimate of the natural rate in Sweden and the estimated natural rates of United States, the Euro Area and the United Kingdom. Al-

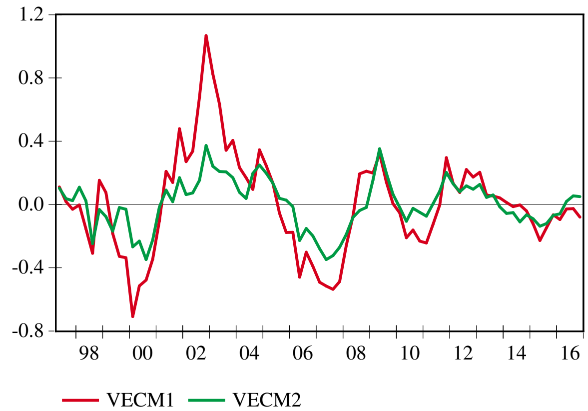

Figure 11. Cointegrating relations for models VECM1 and VECM2.

though the test should be interpreted with some caution, since the time series are themselves estimated, we do find evidence of a cointegrating relation. We also find a significant error correction term for the Swedish natural rate. More than one fifth of disequilibrium from a linear combination of the other rates is expected to be recovered each quarter. Additionally, we find significant influence coming from the natural rate in the United States, which also leads the Swedish natural rate. This could have important implications for monetary policy in Sweden, and in other small open economies. Finally, we also use our natural interest rate estimate to analyze the effect it has on Swedish house prices, and find that when the real rate is separated into the natural rate part and a monetary policy part, the influence of the latter gets stronger while the influence of the natural part is close to zero. These findings thus help shed some light on why it might be necessary to have negative policy rates in an otherwise booming economy.

Appendix A: Technical details

Our estimation method follows, in large, Berger and Kempa (2014). The method is explained in detail in the following subsections.

A1. A state space representation

Linear time series can be cast in state space form

$$x_t = \mu + H\alpha_t + Aw_t + u_t, \tag{A1}$$

$$\alpha_{t+1} = \kappa + T\alpha_t + R\eta_t, \tag{A2}$$

where x_t is the vector of dependent observable variables, w_t is a vector of exogenous observable variables, α_t is a latent state vector, H, A and T are coefficient matrices, R is a selection matrix (usually a subset of the columns of the identity matrix), μ and κ are vector of constants, and u_t and η_t are errors that by assumption are Gaussian: $u_t \sim \mathcal{N}(0, \Sigma_u)$, $\eta_t \sim \mathcal{N}(0, \Sigma_\eta)$. Equations (A1) and (A2) are referred to as the signal equation and state equation, respectively. The latent state vector α_t can be estimated by the Kalman filter and smoother, where the latter is a backward recursion using the output from the filter; see, e.g., Harvey (1989) and Durbin and Koopman (2012). In this paper, we use the smoother to estimate the states.

The state space representation of equations (2)-(12) is outlined as follows. The components are

$$\begin{aligned} x_t &= (y_t, r_t, q_t, \pi_t)', \\ w_t &= (\pi_{t-1}, \Delta q^n_{t-1})', \\ \alpha_t &= (y^*_t, r^*_t, q^*_t, g_t, z_t, \tilde{y}_t, \tilde{r}_t, \tilde{q}_t)'. \end{aligned}$$

The coefficient vectors and matrices are

$$\begin{aligned} \mu &= (0, 0, 0, \delta_1)', \\ \kappa &= (0, 0, 0, (1 - \varphi_2)\varphi_1, 0, 0, 0, 0)', \end{aligned}$$

where φ_2 also enters in (A3),

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \delta_4 & 0 & 0 \end{pmatrix},$$

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \delta_2 & \delta_3 \end{pmatrix},$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varphi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_{11} & \psi_{12} & \psi_{13} \\ 0 & 0 & 0 & 0 & 0 & \psi_{21} & \psi_{22} & \psi_{23} \\ 0 & 0 & 0 & 0 & 0 & \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix}, \tag{A3}$$

and the selection matrix is

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The error u_t is four-dimensional, with independent elements, so that its contemporary covariance matrix is diagonal $\Sigma_u = \mathrm{diag}(0, 0, 0, \sigma^2_\pi)$, and the error η_t is seven-dimensional, with independent elements, so that its contemporary covariance matrix is diagonal with non-zero elements $\Sigma_\eta = \mathrm{diag}(\sigma^2_{y^*}, \sigma^2_{q^*}, \sigma^2_g, \sigma^2_z, \sigma^2_{\tilde{y}}, \sigma^2_{\tilde{r}}, \sigma^2_{\tilde{q}})$.

A2. Parameter estimation and posterior distributions

Let the parameters of the model be collected in the vector θ, and let X denote the stacked vector $X = (x'_1, x'_2, ..., x'_T)'$. Given X, the state vector α_t is estimated numerically for $t = 1, 2, ..., T$ by the Kalman smoother. In this paper, we treat θ and α_t ($t = 1, 2, ..., T$) as random parameter vectors, following, in large, the procedure in Berger and Kempa (2014).

For the sake of brevity, we outline the method of estimation for θ. The estimation of the states follow the same principle. By Bayes' theorem we have that

$$p(\theta)p(X|\theta) \propto p(\theta|X),$$

where $p(\theta)$ denotes the prior density of θ, $p(X|\theta)$ denotes the likelihood function and $p(\theta|X)$ denotes the posterior density of θ. Let $m(\theta)$ be a function of θ such that a moment of the posterior density is obtained by

$$m = \mathrm{E}[m(\theta)|X] = \int m(\theta)p(\theta|X)d\theta. \tag{A4}$$

We use importance sampling (see Särkkä, 2013, for a thorough treatment) with an importance density $i(\theta|X)$ as a proxy for $p(\theta|X)$. Let

$$f(\theta, X) = \frac{p(\theta)p(X|\theta)}{i(\theta|X)} \propto \frac{p(\theta|X)}{i(\theta|X)} \tag{A5}$$

be a weighting function, such that the expectations under $p(\theta|X)$ are the same as under $f(\theta, X)i(\theta|X)$. After some manipulations, Equations (A4) and (A5) follow from

$$m = \frac{\int m(\theta)f(\theta, X)i(\theta, X)d\theta}{\int f(\theta, X)i(\theta|X)d\theta}.$$

We can sample θ from the known importance density a large number of times (n), and produce an estimate of m as

$$\tilde{m} = \sum_{i=1}^{n} w_i m(\theta^{(i)}), \tag{A6}$$

where $\theta^{(i)}$ is the ith draw from $i(\theta|X)$, and

$$w_i = \frac{f(\theta^{(i)}, X)}{\sum_{i=1}^{n} f(\theta^{(i)}, X)}.$$

If $i(\theta|X)$ is proportional to $p(\theta|X)$, then, under some weak regularity conditions (see Geweke, 1989), $\tilde{m}$ is an, almost surely, consistent estimator of m ($\tilde{m} \overset{a.s.}{\to} m$), as $n \to \infty$. As in Berger and Kempa (2014), we choose a normal distribution as the importance density,

$$i(\theta|X) = \mathcal{N}(\theta^{(g)}, \hat{\Omega}^{(g)}),$$

where g denotes the gth step in the sequential updating algorithm. The algorithm starts from $\theta^{(0)} = \mathcal{M}$ and $\hat{\Omega}^{(0)} = 2\mathcal{J}^{-1}$, where $\mathcal{M}$ is the estimated posterior mode and $\mathcal{J}$ is the approximate hessian obtained when maximizing

$$\log p(\theta|X) = \log p(X|\theta) + \log p(\theta) - \log p(X).$$

For this purpose, we use the BFGS-algorithm (see, e.g., Chapter 9 of Dennis and Schnabel, 1983, for details). Since $p(X)$ is not a function of θ, it can be disregarded in the optimization. The inverted hessian is inflated with a factor of 2 to take into account the risk of thicker tails in the posterior distribution, following the suggestion of Bauwens et al. (1999).

We update the importance density using the estimated posterior mean and posterior variance by

$$\hat{\theta}^{(g)} = \tilde{m}^{g-1}; \;\; m(\theta) = \theta,$$

and

$$\hat{\Omega}^{(g)} = \tilde{m}^{g-1}; \;\; m(\theta) = (\theta - \mathrm{E}[\theta|X])(\theta - \mathrm{E}[\theta|X])',$$

until a satisfying precision of $\hat{\theta}^{(g)}$ is obtained. The precision of an element in $\hat{\theta}^{(g)}$ (denoted $\hat{\theta}_j^{(0)}$) is measured by the 95 percent relative error bound given by Bauwens et al. (1999, Eq. 3.34). We update the importance density until this error bound does not exceed 10 percent for any $\hat{\theta}_j^{(0)}$.

The procedure defined above is also applied to estimate the posterior mean for each smoothed state by setting

$$\hat{\alpha}_t = \tilde{m}; \;\; m(\theta) = \alpha_t, \; (t = 1, 2, ..., T),$$

where $\alpha_t^{(i)}$ in equation (A6) is the estimated smoothed state by the Kalman smoother using $\theta^{(i)}$ from the importance density in the last step of the updating algorithm.

Percentiles for the posterior marginal densities can be obtained using the following procedure. Let $F(\theta_j^{(a)|X}) = Pr(\theta_j^{(a)} \leq \theta_j|X)$, where $\theta_j^{(a)}$ is an arbitrary value. An estimate of $F(\theta_j^{(a)|X})$ is obtained by

$$\hat{F}(\theta_j^{(a)|X}) = \tilde{m}; \;\; m(\theta) = I(\theta_j^{(a)}),$$

where $I(\theta_j^{(a)})$ is an indicator function that equals one if $\theta_j^{(a)} \leq \theta_j$ and zero otherwise. An estimate of the bth percentile of the marginal posterior density is thereby equal to the $\theta_j^{(a)}$ such that $\hat{F}(\theta_j^{(a)|X}) = b$.

The percentiles forming the 90 % posterior intervals for the smoothed states are obtained by

$$\alpha_{j,t}^{95\%} = \tilde{m}; \;\; m(\theta^{(i)}) = \alpha_{j,t}^{(i)} + 1.645\sqrt{\hat{U}_{j,t}^{(i)}}, \;\; (t = 1, 2, ..., T),$$
$$\alpha_{j,t}^{5\%} = \tilde{m}; \;\; m(\theta^{(i)}) = \alpha_{j,t}^{(i)} - 1.645\sqrt{\hat{U}_{j,t}^{(i)}}, \;\; (t = 1, 2, ..., T),$$

where $\alpha_{j,t}^{(i)}$ is the jth element of the estimated smoothed state vector and $\hat{U}_{j,t}^{(i)}$ is the jth diagonal element of the estimated smoothed state covariance matrix using $\theta^{(i)}$ from the importance density. These posterior intervals capture both parameter and filter uncertainty.

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