ON HYPOTHESIS TESTING FOR IMPULSE RESPONSE FUNCTION

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The problem of estimation of a stochastic linear system has been a matter of active research for the last years. One of the simplest models considers a 'black box' with some input and giving a certain output. The input may be single or multiple and there is the same choice for the output. This generates a great amount of models that can be considered. The sphere of applications of these models is very extensive, ranging from signal processing and automatic control to econometrics (errors-in-variables models).

An impulse response is the reaction of any dynamic system in response to some external change. The impulse response describes the reaction of the system as a function of time. The dynamic system and its impulse response we consider as mathematical systems of equations describing such objects. Any system in a large class known as linear, time-invariant (LTI) is completely characterized by its impulse response. That is, for any input, the output can be calculated in terms of the input and the impulse response.

Consider a LTI continuous system with a real-valued square integrable impulse response function $H(\tau)$ which is defined on finite domain $\tau \in [0, \Lambda]$. This means that the response of the system to an input signal X(t), which is observed on $t \in [-\Lambda, \Lambda]$, has the following form:

$$Y(\mathrm{t}) = \int_0^{\Lambda} H(\tau)\mathrm{X}(\mathrm{t}-\tau)\,d\tau,\ t \in [0, \Lambda],$$

where $H \in L_2([0, \Lambda])$.

One of the problems arising in the theory of linear systems is to estimate the function H from observations of responses of the system to certain input signals. A sample input–output cross-correlogram is taken as an estimator of the response function

$$\hat{H}(\tau) = \hat{H}_{N,T,\Lambda}(\tau) = \frac{1}{T}\int_0^T Y_N(\mathrm{t})\mathrm{X}_N(\mathrm{t}-\tau)\,\mathrm{dt},$$

where T>0 is a parameter for averaging.

The input processes are supposed to be zero-mean stationary Gaussian process and can be represented in trimmed series of Fourier expansion.

Two criteria on the shape of the impulse response function are given that are based on the rates of convergence of the estimator $\hat{H}(\tau)$ to $H(\tau)$ Banach spaces $C(\mathrm{T}), \mathrm{L}_2(\mathrm{T})$.

References

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