# BAYESIAN A/B/C TESTING

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A/B/C testing is experiment of showing three variants of landing page to website visitors, and statistical analysis is used to determine which one performs better for a given conversion aim.

**Problem formulation.** Bernoulli trials with probabilities $\theta_A, \theta_B, \theta_C$ of success are implemented in three groups of visitors (A, B, C). Probabilities $\theta_A, \theta_B, \theta_C$ of success are unknown random variables. The loss function $L(\theta_A, \theta_B, \theta_C, \cdot)$ describes the loss under decision making about choosing landing page variant which can be published on website. Loss function is $L(\theta_A, \theta_B, \theta_C, A) = max\{\theta_B - \theta_A, \theta_C - \theta_A, 0\}$ for group A, and $L(\theta_A, \theta_B, \theta_C, B) = max\{\theta_A - \theta_B, \theta_C - \theta_B, 0\}$ for group B, and $L(\theta_A, \theta_B, \theta_C, C) = max\{\theta_A - \theta_C, \theta_B - \theta_C, 0\}$ for group C [1]. Our goal is to compute the expected loss $EL(\theta_A, \theta_B, \theta_C, \cdot)$ with analytical approach, to find Bayesian estimators $\hat{\theta}_A, \hat{\theta}_B, \hat{\theta}_C$ for probabilities $\theta_A, \theta_B, \theta_C$ of success concerning the loss function $L(\theta_A, \theta_B, \theta_C, \cdot)$, and compute probabilities $P\{\theta_A > \max\{\theta_B, \theta_C\}\}$, $P\{\theta_B > \max\{\theta_A, \theta_C\}\}$, $P\{\theta_C > \max\{\theta_A, \theta_B\}\}$.

**Results.** The expected loss is computed with analytical approach for groups A, B, C [2].

$$EL(\theta_A, \theta_B, \theta_C, A) =$$
$$= \frac{B(c+1,d)}{B(c,d)}\big(1 - h(a,b,c+1,d)\big) - \frac{B(a+1,b)}{B(a,b)}\big(1 - h(a+1,b,c,d)\big) -$$
$$- \frac{B(c+1,d)}{B(c,d)}\sum_{i=0}^{e-1}\frac{B(c+1+i,d+f)}{(f+i)B(1+i,f)B(c+1,d)}(1 - h(a,b,c+i+1,d+f) +$$
$$+ \frac{B(a+1,b)}{B(a,b)}\sum_{i=0}^{e-1}\frac{B(c+i,d+f)}{(f+i)B(1+i,f)B(c,d)}(1 - h(a+1,b,c+i,d+f) +$$
$$+ \frac{B(e+1,f)}{B(e,f)}\big(1 - h(a,b,e+1,f)\big) - \frac{B(a+1,b)}{B(a,b)}(1 - h(a+1,b,e,f) -$$
$$- \frac{B(e+1,f)}{B(e,f)}\sum_{i=0}^{e}\frac{B(a+i,b+f)}{(i+f)B(1+i,f)B(a,b)}\big(1 - h(a+i,b+f,c,d)\big) +$$
$$+ \frac{B(a+1,b)}{B(a,b)}\sum_{i=0}^{e-1}\frac{B(a+1+i,b+f)}{(i+f)B(1+i,f)B(a+1,b)}(1 - h(a+1+i,b+f,c,d)) +$$
$$+ \frac{B(e+1,f)}{B(e,f)}\sum_{i=0}^{e}\frac{B(c+i,d+f)}{(i+f)B(1+i,f)B(c,d)}(1 - h(a,b,c+i,d+f)) -$$
$$- \frac{B(a+1,b)}{B(a,b)}\sum_{i=0}^{e-1}\frac{B(c+i,d+f)}{(i+f)B(1+i,f)B(c,d)}\big(1 - h(a+1,b,c+i,d+f)\big),$$

where

$$h(a,b,c,d) = 1 - \sum_{i=0}^{c-1}\frac{B(a+i,b+d)}{(d+i)B(1+i,d)B(a,b)}.$$

## References

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