Improvements in calculating the marginal effective tax rate on entrepreneurial investments in a dual tax system: The Swedish case*

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Abstract: This study presents an improvement of the King-Fullerton framework for calculating the marginal effective tax rate (METR) in a dual income tax system. A growing literature points out the importance of entrepreneurial investments for economic growth. The design of the tax system is of significant importance for the allocation of investments and the trade-off between savings and consumption. The established methods for calculating the METR does not fully capture the complexity in the Swedish dual tax system. This study especially extends earlier models to account for the importance of growing legal capital in closely held corporations after investments financed with new share issues. The main finding is that the METR can be calculated with less restrictive assumptions through a more accurate implementation of the legal framework into the model. Earlier studies will overestimate the METR for investments financed with new share issues or rely on more binding restrictions. The model described in the study gives a more complete and flexible toolbox for calculating the METR in a dual tax system, for debt, retained earnings and new share issues as sources of finance.

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1 Introduction

The 1990–1991 tax reform in Sweden introduced a dual income tax system; capital and labor income were taxed separately, with a flat (and lower) tax rate on capital income, and a progressive tax rate on labor income. Special rules for closely held corporations, the so-called 3:12-rules, were introduced to prevent owners of closely held corporations to avoid income shifting. The rules divide the total income into a labor and a capital component, so that the different tax rates can be applied. The rules are complex and unstable; they have almost been reformed on a yearly basis.

Because of the rules' complexity and instability, it has been difficult to incorporate them into standard frameworks for analyzing the effective marginal tax rates. In turn, this makes it difficult to compare the tax burden over time, between sources of finance and investment opportunities. For the same reason it is difficult to analyze the overall effect on the effective taxation of partial changes of the legal framework. Developing a method for calculating the tax burden for investments in closely held corporations that effectively captures main features of the legal framework is of importance both for research, and for policy development.

This study further develops the King-Fullerton framework, which has been used as a standard method to analyze the effect of capital taxation on investments in Sweden and other countries. The King-Fullerton method calculates the marginal effective tax rate (METR) for a marginal investment using three alternative sources of finance: new share issues (NSI), retained earnings (RE) and debt. It takes corporate income taxation, personal income taxation and wealth taxation, and the interactions of these taxes with inflation into account.

It has proven to be difficult to incorporate the rules for closely held corporations in the King-Fullerton framework. How to divide the income into a capital and a labor component is the main difficulty. This turns out to be especially important for investments financed with new share issues. Other complications are the active owners' choice of paying herself/himself wage or dividends, or even accumulate the capital for future capital gains. This study continuous to develops the King-Fullerton (1984) framework after contributions from Lindhe et al (2003), Öberg (2003) and Sørensen (2008).

The purpose of this article is threefold. First, I show different techniques for implementing the King-Fullerton framework in a dual tax system. Depending on what assumptions that are made, there will be several methods, each able to answer different questions about the taxation, for example the evolution of the taxation over time or the difference in taxation between organizational forms. Second, I further develop the framework to improve the accuracy of the METR for closely held corporations, especially by introducing a technique for calculating the tax base for investments financed by new share issues. This source of finance is of special interest since it has a more complicated impact on the taxation, than retained earnings or debt. Finally, I compare the different methods by calculating the different results for the taxation year 2018.

My main findings are that earlier models rest on assumptions that restricts the analysis of the METR to very stylized examples and that those assumptions may lead to an overestimation of the METR for investments financed with new share issues. The model developed in this paper gives researchers and policymakers a flexible tool to evaluate a broad range of policy changes, as well as the necessary technique to construct long time series for the METR.

The rest of the paper is organized as follows. Section 2 briefly describes the taxation of owners of closely held corporations and the optimal tax strategy. Section 3 presents the King-Fullerton framework, earlier extensions and suggests improvements in the calculations when the source of finance is new share issues. Section 4 shows the results for new share issues and Section 5 concludes. Appendix A provides a formal presentation of the METR and extends the calculations to financing with debt and retained earnings, which is more straightforward. Appendix B presents a formal proof of the present value maximization problem. Appendix C deepens the description of the difference between the methods. Appendix D describes tax rates, interest rates and inflations rates used in the numerical examples, and finally Appendix E describes how to incorporate special tax rules into the model.

2 Capital income tax for owners of closely held corporations

The principal rule concerning the definition of closely held corporations is that four or fewer owners have to control more than 50 percent of the ultimate voting rights in the corporation.[1] The rules distinguish between an active owner that invests both capital and labor, i.e., take active part in the governance and the development of the corporation, by calling her shares qualified, and a passive owner passively providing capital by calling her shares unqualified. An owner is regarded active, or synonymously qualified, if (s)he or a close family member is, or during the past five years has been, active in the income generation of the corporation to a

[1] SFS, 1999: 1229, Ch. 56, §3. See Bjuggren et al (2011) for a detailed analysis.

"considerable extent" (Alstadsæter and Jacob, 2016). Otherwise (s)he is called passive, or synonymously unqualified. This paper focuses exclusively on active owners, since the difficulties in calculating the METR are caused by the legislator's intention to prevent income shifting. Passive owners' income is always classified as capital income, and hence not subject to any measures against income shifting.

Since there is no objective way to determine whether the source of the profit in a closely held corporation stems from an active owner's invested capital or work effort, there must be a rule based division between labor and capital income. Lindhe et al (2003) refers to this as the Achilles' heel of the dual tax system.

This division between capital and labor income is the main aim of the rules for closely held corporations. Since owners' income from dividends, capital gains and interest payments are taxed differently, a further division of the total income is necessary to accurately calculate effective tax rates. For an owner of a closely held corporation it is also always possible to distribute the surplus as wage payments to oneself, and as will be seen, the size of the wage payment itself governs how to classify the total income; the income division is complex and dynamic. This paper gives a framework for how this complexity can be dealt with mathematically when calculating the METR (for an exhaustive description of the tax rules and rates, see Wykman 2019b).

2.1 The division between capital and labor income taxation

The tax rules for closely held corporations divides the owner's dividend income and/or capital gains into capital or labor income taxation by the so-called dividend allowance (DA, *gränsbeloppet*). Dividends and/or capital gains below the limit for the dividend allowance are taxed as capital income, while dividends and/or capital gains exceeding the limit are taxed as labor income[2]. A closely held corporation can pay the owner an interest, but only on market conditions, as if the owner was an arbitrary debt holder. Thus, interest payments are not an equivalent choice to dividends or wage payments.[3] Interests payments are taxed separately

[2] Before 2006 and between 2007 and 2009, capital gains were split between capital income and labor income (the so-called "split-rule", *klyvningsregeln*), i.e., half of the gains was taxed as labor income and the other half as capital income, with the exception of 1994 when 70 percent were taxed as labor income (Wykman 2019b).

[3] Interest payments are deductible at corporation level and interest payments are not included in the dividend allowance. However, there are rules that restricts the corporation to pay the owner an interest of choice, and hence avoiding progressive income taxation at personal level. There is no fixed regulation of how high interest the corporation is allowed to pay the owner. It should be on market conditions. The Tax Authority always accepts the government borrowing rate (*statslåneräntan*, SLR) plus 3 percentage points.

and to pay the owner an interest (s)he must be a debt holder. Dividends within the dividend allowance are often called normal dividends, while dividends above are denoted excess dividends.

The way that the dividend allowance is calculated has changed over time. It was originally calculated as an imputed return on the equity base (defined below). The imputed return is the government borrowing rate (*statslåneräntan*, SLR) plus a mark up of originally 5 percentage points, increased to 9 percentage points in 2019.[4] The equity base is calculated as:

$$Equity\ base = Acquisition\ cost\ of\ shares + capital\ injections \tag{1}$$

In 1994, the dividend allowance became more generous by adding part of the wages, a so-called wage base (*lönesummetillägg*), to the equity base (for a more detailed description, see Wykman 2019b). Since 2006 the dividend allowance has two separate parts, the imputed share of the equity base and the wage based allowance (WBA; *lönebaserat utdelningsutrymme*). The WBA is calculated as a share of the wage sum. All together, the dividend allowance is calculated as:

$$\begin{aligned} Dividend\ allowance = {} & Imputed\ return \times equity\ base + \\ & + wage\ based\ allowance \end{aligned} \tag{2}$$

Unused dividend allowances from previous years can be added to the dividend allowance with a separate imputed return, the so-called carry forward interest rate, so that:

$$\begin{aligned} & Dividend\ allowance = This\ year's\ dividend\ allowance + \\ & carry\ forward\ interest\ rate \times unused\ dividend\ allowance \end{aligned} \tag{3}$$

The total tax burden on normal dividends is corporate income tax and personal capital income tax. The total tax burden on excess dividends is corporate income tax and labor income tax. The owner can always withdraw the whole profit from the corporation by paying herself/himself wage, subject to social security contributions and labor income tax.

[4] The government borrowing rate consists of the average market yield on government bonds with a remaining maturity of at least five years (Swedish national debt office, *Riksgäldskontoret*). The actual rate from the 30th November the year before is to be used.

Any surplus withdrawn from the corporation will be taxed in one of three categories, where the tax rates on wage and outside the dividend allowance vary due to level of income:

Wage:

$$\frac{\sigma+\tau_w}{(1+\sigma)} \tag{4}$$

Dividends within the dividend allowance:

$$\tau + (1-\tau)\tau_d \tag{5}$$

Dividends outside the dividend allowance:

$$\tau + (1-\tau)\tau_w \tag{6}$$

Where σ is the social security contribution[5], τ_w is the labor income tax, τ is the corporate income tax and τ_d is the capital income tax on dividends within the dividend allowance. For surpluses that exceed certain thresholds the tax rate in (6) changes to $\tau + (1-\tau)\tau_c$, where τ_c is the statutory capital income tax, which is lower than the labor income tax τ_w, but higher than τ_d, the capital income tax within the dividend allowance. As described above capital gains within the dividend allowance may be taxed at rate τ_d at a personal level, consequently the tax rate in (5) remains the same under dividend payments and capital gains as long as $\tau_d < \tau_c$. Since the Swedish labor income tax system is progressive with three different tax rates there is eight[6] different statutory marginal tax rates facing an active owner of a closely held corporation.

Because of the several marginal tax rates, the earlier work, such as Lindhe et al (2003) and Öberg (2003) relies on a pecking order characterized by the inequality:

$$(1-\tau)(1-\tau_d) > \frac{1-\tau_w}{(1+\sigma)} > (1-\tau)(1-\tau_w) \tag{7}$$

The pecking order in (7) states that dividend payments within the dividend allowance is always preferred to wage payments, and that wage payments is always preferred to dividend payments outside the dividend allowance (but below the threshold). The inequality holds for

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[5] Since there are ceilings for the social benefits, one could assume that the whole social security contribution is a tax without any loss of generality.

[6] Three rates of (4), one of (5) and four of (6)

2018 tax rates, and is discussed for the whole period in Wykman (2019b). For the purpose of this paper, Inequality (7) is taken as given, and hence the optimal tax strategy reduces to a choice (division) between normal dividend payments and wage payments. Situations where wage payments are preferred to dividends within the dividend allowance or when dividends outside the dividend allowance are preferred to wage payments could occur, and can be incorporated in the model derived below by simply changing tax rates and/or payment flows.

Since (7) holds, the optimal tax strategy is straight forward, pay dividends until the whole dividend allowance is used and thereafter withdraw any surplus as wage. The dividend allowance however, depends on the wage payment, which implicates that the tax optimal division of dividends, *D*, and wage, *W*, must be the solution to the owner's after-tax profit maximization problem:

$$Max\ (1-\tau_w)W + \ (1-\tau_d)D \tag{8}$$

$$s.t.\ \ (1+\sigma)W + \frac{1}{1-\tau}D = GE \tag{9}$$

Where *GE* is the corporation's gross earnings. Assuming that the equity base is unity, the surplus is large enough and that there is no unused dividend allowance, the dividends *D* will equal the dividend allowance in Equation (2) and hence:

$$D = \beta + \varphi W \tag{10}$$

Where β is the imputed return[7] on the dividend allowance and φ is the imputed return on the wage base.

Solving (8) for *W* and *D* gives the tax optimal division:

$$W = \frac{(1-\tau)GE-\beta}{\varphi+(1+\sigma)(1-\tau)} \tag{11}$$

$$D = \ \beta + \varphi \frac{(1-\tau)GE-\beta}{\varphi+(1+\sigma)((1-\tau)} \tag{12}$$

[7] Since β is a function of the government-borrowing rate, it could be calculated within the model presented below. Earlier work however takes it as exogenously given and this paper confirms to that standard. Section 4 briefly discusses the endogenous situation where $\beta = i + statutory\ imputed\ return$.

This division of income is in line with Öberg (2003) and is necessary in order to estimate the marginal effective tax rate on investments financed by new share issues. If the investment is financed with retained earnings, the equity base remains constant and the dividend allowance in Equation (10) reduces to:

$$D = \varphi W \tag{13}$$

With a clear pecking order and a correct division of the total income, it is possible to determine the effective marginal tax rate on an investment with the King-Fullerton (1984) framework.

3 Measuring the METR

3.1 The original King-Fullerton framework

The aim of King-Fullerton (1984) is to calculate the METR on investment projects in the nonfinancial corporate sector using a framework that takes all personal capital income taxes, corporate taxes, wealth taxes, depreciation allowances and inflation into account. Below I give a short description of the framework and the intuition behind the model. The original King-Fullerton model examines the effective marginal tax rates on investments from different sources of finance (new share issues, retained earnings and debt), from different groups of savers (household, tax-exempt institutions and insurance companies), in different assets (machinery, buildings and inventories) and in different industries (manufacturing, commerce and “other”). This study analyzes the METR for closely held corporations and hence only households’ saving. There are no differences in taxation between industries and in line with this study’s purpose the analysis is restricted to what in the original model is referred to as machinery.

The framework has been extended in several ways, for example by accounting for risk. In the following, I refrain from such extensions and focus on how to incorporate the special features of the rules for closely held corporations in the original model. Until now, it has not been a complete solution how to incorporate the present rules in the King-Fullerton framework, which will be done below. Lindhe et al (2003), Öberg (2003) and Sørensen (2008) have made important contributions, on which these further extensions rely.

Now, as the starting point for the analysis, a saver can either lend her/his capital to the capital market at the market interest rate or invest in an investment project. The project needs to generate a real rate of return after taxes that at least equals the real interest rate after taxes. The minimum rate of return that an investment must yield before taxes to provide the saver with the same net of tax return that (s)he would receive from lending at the market interest rate is called the cost of capital and denoted by p. A necessary condition for any investment project is that its profitability is at least as high as the cost of capital.

Taxes drive a wedge between the pretax rate of return on the investment project and the net return received by savers. As taxation is normally based on nominal income, both the real rate of return and the inflation compensation are taxed. The inflation rate hence influences the amount of tax paid, and in order to capture this effect, the tax wedge is calculated in real terms where the real tax wedge increases with inflation.

The marginal tax wedge, *w*, is defined as:

$$w = p - s \tag{14}$$

where *p* is the pretax real rate of return on a marginal investment and *s* the post-tax real rate of return to the saver.

The METR is defined as:

$$METR = \frac{w}{p} = \frac{p-s}{s} \tag{15}$$

The METR is, hence, the ratio of the marginal tax wedge, *w*, to the pretax real rate of return, *p*, which is a measure of the distortion caused by the tax system. Either the saver invests in business activity that generates the pretax real rate of return *p*, or the saver buys government bonds and receive the real post tax return *s*. The interest rate *r* is an intermediate between the two options. The METR is a theoretical value calculated based on an equilibrium model.

The METR can be calculated either given a fixed pretax real rate of return, *p*, or given a fixed r, both methods are widely used. King-Fullerton base their calculations on a fixed pretax real rate of return, *p*. To conform to the original model, a fixed-p approach is used here as well, and even though the methods give different insights about the tax system, the models can easily be transformed into one another by minor modifications and rearrangement in the system of equations.

With a fixed pretax real rate of return, *p,* all investments must have the same sum of marginal rate of return, MRR and capital depreciation rate δ, so that[8]:

$$p = MRR - \delta \tag{16}$$

The real post tax return to the saver, *s*, is defined as:

$$s = (1 - m)i - \pi - w \tag{17}$$

where *m* is the tax on interest income, *i* is the nominal interest rate, π is the rate of inflation and *w* is the wealth tax.

Now, the only remaining part is to calculate the interest rate, *i*, that on the marginal makes the saver willing to invest in a closely held corporation that generates the return, *p*, given the tax system and the inflation rate.

The interest, *i*, is calculated through what is generally defined as the discount factor for the corporation. In the original King-Fullerton setting this discount factor (normally ρ in the literature) is calculated for any given set of (corporate level) tax rules. Then, based on the source of finance, new share issue, retained earnings or debt, the interest, *i*, is calculated as the after-tax value of ρ for the investor.

In a perfectly competitive economy, the discount factor must be such that an equilibrium and non-arbitrage condition hold. If the size of the investment financed by new share issues is unity[9], the equilibrium condition implies that present value of the investment project *V(0)* must be 1, so that:

$$V(0) = 1 \tag{18}$$

The non-arbitrage condition is implicit in the original King-Fullerton model. The corporation's discount rate ρ is by definition the compensation the corporation has to pay the

[8] King-Fullerton (1984) assumes *p*=10%, to conform to this I assume δ=10% and *MRR*=20%
[9] For investments financed with retained earnings the present value is defined as the income (at personal level) the investor has to give up to do the investment, compared to the after tax value of the investment. For this, and debt financing, see Appendix A.

investor. For an investment to be attractive, this compensation must, after taxes, be at least as high as lending to the market at rate i, so that:

$$\left(1-m_{cf}\right)\rho=(1-m)i \leftrightarrow \rho=i\frac{(1-m)}{\left(1-m_{cf}\right)} \qquad (19)$$

In Equation (19) m_{cf} is the tax on the cash flow from the corporation to the investor. If the corporation pays dividends it will be τ_d and if the investor sells shares it will be the effective[10] capital gain tax, τ_c.

To complete the model, the total after corporate tax value of an investment can be described as the yearly marginal rate of return, in infinite time[11]:

$$V(0)=\int_0^{\infty}(1-\tau)MRRe^{-(\rho+\delta-\pi)t}dt=\frac{(1-\tau)MRR}{\rho+\delta-\pi} \qquad (20)$$

The nominal value in Equation (20) is straightforward; it increases in marginal return and inflation, and decreases in the discount rate and capital depreciation.

Investments are normally subject to write-offs and/or other grants, so that the cost C (which in equilibrium must equal the value V) of the investment rather can be described as:

$$C=1-A \qquad (21)$$

Where A is any grants or allowances. A standard assumption is that tax depreciation is a continuous exponential function decreasing at rate a, so that:

$$A=\int_0^{\infty}\tau ae^{-(a+\rho)t}dt=\frac{\tau a}{a+\rho} \qquad (22)$$

By combining Equations (18), (20), (21) and (22) the relationship between the cost of capital p and the discount rate ρ can be described as:

$$p=\left(\frac{1-A}{1-\tau}\right)(\rho+\delta-\pi)-\delta \qquad (23)$$

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[10] The effective capital tax can be derived endogenously in the model. However this further complicates the calculations and depends on assumptions of the average holding period. For simplicity, I assume the effective capital tax to be half of the statutory rate. The same assumption is made by King-Fullerton (1984, p. 146).

[11] The model can be expressed in continuous or discrete time, both approaches are frequently used. The original model was expressed in continuous time and the same approach is used here.

Now Equation (23) can be solved for ρ and using this result, Equation (19) can be solved for i, which in turn can be used to solve Equation (17) for s, which finally allow the calculation of the METR in Equation (15). Depending on the complexity of A, this can be done by exact calculations or by simulations. From Equation (23) it is clear that the relationship between p and ρ may not be linear. A unique solution is however guaranteed by economic reasoning and the construction of the model, as ρ is the largest compensation the corporation can pay the investor, and since the investor can chose between investment projects, the marginal investment will, in equilibrium, be carried out in the corporation paying the highest compensation. Hence, the largest ρ that solves Equation (23) must be used.

This original King-Fullerton model does however not allow for any division of income, which is the core of the tax rules for closely held corporations, nor does it address the very foundation of a closely held corporation, namely the fact that the investor is also the active owner of the corporation. The rest of this section deals with the incorporation of those special features in the framework described above.

In the following the term ρ will not be used, instead the interest rate i with the associated tax rates will be written out in full, to facilitate understanding.

3.2 Earlier extensions

In this section Lindhe et al (2003), Öberg (2003) and Sørensen (2008) extensions of the original King-Fullerton model are explained. It is difficult to incorporate the tax rules for closely held corporations into the King-Fullerton framework for three reasons:

1. The division of the owner's surplus into capital and labor taxation is not included in the original model.

2. The division depends on the equity base, which must be determined within the model and depends on the source of finance.

3. Depending on the division of income, the investor faces different tax rates[12].

The definition of the equity base is crucial, since it governs the dividend allowance, which in turn defines how much of the profit that will be taxed at a lower, flat tax rate, and how much

[12]The many tax rates in themselves makes it demanding to determine the METR, but is not a complication for deriving the expression for the METR.

that will be taxed at a higher, progressive tax rate. The equity base is however not defined at market value, but as the acquisition cost of the shares and capital injections, as described in Equation (1). The pretax real rate of return, *p* or the marginal rate of return, *MRR* as defined in Equation (16) are calculated on fixed assets[13], such as machinery, which will depreciate at rate δ over time. The equity base governs the taxation and the fixed assets the earning in the model. At the start of an investment project, the equity base and the fixed assets will be the same, but with time the fixed assets will depreciate, which the equity base will not.

This difference between the equity base and the fixed assets gives rise to problems that until now has not been dealt with. This is explained in detail in Section 3.3. Finally, the original King-Fullerton model is general in the sense that the investor receives all surplus as capital income (dividends, capital gains and interest). In a closely held corporation, not only can capital income be substituted by wage, the owner can also choose to distribute the whole surplus as an increase in share value. Öberg (2003) solves this by introducing a more rigorous definition of the non-arbitrage condition:

$$i(1-m)V(t) = \left(1-m_{cf}\right)CF(t) + (1-\tau_c)R(t) + (1-\tau_c)\frac{d}{dt}V(t) \qquad (24)$$

The right hand side of Equation (24) is any after tax profit at time *t*, for investing in the corporation. The investor can receive this profit either as cash flow *CF*, repurchased shares *R(t)* or as an increase in the corporation value $\frac{d}{dt}V(t)$. Under non-arbitrage conditions this value must equal holding the corporation value $V(t)$ in government bonds at interest *i* and capital income tax *m*.

Solving differential Equation (24) for the maximum present value *V(0)* gives:

$$V(0) = \int_0^{\infty}\left(\frac{\left(1-m_{cf}\right)}{(1-\tau_c)}CF(t) + R(t)\right)e^{-\frac{i(1-m)t}{(1-\tau_c)}}dt \qquad (25)$$

Depending on how complex the cash-flow structure is, Equation (25) can be combined with the equilibrium condition in Equation (18) and the interest rate can be solved for, either by calculus or by simulations. These expressions are however often tedious and uninformative[14].

[13] Financial assets are not taken into consideration

[14] See Appendix B for a proof that (25) is the maximum value of *V(0)*, from solving Equation (24).

Now, by replacing the general cash-flow term CF, with the income division described in Equations (11) and (12), the maximum present value of a marginal investment in a closely held corporation can be described as:

$$V(0) = \int_0^\infty \left(\frac{(1-\tau_w)}{(1-\tau_c)} W(t) + \frac{(1-\tau_d)}{(1-\tau_c)} D(t) + R(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \tag{26}$$

The last step is to define the functions *W(t)* and *D(t)*. With Öberg's (2003) assumption of repurchase of shares at a pace of $R(t) = (\delta - \pi)e^{-(\delta-\pi)t}$, the division of income in Equations (11) and (12) together with the tax depreciation in Equation (22) will imply that a marginal investment will have the following impact on the marginal cash flow, from the corporation to the owner:

$$W(t) = \frac{(1-\tau)MRRe^{-(\delta-\pi)t} - \beta e^{-(\delta-\pi)t} - (\delta-\pi)e^{-(\delta-\pi)t} + \tau a e^{-at}}{\varphi + (1-\tau)(1+\sigma)} \tag{27}$$

$$D(t) = \beta e^{-(\delta-\pi)t} + \varphi W(t) \tag{28}$$

Since Öberg (2003) assumes the same pace of repurchases as the depreciation rate of capital, $(\delta - \pi)$, the equity base on which the waged based allowance is calculated "depreciate" at the same rate as the marginal rate of return. Under this assumption, both the wage and dividend payments in Equations (27) and (28) will go to zero as time goes to infinity. If the repurchase pace is lower than the real depreciation rate, the modelled wage payments will with time be negative and the dividend payments will exceed the total income from the investment. Hence the *i*, that solves Equation (26) would lack economic meaning. This problem occurs because the fixed assets depreciate with time, while the equity base is constant if there is no repurchase of shares.

Öberg (2003) avoids this problem by a repurchase-assumption as stated above. Another way of avoiding this problem is to assume that the fixed assets does not depreciate with time. This is the approach by Lindhe et al (2003). The technique in Lindhe et al (2003) is also different because the cost of capital function is derived by a utility maximization problem where the corporation's owner supply labor, hire labor and choses an optimal capital stock. Sørensen (2008) simplifies the model by assuming that capital is the only input and derives the result in a static setting.

Relating to the notation above Sørensen (2008) refers to the gross earning *GE* as *Y*, and the marginal rate of return as *Y´*. Since capital is the only input, it follows that:

$$Y = F(K) \tag{29}$$

with the standard assumptions $F'(K) > 0$ and $F''(K) < 0$.

The tax payment is:

$$T(Y), \text{ where } T'(Y) > 0 \tag{30}$$

Which generates an after-tax profit equal to:

$$\Pi = F(K) - r^*K - T(Y) \tag{31}$$

where r^* is the opportunity cost of capital.

Now, under maximization conditions, $\frac{\partial \Pi}{\partial K} = 0$, which gives:

$$F'(K) - r^* = T'\big(F(K)\big)F'(K) \leftrightarrow F'(K) = \frac{r^*}{(1 - T'\big(F(K)\big)} \tag{32}$$

Now, $T'\big(F(K)\big)$ is the marginal tax rate *m* and in equilibrium the marginal productivity of the capital is the cost of capital, so that $F'(K) = p$. Together this gives the cost of capital function:

$$p = \frac{r^*}{1-m} \tag{33}$$

Now, with the *fixed-r* definition of the METR:

$$METR = \frac{p - r^*}{p} \tag{34}$$

It follows that:

$$METR = m \tag{35}$$

Note that Equations (15) and (34) conceptually are the same, but in a fixed-p model it is more convenient to use (15) and in a fixed-r model it is more straightforward to use (34). The results are however the same. With the standard notation, *r* in the fixed-r model corresponds to *s* in the fixed-p model.

The derivation of the METR in Equations (29)–(35) is useful for understanding the METR concept but uninformative about the size of the METR.

Equation (29) must, for an active owner, be specified as:

$Y = F(K, L)$ (36)

This is in itself not a complication since one can, without any loss in generality, assume that the entrepreneur only works with his/her corporation.

Hence, the profit described in Equation (31) holds, except for the last term $T(Y)$. As described above the tax payment, T, will depend on Y, but not solely. Assuming that the division between dividends and wage payment is captured in the specification of the function T, it is still necessary to re-write (31) as:

$T(Y, EB)$, where $\frac{\partial T}{\partial Y} > 0$ and $\frac{\partial T}{\partial EB} < 0$ (37).

Where EB is the equity base. The sign of the second partial derivative in (37) holds under the assumption that pecking order in Equation 7 holds and that T divides as least some of the total income Y to wage taxation.

In a static setting as described here, (36) will not complicate the calculations of the METR since a unit investment will increase both Y and EB. But as described in Section 3.1, the King-Fullerton model evaluates the METR in an infinite time setting. With time, the invested capital K will depreciate with an ever-lower impact on Y, while the investment's impact on the EB will remain unchanged.

While Lindhe et al (2003) and Öberg (2003) make a contribution by incorporating the dual tax system's division of income into the King-Fullerton framework, they disregard the importance of the distinction between the equity base and the fixed assets. Lindhe et al (2003) circumvents the problem by assuming that the investment does not depreciate, and Öberg (2003) by letting the corporations repurchases shares at the same pace as the capital depreciate.

The fixed assets will generate the return on investment, while the equity base will determine the division of income between labor and capital, and hence the taxation. With the assumptions made by Lindhe et al (2003), used in Sørensen (2008), and by Öberg (2003) the impact on the METR of this important distinction will not be analyzed[15].

An important feature of Öberg's (2003) model, discussed in Section 4, is the taxation of the repurchases of shares. In Equation (24) repurchases are specified as a separate income $R(t)$ and taxed at rate τ_c. However, as described in Section 2.1 all income above β will according to the tax rules be taxed as wage τ_w at a personal level. According to tax law the capital gain must either be taxed highly and progressively at τ_w or $R(t)$ must be considered as a continuous approximation for a future exit, where capital gains could be taxed either at 25 percent capital income tax after five years of inactivity in the firm, or at 30 percent if the total income exceeds a certain threshold. See Wykman (2019b) for a detailed description.

In the following, this paper focus on extending the Öberg (2003) model. Öberg (2003) uses a fixed-p model, which could be argued to be more relevant than a fixed-r model when analyzing the METR for closely held corporations, especially since it allows to analyze the METR for different level of returns. Sørensen (2008) extends the model introduced by Lindhe et al (2003) in a fixed-r setting. Since the tax rules for owners of closely held corporations are progressive, and the King-Fullerton model will calculate the METR for an investment that is barely profitable, the fixed-r approach will calculate the METR for investments with potentially very low profitability that, hence, face low statutory tax rates at personal level.

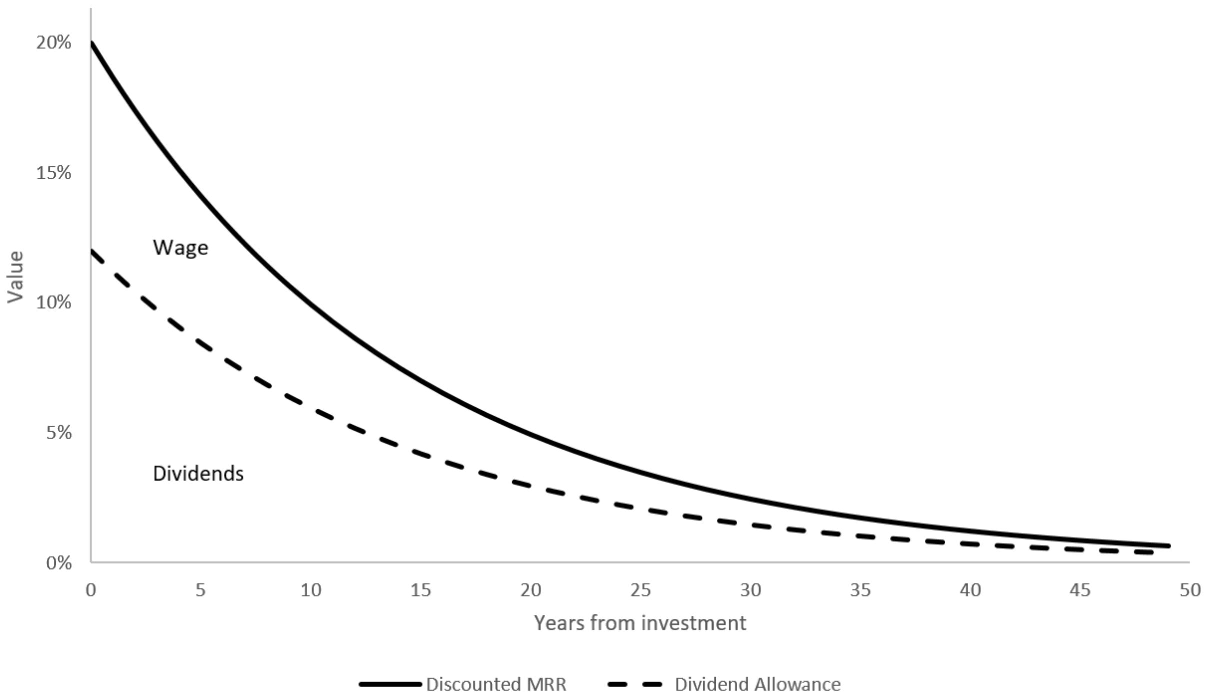
3.3 Calculating the METR with respect to the difference between the equity base and the fixed assets

From the above, it is clear that real world conditions will alter the division of income over time with tax consequences that is ignored in previous work, either by assumptions about non-depreciating investments, or by construction of repurchases of shares.

Excluding taxes, the wage and dividend payments in Equations (27) and (28) together with the discount rate used in Equation (26) could be described by Figure 1. The solid line represents the marginal rate of return and the dashed line the dividend allowance.

[15] Sørensen (2008) notes this problem and makes a separate analysis based on a two-year example, but without extending the model.

Figure 1. The division of income between wage and dividends from an investment (MRR=20%) financed by new share issues, with repurchases of shares



Note: MRR is the marginal rate of return on an investment.
Source: Own figure

The present pre-tax value (of the first 50 years) of the return on the investment will be the area under the solid line. The present value of the pre-tax wage payments will be the area under the solid line and above the dashed line, and the pre-tax value of the dividend payments will be the area under the dashed line. But what would be the corresponding payment sheme without the assumption of repuchases of shares?

This could be solved by changing the non-arbitrage condition in Equation (24) to:

$$i(1-m)V(t) = \left(1-\tau_p\right)W(t) + (1-\tau_d)D(t) + (1-\tau_c)\frac{d}{dt}V(t) \qquad (38)$$

Now, as above, solve the differential Equation (38) for the maximum present value:

$$V(0) = \int_0^{\infty}\left(\frac{(1-\tau_p)}{(1-\tau_c)}W(t) + \frac{(1-\tau_d)}{(1-\tau_c)}D(t)\right)e^{-\frac{i(1-m)t}{(1-\tau_c)}}dt \quad (39)$$

Without any repurchases of shares the owner of the corporation will, from a legal perspective always be allowed to withdraw at least β as dividend payment, so Equation (28) now takes the form:

$D(t) = \beta + \varphi W(t)$ (40)

Which in turn alters the wage payment in Equation (27) to:

$$W(t) = \frac{(1-\tau)MRRe^{-(\delta-\pi)t} - \beta + \tau a e^{-at}}{\varphi + (1-\tau)(1+\sigma)} \qquad (41)$$

In practice the difference is that β is now not discounted with $e^{-(\delta-\pi)t}$. This however allows for the wage in Equation (41) to become negative, which would make no sense. Hence a restriction is necessary, so that the minimum of $W(t)$ is 0. This means that wage will only be paid up to a time t_d, where:

$$(1-\tau)MRRe^{-(\delta-\pi)t_d} + \tau a e^{-at_d} = \beta \quad (42)$$

For any $t > t_d$, the wage $W(t) < 0$, and the model would lack economic meaning.

With this definition of a breakpoint t_d, the present value function in (39) must be reforumlated as:

$$V(0) = \int_0^{t_d} \left(\frac{(1-\tau_{pw})}{(1-\tau_c)} W(t) + \frac{(1-\tau_{dp})}{(1-\tau_c)} D(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt + \int_{t_d}^{\infty} \frac{(1-\tau_{dp})}{(1-\tau_c)} D^*(t) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \ (43)$$

where

$$D^*(t) = (1-\tau)MRRe^{-(\delta-\pi)t} + \tau a e^{-at} \qquad (44)$$

Further, the King-Fullerton framework relies on the assumption that all tax advantages are used, otherwise the agents in the model would not maximize their profit or utility. With a constant, β, a marginal investment will give raise to a tax credit that can be used to transform income from labor to capital from time, t_d, and onwards. Highly taxed labor income, *W*, could be shifted to lower taxed dividends income, *D*. The tax credit will be equal to:

$$TC = \int_{t_d}^{\infty} (\beta - D^*(t)) (\frac{1}{1+\varphi}) (\left(\sigma + \frac{\tau_{pw}}{1+\sigma}\right) - (\tau + \tau_d - \tau\tau_d)) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad (45)$$

The term $\frac{1}{1+\varphi}$ must be included due to the wage based allowance. The tax cost of wage payments in terms of dividend payments is less when the wage is included in the dividend allowance.

So finally, the present value of the investment takes the form:

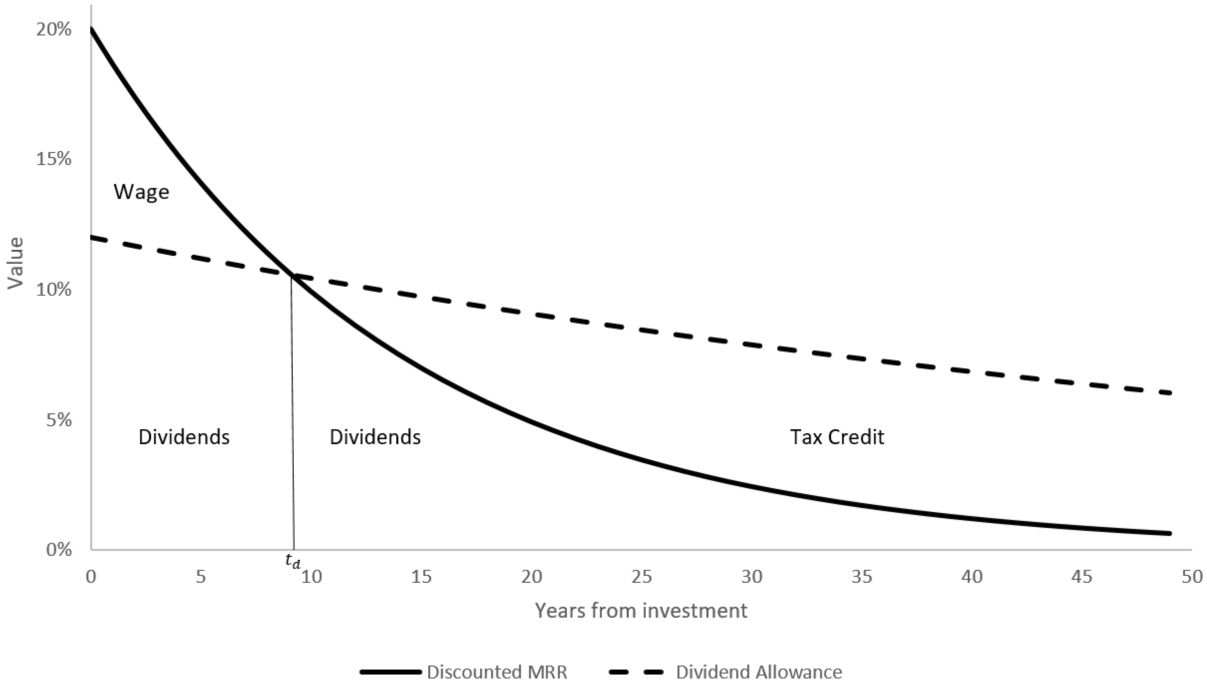
$$V(0) - \frac{TC}{(1-\tau_c)} =$$

$$= \int_0^{t_d} \left(\frac{(1-\tau_{pw})}{(1-\tau_c)} W(t) + \frac{(1-\tau_d)}{(1-\tau_c)} D(t) \right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt + \int_{t_d}^{\infty} \frac{(1-\tau_d)}{(1-\tau_c)} D^*(t) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad (46)$$

The equilibrium condition in Equation (18), $V(0) = 1$, must still hold. By simulation, an *i* can be found, such that Equation (46) fulfils the equilibrium condition.

This new division of income with the additional tax credit could also be described graphically, similarly to the original Öberg (2003) model (without repurchases) above.

Figure 2. The division of income after an investment (MRR=20%) financed by new share issues, without repurchases of shares



Note: MRR is the marginal rate of return on an investment.
Source: Own figure

Comparing Figure 1 and Figure 2, it is clear that the payment scheme, and hence the taxation, will be significantly different depending on whether or not one assumes repurchases of shares. Without repurchases, wage payments will be lower, dividend payments higher, and if there exist other incomes in the corporation, the new investment will potentially transform part of these (or all) from high taxed labor income, to low taxed capital income.

To calculate the difference in METR between the different approaches: repurchases of shares (Equation 26), no repurchases of shares without possibility to utilize the tax credit (Equation 43) and no repurchases of shares with the possibility to use the tax credit (Equation 46), the equilibrium condition in Equation (18) is used to solve for the three different *i:s*. Finally, Equations (15) and (17) are used to calculate the different METRs.

4 Results

The METRs for investments financed with new share issues reported in this paper are based on the tax rates in 2018, and calculated without respect to various special rules in the tax code (for incorporation of those rules see Appendix E). The results from the different models cannot be compared without considering the different assumptions, simplifications and techniques behind them. It is however useful to compare the results and to establish whether the different approaches affect the result in a logical way and whether the differences are substantial. If so, all models can have their unique advantages, either because of how simple they are to use, or because different questions should be answered. For example, when Sørensen (2008) updated the Lindhe et al (2003) model, it was with the main purpose to analyze tax discrimination between investments in different organizational forms. Since the difference in itself was the focus, some shortcomings that affected all organizational forms equally was not a problem. Table 1 reports the results[16].

Table 1. Marginal effective tax rate (METR) financed with new share issues, different methods, 2018

Method	**METR (%)**
King-Fullerton (1984)	34.5, 73.2
Öberg (2003)	36.5, 51.5
Lindhe et al (2003) and Sørensen (2008)[a]	40.2
Extended model (2019)	33.6, 41.4

Source: Own calculations.

Note: For reasons of comparison the nominal interest rate in the fixed-r model by Lindhe (2003) and Sørensen (2008) is the same as the rate that gives the METR of 33.6 percent in the extended model. This corresponds to a cost of capital at 11.1 percent. The original King-Fullerton model is calculated with a fixed-p approach, but could as well be derived with a fixed-r.

[a] Using the cost of capital function in equation 4.25 in Sørensen (2008, p. 155) which corresponds to equation (8) in Lindhe et al (2003, p. 9).

For three of the models there are a low and a high case. The King-Fullerton (1984) has a low result when the whole surplus is assumed to be distributed within the dividend allowance and a high case when the whole surplus is subject to the highest labor income tax. The Öberg (2003) has a low case when repurchases outside the dividend allowance are taxed at a flat capital tax (in line with her own calculations, but without the abolished split-rule) and a high case when all income outside the dividend allowance is taxed as labor income. The extended

[16] I use tax rates, inflation rate and government borrowing rate from 2018 to illustrate the METR for the different models, see Appendix D for specification.

model has a low case when the corporation can take full tax advantage of the larger equity base, and a higher case when no income shifting is possible.

At a first glance, the different METRs in Table 1 might raise the question whether any of the methods are reliable. It is though important to hold two things in mind. Firstly, the rules are complex and often criticized for unpredictable tax outcomes. Hence, it should be no surprise that slightly different methods could give substantially different outcomes. Small variations in, e.g., the level of income and the rate of return will also in real life give rise to potentially different tax outcomes on the margin.

In addition, the different methods have different shortcomings and assumptions that will affect the result. Most obvious is that the original King-Fullerton model does not have any tool to divide the total income into labor and capital, hence the whole income will be taxed as either or. As stated above the lower METR (34.5%) is when all income is taxed as capital (at 20% within the dividend allowance) and the higher METR (73.2%) is when all income is taxed at the highest labor tax. This model is too crude to capture the main idea of the rules for closely held corporations and these results function only as a benchmark and an upper limit of the taxation.

The lowest METR (36.5%) in the Öberg (2003) model reflects a situation with repurchases of shares. Öberg also assumes quite a high pace for repurchases, which will allow a substantial share of the income to be taxed at a relatively low effective capital gains tax. If, on the other hand, the active owner should rely on wage and dividend payments (and capital gains outside the dividend allowance) the METR will be substantially higher (51.5%).[17]

The Sørensen (2008) and Lindhe et al (2003) models are different in the sense that they use a fixed-r approach. Hence, it will report the METR for an investment with (slightly) different marginal rate of return. As reported in the note of Table 1, the difference in the interest rate is however negligible, or at least very small. Another difference is that this model does not account for any depreciation nor does it include any tax depreciation allowances. Without repurchases of shares, the model calculates the METR to 40.2% instead of the Öberg model's

[17] Note that Öberg herself never used her model to calculate the METR without repurchases, that is an extension made in this paper for reasons of comparison.

51.5%. The model is however inflexible since it cannot account for different depreciation rates or allowances, nor can it capture the effect of income shifting.

The extended model reports the lowest METR (33.6 %), and this is in a situation where all income is from wage or dividend payments. This result is even lower than the METR from the original King-Fullerton model, when the whole income is taxed as dividends. This paradox makes however perfect sense, since the extended model captures the full tax effect, which also include the corporation's possibility to shift other, highly taxed income, into lower taxed dividend income, due to the higher dividend allowance induced by the new investment. The higher METR (41.4) in the extended model is from a situation where the corporation cannot utilize the whole dividend allowance (the tax credit) that follows from a raise in the equity base (cannot shift other incomes, not generated from the investment, from the higher progressive tax to the lower flat tax). One should note that this METR is very close to the METR in the Sørensen (2008) and Lindhe et al (2003) model, which is reasonable under the assumed depreciation rate and depreciations allowances, since none of the models account for the potential income shifting.

Overall, this comparison gives evidence for the reliability in the extended model. It captures more aspects of the tax system than earlier models and gives greater degrees of freedom when analyzing different policy changes, or evaluating the tax system over time. Especially this model gives a lower METR than predicted by earlier models, which is due to the new integration of the difference between the equity base and the fixed assets. In itself, this deepens the understanding of the popular concept of income shifting.

The results coincide when it is reasonable to expect that they should. For example, the original King-Fullerton model (relying on the simplification that the whole surplus could be withdrawn as capital) estimates the METR only 2 percentage points lower than the Öberg model (relying on the assumption of repurchases of shares). In turn, both models only overestimate the METR moderately compared to the extended model. The extended model does, however, not rely on the same strong assumptions as the other two. But maybe more important, the work of this paper shows that the King-Fullerton framework in different approaches is a robust tool, also for evaluating the most complex parts of a dual tax system – closely held corporations.

The King-Fullerton model calculates the METR for different sources of finance. For a description of how to calculate the METR for debt and retained earnings, see Appendix A. Here it is worth noting that the METR is lower for new share issues than retained earnings, which is due to the large impact of the enlargement of the equity base from investments financed with new share issues.

Table 2. The marginal effective tax rate (METR), different sources of finance, extended model.

Source of finance	**METR (%)**
New share issues	33.6
Retained earnings	42.2
Debt	24.3

Source: Own calculations.

Note: The baseline scenario is that all tax advantages could be utilized, hence the lower METR for new share issues from Table 1 is reported.

Finally, a comment about exogenous and endogenous determination of β, or the imputed return on the dividend allowance. The results above rest on an exogenous given β, which is what owners of closely held corporations in practice faces. Since β consists of two components, the government borrowing rate and a risk premium decided by law, the first part could be simulated together with the general simulated interest rate in a fixed-p model. In a fixed-r model, the chosen r could be the first component of β.

Except for the fact that β is exogenously given for closely held corporations and mathematical complications, there are strong reasons against an endogenous approach. Firstly, the size of β is a regulation, and a model that simulates the legal framework as such, cannot simultaneously simulate the consequences of the same framework. Secondly, if β is simulated within the model, the different METRs will not reflect different tax rates and rules, it will to some extent reflect different legal frameworks. Lastly, and partly connected to some of the many mathematical problems with an endogenous approach, a simulation of β will, within any meaningful model specification, give raise to an as-high-as-possible β. In practice, this can only generate a situation where the whole surplus is paid to the owner as dividends within the dividend allowance.

5 Concluding remarks

The King-Fullerton framework is a standard way of measuring METR. The model is complicated in itself since it aims to measure the marginal taxation in economic equilibrium, with respect to different sources of finance, depreciations rates, inflation and allowances, rather than just summing up tax rats. To introduce the tax rules for closely held corporations within the King-Fullerton framework is demanding. The main reason is the division of capital income into labor and capital income with their own tax rates and rules. The marginal effective tax rate on capital income (METR) of investments depends on, e.g., corporation specific characteristics such as profit and profitability, the wage sum, owner's wage and source of finance. A major complication of the legal framework is a mixture of real economic variables and booked values, that evolves differently over time. This difference gives rise to (significant) tax consequences, which has not been studied in detail before this study.

All different methods discussed in this paper rely on assumptions and simplifications. The method developed in Section 3.3 is unique since it distinguishes between the equity base and fixed assets. This distinction is of great importance for the tax rules for closely held corporations and allows an incorporation into the King-Fullerton framework, without the earlier models' assumptions of non-depreciating investments or repurchase programs. In turn, this gives the extended model more flexibility for analyzing the total tax effect of different policy changes, or even different investment objects, such as different depreciations allowances or investments with different lifespans.

Another advantage of the extended model is that it better reflects real life conditions for a large share of investments in closely held corporations. The active owner may have a long time commitment and hence does not wish to sell shares, instead (s)he relies on wage payments and dividends generated by the investment. The extended model can easily analyze situations where the return to the active owner is wage, dividends, capital gains or a combination of any of them. Repurchases of shares are rare in closely held corporations and even more so, it is unlikely that they coincide perfectly with the depreciation rate. The extended model does not depend on those assumptions and therefore capture reality better.

Further, my results provide relevant information how to compare METRs calculated with different techniques. This gives a better understanding for earlier research and a powerful tool to evaluate future research as well as policy evaluation.

Finally, the model introduced in this paper, together with the incorporation of special tax rules enables researches and policy makers to construct time series over the marginal effective taxation as well as evaluate a broad range of policy changes.

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Appendix A. Retained earnings and debt financing

The tax wedge is defined as:

$$METR = \frac{p-s}{p} \qquad \text{(A1)}$$

In equation (A1), p is the pre-tax rate of return on real investment and s is the post-tax rate of return received by the saver.

The relation between the interest rate and the return to the saver is defined as:

$$s = (1-m)i - \pi - w_p \qquad \text{(A2)}$$

In equation (A2) m is the personal tax rate on interest income, i is the nominal interest rate, π is the inflation rate and w_p is the personal wealth tax.

With a fixed pretax real rate of return, p, all investments must have the same sum of marginal rate of return, MRR and capital depreciation rate, δ, so that

$$p = MRR - \delta \qquad \text{(A3)}$$

A1. Debt

From the perspective of the corporation, the most tax efficient way to finance an investment is by debt, since interest is deductible. The calculation of the METR is then straightforward.

Assuming a repayment pace of the debt at the rate of real capital depreciation, the debt at time t is:

$$Debt(t) = e^{-(\delta-\pi)t} \qquad \text{(A4)}$$

Since interest payments are deductible, the total tax liability for the corporation is

$$Tax(t) = \tau\left(MRRe^{-(\delta-\pi)t} - iDebt(t) - ae^{-at}\right) \qquad \text{(A5)}$$

The after-tax cash flow at time, t, will be:

$$CF(t) = MRRe^{-(\delta-\pi)t} - iDebt(t) - (\delta-\pi)e^{-(\delta-\pi)t} - Tax(t) \qquad \text{(A6)}$$

For the investment to be carried out, the present value of the cash flow must be at least zero.

$$\int_0^{\infty} CF(t)\, e^{-i(1-\tau)t} dt = 0 \quad \text{(A7)}$$

Combining equations (A1), (A2), (A3) and (A7), the METR can be computed.

A2. Retained Earnings

Another option to finance an investment is for the owner to use retained earnings and withdraw wage or dividend payments. As described above, there is no clear pecking order which reduction to make over time. It is, however, important to notice that after the introduction of the wage based allowance, a reduction of wage payments will induce lower dividend payments.

Slightly redefining W and D to the amounts withdrawn as the owner's wage and dividends income, the non-arbitrage condition in Equation (24) must hold[18]. Since retained earnings from the owner's perspective means giving up income here and now, for a future return the present value of the investment must equal the value of the payments withdrawn.

$$V(0) = \int_0^{\infty} \left(\frac{(1-\tau_w)}{(1-\tau_c)} W(t) + \frac{(1-\tau_d)}{(1-\tau_c)} D(t) \right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \quad \text{(A8)}$$

In equilibrium, one unit spent on the investment project must have the same after-tax value as the same amount withdrawn from wage or dividend payments.[19]

$$(1-\tau_c)V(0) = \left(\frac{1}{1+\varphi}\right) \frac{1-\tau_w}{(1-\tau)(1+\sigma)} + \left(\frac{\varphi}{1+\varphi}\right)(1-\tau_d) \qquad \text{(A9)}$$

In equation (A9), $\rho\varphi$ determines the share of dividends withdrawn. Ignoring the weights, the first term on the right hand side is the total after tax value of one unit in wage payments, and the second term is the total after tax value of one unit in dividends income.

Since the withdrawal of wage and dividend payments does not affect the capital or tax base of the corporation, the equation pair (40) and (41) now take a slightly less complicated form:

$$W(t) = \frac{(1-\tau)MRRe^{-(\delta-\pi)t} + \tau a e^{-at}}{\rho\varphi + (1-\tau)(1+\sigma)} \qquad \text{(A10)}$$

$$D(t) = \rho\varphi W(t) \qquad \text{(A11)}$$

[18] Here I assume a case without repurchases of shares

[19] See the section on pecking order.

Combining equations (A1), (A2), (A3), (A8), (A9), (A10) and (A11) the METR can be computed.

A.3 Active owner and debt

The debt calculation presented in Section A1 is the standard King-Fullerton approach. As mentioned, the rules for closely held corporations regulates the interest payments, when the owner is the debtholder. This is to prevent the active owner to shift income, from progressively taxed labor income, to flat rate taxed interest income. In cases where the regulation becomes binding, other ways of calculating the METR might be more accurate.

In such a situation, it is necessary to include dividends and wage payments in the analysis, since the size of the interest payments is regulated, and not necessarily is enough to distribute the whole surplus to the debt holder (who is also the active owner). Which level of interest rate that is acceptable is a case-by case decision[20] by the Tax Authority, but the interest on government bonds+3 percentage points is always accepted.

Assume that no repayments of the debt are made, then the debt relationship between the corporation and the owner persists over time, so the value of the investment equals the debt:

$$V(0) = 1 \qquad \text{(A12)}$$

The surplus will be distributed to the owner as interest payments, *I* together with wage and dividend payments, as described earlier. The present value function takes the form:

$$V(0) = \int_0^\infty \left(\frac{(1-\tau_w)}{(1-\tau_c)} W(t) + \frac{(1-\tau_d)}{(1-\tau_c)} D(t) + \frac{(1-m)}{(1-\tau_c)} I(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad \text{(A13)}$$

Let γ denote the allowed return to the owner of the debt (hence the allowed deductible amount for the corporation) and note that the corporation's capital stock has not increased, and hence the dividend allowance only grown by the increase in wage payments.

$$W(t) = \frac{(1-\tau)MRRe^{-(\delta-\pi)t} - (1-\tau)\gamma + \tau a e^{-at}}{\varphi + (1-\tau)(1+\sigma)} \qquad \text{(A14)}$$

[20] It must be motivated by the market interest rate.

$$D(t) = \varphi W(t) \quad \text{(A15)}$$

and for clarity:

$$I(t) = \gamma \quad \text{(A16)}$$

The METR can now be calculated as above.

Appendix B. The maximization problem

In Section 3.2 , I claim that the non-arbitrage condition:

$$i(1-m)V(t) = (1-\tau_d)\left(MRRe^{-(\delta-\pi)t} + ae^{-at}\right) + (1-\tau_c)V'(t) \qquad \text{(B1)}$$

implies the maximum present value:

$$V(0) = \int_0^{\infty} \left(\frac{(1-\tau_d)}{(1-\tau_c)}\left(MRRe^{-(\delta-\pi)t} + ae^{-at}\right)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad \text{(B2)}$$

This can be proven by rewriting (B1) as:

$$V^{'}(t) - AV(t) = -BU(t) \quad \text{(B3)}$$

where

$$A = \frac{i(1-m)}{(1-\tau_c)},\ B = \frac{(1-\tau_d)}{(1-\tau_c)},\ U(t) = \ MRRe^{-(\delta-\pi)t} + ae^{-at}$$

Standard technique for solving differential equations gives:

$$(B3) \leftrightarrow e^{-At}\left(V^{'}(t) - AV(t)\right) = -BU(t)e^{-At} \leftrightarrow \frac{d}{dt}\left(e^{-At}V(t)\right) = -BU(t)e^{-At} \qquad \text{(B4)}$$

Integration gives:

$$e^{-At}V(t) = -B\int_0^t U(x)\, e^{-Ax}dx + C \qquad \text{(B5)}$$

Where C is a constant.

Since $e^{-At}V(t) \to 0\ when\ t \to \infty$

$$C = \ B\int_0^{\infty} U(x)\, e^{-Ax}dx \quad \text{(B6)}$$

(B5) and (B6) gives:

$$e^{-At}V(t) = -B\int_0^t U(x)\, e^{-Ax}dx + \ B\int_0^{\infty} U(x)\, e^{-Ax}dx \qquad \text{(B7)}$$

which is equivalent to:

$V(t) = Be^{At} \int_t^{\infty} U(x)\, e^{-Ax} dx$ (B8)

Solving Equation (B8) gives:

$$Be^{At} \int_t^{\infty} U(x)\, e^{-Ax} dx = [MRR = b, c = \delta - \pi]$$

$$= Be^{At} \left(\int_t^{\infty} be^{-(c+A)t} dt + \int_t^{\infty} ae^{-(a+A)t} dt \right) =$$

$= \frac{Bb}{c+A} e^{-ct} + \frac{Ba}{a+A} e^{-at}$ (B9)

Equation (B9) is strictly declining in *t*, and, hence, the largest value for *V* must be in *t=0*. Inserting *t=0* in Equation (B8) gives Equation (B2). QED

Appendix C. Differences between models

Öberg introduces a more complicated non-arbitrage condition, which is also used in the extended model in this paper. This appendix highlights the differences between those models and the original King-Fullerton model.

The METR is in both cases defined equally:

$$METR = \frac{p-s}{p} \qquad \text{(C1)}$$

$$s = (1-m)i - \pi - w_p \qquad \text{(C2)}$$

In Equation (C1), *p* is a fixed value. For simplicity I disregard any tax depreciation (*a*=0). To be able to compare the models it is necessary to assume that all surplus can be withdrawn as dividend payments and that a reduction in wage payments does not affect the dividend payments. Even though those assumptions limit the comparison, it still gives general insights in how the different approaches affect the result. Especially it is notable that the analysis gives a complete description of the difference for widely held corporations.

C.1 Öberg (2003) model

$$V(0) = \int_0^\infty \left(\frac{(1-\tau_d)}{(1-\tau_c)} D(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad \text{(C3)}$$

$$D(t) = (1-\tau)MRRe^{-(\delta-\pi)t} \qquad \text{(C4)}$$

(C3) and (C4) =>

$$\frac{(1-\tau_d)(1-\tau)MRR}{(1-\tau_c)} \int_0^\infty e^{-\left(\frac{i(1-m)}{(1-\tau_c)}+(\delta-\pi)\right)t} dt = \frac{(1-\tau_d)(1-\tau)MRR}{(1-\tau_c)} \times \frac{1}{\frac{i(1-m)}{(1-\tau_c)}+(\delta-\pi)} \qquad \text{(C5)}$$

Now, for new share issues the equilibrium condition implies:

$$\text{(C5)} = 1 \rightarrow$$

$$\rightarrow \frac{(1-\tau_d)(1-\tau)MRR}{(1-\tau_c)} = \frac{i(1-m)}{(1-\tau_c)} + (\delta-\pi) \leftrightarrow i = \frac{(1-\tau_d)(1-\tau)MRR}{(1-\text{m})} - \frac{(1-\tau_c)}{(1-\text{m})}(\delta-\pi) \qquad \text{(C6)}$$

(C6) in (C2) =>

$$s = (1-\tau_d)(1-\tau)MRR - (1-\tau_c)(\delta-\pi) - \pi - w_p \qquad \text{(C7)}$$

Now, $p = MRR + \delta \rightarrow (\text{C1}) = 1 - \frac{s}{MRR+\delta}$ (C8)

(C7) and (C8) =>

$$METR = 1 - \frac{(1-\tau_d)(1-\tau)MRR-(1-\tau_c)(\delta-\pi)-\pi-w_p}{MRR+\delta} \quad \text{(C9)}$$

Now, how do changes in the variables affect the METR for new share issues?

$$\frac{\partial METR}{\partial \tau_d} = \frac{(1-\tau)MRR}{MRR+\delta} > 0 \qquad \text{(C10)}$$

$$\frac{\partial METR}{\partial \tau} = \frac{(1-\tau_d)MRR}{MRR+\delta} > 0 \qquad \text{(C11)}$$

$$\frac{\partial METR}{\partial \tau_c} = \frac{\pi-\delta}{MRR+\delta} \begin{matrix} < 0 \; if \; \delta > \pi \\ 0 \; if \; \delta = \pi \\ > 0 \; if \; \delta < \pi \end{matrix} \qquad \text{(C12)}$$

For retained earnings (financed with withdrawn wage) Equations (C3) and (C4) are replaced with:

$$V(0) = \int_0^\infty \left(\frac{(1-\tau_w)}{(1-\tau_c)} W(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \quad \text{(C13)}$$

$$W(t) = \frac{MRRe^{-(\delta-\pi)t}}{(1+\sigma)} \qquad \text{(C14)}$$

(C13) and (C14) =>

$$\frac{(1-\tau_w)MRR}{(1-\tau_c)(1+\sigma)} \int_0^\infty e^{-\left(\frac{i(1-m)}{(1-\tau_c)}+(\delta-\pi)\right)t} dt = \frac{(1-\tau_w)MRR}{(1-\tau_c)(1+\sigma)} \times \frac{1}{\frac{i(1-m)}{(1-\tau_c)}+(\delta-\pi)} \qquad \text{(C15)}$$

The equilibrium condition for retained earnings implies (compare with Equation A9 in Appendix A):

$$C13 = \frac{1-\tau_w}{(1-\tau)(1+\sigma)(1-\tau_c)} \rightarrow$$

$$\rightarrow \frac{(1-\tau_w)MRR}{(1-\tau_c)(1+\sigma)} = \left(\frac{i(1-m)}{(1-\tau_c)} + (\delta - \pi)\right) \times \frac{1-\tau_w}{(1-\tau)(1+\sigma)(1-\tau_c)} \leftrightarrow$$

$$\leftrightarrow i = \frac{(1-\tau_c)(1-\tau)MRR}{(1-\mathrm{m})} - \frac{(1-\tau_c)}{(1-\mathrm{m})}(\delta - \pi) \quad \text{(C16)}$$

(C16) in (C2) =>

$$s = (1-\tau_c)(1-\tau)MRR - (1-\tau_c)(\delta - \pi) - \pi - w_p \qquad \text{(C17)}$$

(C8) and (C17) =>

$$METR = 1 - \frac{(1-\tau_c)(1-\tau)MRR - (1-\tau_c)(\delta-\pi) - \pi - w_p}{MRR+\delta} \quad \text{(C18)}$$

Now, how do changes in the variables affect the METR for retained earnings?

$$\frac{\partial METR}{\partial \tau_c} = \frac{(1-\tau)MRR - (\delta-\pi)}{MRR+\delta} \begin{matrix} < 0 \; if \; (\delta - \pi) > (1-\tau)MRR \\ 0 \; if \; (\delta - \pi) = (1-\tau)MRR \\ > 0 \; if (\delta - \pi) < (1-\tau)MRR \end{matrix} \qquad \text{(C19)}$$

$$\frac{\partial METR}{\partial \tau} = \frac{(1-\tau_c)MRR}{MRR+\delta} > 0 \qquad \text{(C20)}$$

For retained earnings, one should note that the METR is independent of wage and dividend tax. Further, the METR is increasing in both corporate tax τ and capital gain tax τ_c, the latter holds as long as an investment generates more after tax surplus, than it loses in real value, which is a necessary condition for the investment to be carried out.

C.2. Original King-Fullerton (1984) model

The present value for a marginal investment financed with new share issues:

$$V(0) = \frac{(1-\tau)MRR}{\rho + (\delta - \pi)} \qquad \text{(C21)}$$

For investments financed with new share issues:

$$\rho = i\frac{(1-m)}{(1-\tau_{d})} \qquad \text{(C22)}$$

Since $V(0) = 1$ we solve for i:

$$\frac{(1-\tau)MRR}{i\frac{(1-m)}{(1-\tau_d)}+(\delta-\pi)} = 1 \leftrightarrow i = \frac{(1-\tau_d)}{(1-\mathrm{m})}\big((1-\tau)MRR-(\delta-\pi)\big) \qquad \text{(C23)}$$

Insert (C23) in (C2) =>

$$s = (1-\tau_d)((1-\tau)MRR-(\delta-\pi))-\pi-w_p \qquad \text{(C24)}$$

Insert (C24) in (C8) =>

$$METR = 1-\frac{(1-\tau_d)((1-\tau)MRR-(\delta-\pi))-\pi-w_p}{MRR+\delta} \qquad \text{(C25)}$$

$$\frac{\partial METR}{\partial \tau_d} = \frac{(1-\tau)MRR-(\delta-\pi)}{MRR+\delta} \begin{matrix} < 0 \; if \; (\delta-\pi) > (1-\tau)MRR \\ = 0 \; if \; (\delta-\pi) = (1-\tau)MRR \\ > 0 \; if \; (\delta-\pi) < (1-\tau)MRR \end{matrix} \qquad \text{(C26)}$$

$$\frac{\partial METR}{\partial \tau} = \frac{(1-\tau_d)MRR}{MRR+\delta} > 0 \qquad \text{(C27)}$$

The original King-Fullerton model derives the METR for new share issues and retained earnings in the same way, except for the relationship between ρ and i:

$$\rho = i\frac{(1-m)}{(1-\tau_{c})} \qquad \text{(C28)}$$

Replacing Equation (C22) with (C28) leaves Equations (C25) and (C26) unchanged, but with τ_d replaced with τ_c.

C.2. Comments about the differences

For new share issues the main difference is that capital gain taxation affects the METR in the Öberg (2003) model, and hence in the extended model derived in this paper. Since Öberg includes the growth of value in shares, it is natural that the taxation of capital gains will play a significant role in her model, also for new share issues. As can be seen, the taxation of capital gains will not have an effect on the METR as long as the capital depreciate at the rate of inflation. This is an effect of the nominal taxation of capital gains.

The effect of a change in corporate tax rate is the same in both models when investments are financed with new share issues. As long as $(\delta - \pi) < (1 - \tau)MRR$ holds, a change in the taxation of dividends changes the METR in the same direction in both models, even though the magnitude differs. As stated above this situation is also the one of interest for an analysis.

For retained earnings there is no difference between the models, which is clear from the identical form of Equations (C18) and (C25).

One should keep in mind that this analysis is done under the assumptions of no depreciation allowances and no division of income between labor and capital income. The extended model introduced in this paper is more complicated, but introduced growth in corporation value in the same way as Öberg (2003), hence the principle differences and similarities with the original King-Fullerton (1984) model will be the same.

Appendix D. Tax rates, inflation and government borrowing rate 2018

The calculations in the paper are based on the rates in Table D1.

Table D1. Rates used in the calculations in the paper.

Rate	**Percent**
Corporate tax rate, $\boldsymbol{\tau}$	22
Dividend tax rate, $\boldsymbol{\tau_d}$	20
Labor tax rate, $\boldsymbol{\tau_w}$	57.1
Capital tax rate outside DA, $\boldsymbol{\tau_c}$	57.1
Interest tax rate, *m*	30
Social security contributions, $\boldsymbol{\sigma}$	31.42
Inflation rate, $\boldsymbol{\pi}$	1.8
Government borrowing rate	0.049

Source: Own table after SKV 292, Statistic Sweden (SCB) and Swedish National Debt Office

Note: The labor income tax refers to the highest marginal income tax. The capital tax is the statutory capital tax rate outside the dividend allowance (DA), which is the same as the labor income tax. For simplicity, we assume the effective capital tax to be half of the statutory rate. The same assumption is made by King-Fullerton (1984, p. 146). In reality, the active owner of a closely held corporation could face very different tax rates depending on the level of income and whether the capital gain is inside the dividend allowance or not. For a detailed description, see Wykman (2019).

Appendix E. Some special rules

The results in Section 4 are calculated without considering any special rules. Since the purpose is to compare different models, this simplification does not affect the result. To calculate time series for the METR from the tax reform in 1990–1991 and onwards there are a few special tax rules that should be considered. Below I explain how they are included in the model.

E1. The Annell Deduction

The Annell deduction[21] was introduced in 1960 and abolished in 1994, together with the double taxation. When the double taxation was reintroduced in 1995, the Annell deduction was not. The deduction is a relief in the double taxation with up to 10 percent on new investments under a time period of 10 years (SOU 2009:33).[22]

Following Öberg (2003) the value of the tax reduction, from the investor's perspective, can be calculated as:

$$E = y\tau \int_0^z e^{-i(1-m)t} dt \qquad \text{(E1)}$$

In equation (E1), *y* is the size (as share of the new investment) and *z* the length (in years) of the deduction. For example and according to the information stated above *y=0.1* and *z=10*.

In equilibrium, the necessary present value of an investment project must now be:

$$V(0) = \frac{1}{1+E} \qquad \text{(E2)}$$

E2. The SURV

During 1991, 1992 and 1993 it was possible to set aside some funds free of tax. Öberg (2003) incorporate this by an annual tax credit of

[21] Named after the investigator who suggested the deduction.
[22] Earlier years other percentages and time spans were in use.

$0.3\tau\frac{i(1-m)}{(1-\tau_c)}e^{-at}$ (E3)

This is incorporated just as the depreciation allowances. For calculus reasons a reasonable assumption like $i = 0.08$ in equation (E3) is preferable to exact calculations.[23]

E3. Periodization Fund

Since 1994 periodization funds allows 25 percent of the corporate profit to be taxed within a period of 6 years. This will lower the effective tax rate, and the higher nominal interest rate, the larger tax relief (SOU 2002:52). However, for an investor with a forward looking perspective it is unclear what nominal interest rate to assume for the coming six years.

The periodization fund may also lower the effective tax rate due to a future cut in corporate tax. Even though that has become the case, it has been unknown for the investor at the time for the investment. As in all other cases, and in line with the King-Fullerton model, we assume that the investor takes the statutory corporate tax and the nominal interest rate as given, at the present level.

Mathematically it is straightforward to introduce the periodization fund in the model by allowing for an effective corporate tax rate calculated as a weighted average of the present corporate tax and the future (six years at most) corporate tax rate, with respect to a discount factor equal to the post tax nominal interest rate.

However, since the tax effect of the periodization fund is uncertain, and depends on the alternative use of the present tax savings, it is not possible to determine the exact effect.

The precise measure as used in Öberg (2003):

$\tau_e = 0.75\tau_{s,t=1} + 0.25e^{-6(1-m)i}\tau_{s,t=6}$ (E4)

could be simplified by using the same approximation as above ($i = 0.08$). Since $m = 0.3$ throughout the period (E4) simplifies to:

[23]Without approximation the model would end up in the complicated situation where the METR depends on an integral of the form: $\int_0^{y(i)} x(i)e^{-it}dt$. The result is not sensitive for this simplification, a percentage point change in *i* induces approximately a 0.2 percentage point change in the METR.

$\tau_e = 0.93\tau_s$ (E5)

Equation (E5) states that the periodization fund reduces the statutory corporate tax by 7 percentage points.[24]

E4. The relief amount

A part of the dividend allowance was completely exempted from taxation between 1997 and 2006. The reduction is called the relief amount (*lättnadsbelopp*). Within the relief amount dividends were exempt from capital income taxation. The relief amount was calculated in the same way as the dividend allowance, but with a lower imputed rate of return, first at 65 percent of SLR and from 1998 and onward 70 percent. The relief amount was a subset of the dividend allowance, and did not increase the total share of the surplus subject to capital income taxation.

To incorporate the relief amount Equation (43) has to be modified to:

$$V(0) =$$

$$= \int_0^{t_d} \left(\frac{(1-\tau_w)}{(1-\tau_c)} W(t) + \frac{(1-\tau_d)}{(1-\tau_c)} D(t) + \frac{1}{(1-\tau_c)}\ R(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt + \int_{t_d}^{t_f} \left(\frac{(1-\tau_d)}{(1-\tau_c)} D^*(t) + \right.$$
$$\left. + \frac{1}{(1-\tau_c)}\ R(t)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt + \int_{t_f}^{\infty} \frac{1}{(1-\tau_c)}\ R^*(t) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad \text{(E5)}$$

Consequently, the tax credit in Equation (45) takes a more complicated expression:

$$TC = \int_{t_d}^{t_f} (\beta - \alpha - D^*(t)))\left(\frac{1}{1+\varphi}\right)\left(\left(\sigma + \frac{\tau_w}{1+\sigma}\right) - (\tau + \tau_d - \tau\tau_d)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt +$$
$$\int_{t_f}^{\infty} (\beta - \alpha)\left(\frac{1}{1+\varphi}\right)\left(\left(\sigma + \frac{\tau_w}{1+\sigma}\right) - (\tau + \tau_d - \tau\tau_d)\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt + \int_{t_f}^{\infty} (\alpha - R^*(t)) *$$
$$\left(\left(\sigma + \frac{\tau_{pw}}{1+\sigma}\right) - \tau\right) e^{-\frac{i(1-m)t}{(1-\tau_c)}} dt \qquad \text{(E6)}$$

In Equations (E5) and (E6) the relief amount R is a fixed amount:

[24] The exact assumption on i plays a minor role and the results will be robust (The maximum differences between any two reasonable assumptions are approximately 1 percentage point on the corporate income tax.)

$R(t) = \alpha$ (E7)

and a part of the total dividend allowance, so that:

$D(t) = \beta - \alpha$ (E8)

And:

$D^{*}(t) = (1 - \tau)MRRe^{-(\delta - \pi)t} - \alpha + \tau a e^{-at}$ (E9)

and finally

$R^{*}(t) = (1 - \tau)MRRe^{-(\delta - \pi)t} + \tau a e^{-at}$ (E10)

The time limits are defined as:

d is the value for *t*, such that

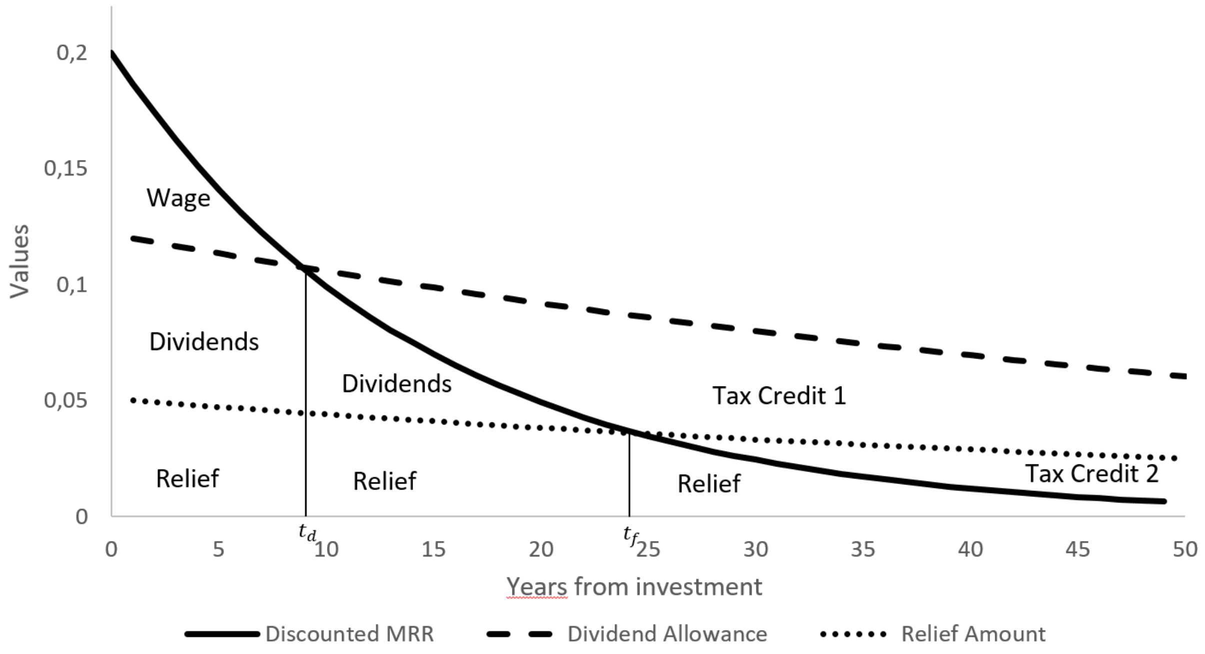
$(1 - \tau)MRRe^{-(\delta - \pi)t} - \alpha + \tau a e^{-at} = \beta$ (E11)

f is the value for *t* such that

$(1 - \tau)MRRe^{-(\delta - \pi)t} + \tau a e^{-at} = \alpha$ (E12)

This is illustrated in Figure E1, which corresponds to Figure 2 in Section 4. The first two integrals on the right hand side of Equation (E6) is called *Tax Credit 1* and the last integral *Tax Credit 2*.

Figure E1. The division of income after an investment (MRR=20%) financed by new share issues, with repurchases of shares and relief amount



Note: MRR is the marginal rate of return on an investment.
Source: Own figure.