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Cream-skimming entry in railway passenger services?

by

Lena Wieweg

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SE-701 82 Örebro
SWEDEN

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Cream-skimming entry in railway passenger services?

1. Introduction

One argument against liberalising entry into railway passenger services is that entry on profitable routes may undermine the possibility to supply less profitable routes, which eventually implies a reduced network or increased cost for subsidies. In Great Britain this is manifested by a very restrictive use of open access operators, based on the argument that entry on the most profitable parts of a franchise will increase the subsidy needed to keep the remainder running (Nash, 2002).

The purpose of this paper is to analyse changes in revenue on low profitable routes as a result of entry on high profitable routes. This will be examined in a setting of a very simple network, consisting of three products, where two can be regarded as components and one as a composite good. To start with, in section 2, the analysis will be in general terms where the products are not necessarily specific to railway passenger services. Thereafter, in section 3, some of the specific features inherent in the railway passenger services will be discussed to see to what extent the general results can be applied in this specific case.

2. The general case

2.1 The model

There are three products, labelled A, B and AB where A and B can be regarded as components and AB as a composite product. Consumers however demand all three products so that A and B are not only components but can be used on their own. Consumers of AB have the opportunity to buy it as a composite product or to buy the two components A and B separately.

The inverse demand functions for the three products are assumed to be linear:

$p_i(q_i) = a_i - b_i q_i$, i = A, B, AB

With regard to the cost function there are two cases depending on whether production consists of A and B (and AB is a “by-product”) or if production consists of AB (and A and B are the “by-products”).

$C(A, B, AB) = F_A + F_B + c_A(q_A) + c_B(q_B) + c_{AB}(q_{AB})$ A and B are the main products

$C(A, B, AB) = F_{AB} + c_A(q_A) + c_B(q_B) + c_{AB}(q_{AB})$ AB is the main product

2.2 Prices

The prices of the three products cannot be set independently of each other. There are two restrictions generated by the demand side:

1) $p_{AB} \geq \max(p_A, p_B)$

If the price p_{AB} were lower than one or both of the components prices, consumers of that component would prefer to pay the lower composite price. Therefore the price of the composite product is bounded from below to be at least equal to the highest of the other prices.

2) $p_{AB} \leq p_A + p_B$

Since it is possible for consumers to buy components separately, the price of the composite product is bounded from above of the sum of the prices of the components.

The first restriction is binding when demand for the composite product AB is low relative to one or both of the components. The second restriction is binding when demand for the composite product is large relative to demand for the components.

2.3 Pre-entry market conditions

The main interest in this paper is to study changes in profitability on low profit markets as a result of entry on high profit markets. For this reason, we need to establish suitable market conditions.

In the pre-entry case, one single firm produces all three products. Demand for component A is high relative to demand for B and AB ($a_A > a_B, a_{AB}$). Production of A is profitable on a stand-alone basis, $p(A) > 0$. Total profits from production of both A and B (and therefore also of AB) are however at least as high as profits from A alone, $p(A, B, AB) \geq p(A)$. Production of B may or may not be profitable on a stand alone basis.

With regard to the cost function, the production consists of A and B while AB is just the sum of A and B. This means that there are fixed costs only for A and B.

$$C = F_A + F_B + c_A(q_A) + c_B(q_B) + c_{AB}(q_{AB})$$

It is further assumed that marginal costs are constants and very low; $c_A(q_A) = c_A \cdot q_A$, $c_B(q_B) = c_B \cdot q_B$ and $c_{AB}(q_{AB}) = c_{AB} \cdot q_{AB}$. Without loss of generality $c = 0$ will be used (prices can be interpreted as prices over marginal cost).

If the three products were independent of each other, or if no restriction were binding, profit maximisation simply gives the unrestricted monopoly prices p^*. In this case however, restriction 1 will be binding since $p^*_{AB} < \max(p^*_A, p^*_B) = p^*_A$.

The expression for profit maximisation under the assumption of zero marginal costs is shown in (1) below. Under the assumptions regarding market conditions, profit maximisation is done subject to restriction 1 only.

$$Max\, p\,(A,B,AB) = p_A \cdot q_A(p_A) + p_B \cdot q_B(p_B) + p_{AB} \cdot q_{AB}(p_{AB}) - F_A - F_B \tag{1}$$

$$\begin{aligned} s.t. \quad & p_{AB} \geq p_A \\ & p_{AB} \geq p_B \end{aligned}$$

In terms of the parameters, the corresponding Langrangian is as shown in (2) below.

$$L = p_A\left(\frac{a_A - p_A}{b_A}\right) + p_B\left(\frac{a_B - p_B}{b_B}\right) + p_{AB}\left(\frac{a_{AB} - p_{AB}}{b_{AB}}\right) - m_1(p_{AB} - p_A) - m_2(p_{AB} - p_B) - F_A - F_B \tag{2}$$

The reason for m_2 to apply, although it has been stated in the pre-entry market conditions that $p^*_A > p^*_B$, will become clear soon. In short, since the first restriction will be binding ($m_1 > 0$) the price of both A and AB may be adjusted. Therefore, it has to be checked that price of AB is at least as high as the price of B.

Case 1) $m_1 > 0$, $m_2 = 0$

$m_1 > 0$ implies that the restriction $p_{AB} \geq p_A$ is binding with equality. The implication of $m_2 = 0$ is that the unrestricted price of B is lower than the resulting profit maximising price of AB (shown in (3) below)

The first order condition for profit maximisation gives the following optimal prices:

$$p_A = p_{AB} = \bar{p}_{A,\,AB} = \frac{a_A b_{AB} + a_{AB} b_A}{2(b_A + b_{AB})} \tag{3}$$

$$p^*_B = \frac{a_B}{2} \leq \bar{p}_{A,\,AB} \text{ since } m_2 = 0$$

The common price $\bar{p}_{A,\,AB}$ will only be profit maximising if demand of AB is not too low. It can be shown that if demand of AB, in terms of a_{AB}, is lower than (4) below, profits will be higher by charging p^*_A for both A and AB.

$$a_{AB} = \frac{a_A}{b_A}\left(\sqrt{(b_A + b_{AB})b_{AB}} - b_{AB}\right) \tag{4}$$

The break-even level of a_{AB} is derived by comparing the two strategies $p_{A,\,AB}(p*_A)$ and $p_{A,\,AB}(\overline{p}_{A,\,AB})$.

$$p_{A,\,AB}\left(p*_A\right) = \frac{a_A(a_A b_{AB} + b_A(2a_{AB} - a_A))}{4b_A b_{AB}}$$

$$p_{A,\,AB}\left(\overline{p}_{A,\,AB}\right) = \frac{\left(a_A b_{AB} + a_{AB} b_A\right)^2}{4b_A b_{AB}(b_A + b_{AB})}$$

When a_{AB} is equal to (4), the two strategies gives equal profit.

It can easily be seen that if $a_{AB} > \frac{a_A}{2}$ strategy $\overline{p}_{A,\,AB}$ will always yield higher profit than $p*_A$ (with the exception that when demand for A and AB is equal the two strategies will yield equal profit). It is therefore sufficient to compare the two strategies when $a_{AB} \leq \frac{a_A}{2}$ which implies $q_{AB}\left(p*_A\right) = 0$.

When a_{AB} is higher than or equal to (4) the common price $\overline{p}_{A,\,AB}$ will be profit maximising. The expression (4) depends on both b_A and b_{AB}. The limit of the break-even size of a_{AB} as the values of b_A and b_{AB} approaches the extreme values 0 and ∞ (can never be negative) is

$$\lim_{b_A \to 0} a_{AB} = 0{,}5a_A$$

$$\lim_{b_A \to \infty} a_{AB} = 0$$

$$\lim_{b_{AB} \to 0} a_{AB} = 0$$

$$\lim_{b_A \to \infty} a_{AB} = 0{,}5a_A$$

Case 2) $m_1 > 0$, $m_2 > 0$

The implication of $m_2 > 0$ is that the unrestricted price of B is higher than the profit maximising price of AB resulting from $m_1 > 0$. Together this means that profit maximising gives a common price for all three products.

$$p_A = p_B = p_{AB} = \overline{p}_{A,\,B,\,AB} = \frac{a_A b_B b_{AB} + a_B b_A b_{AB} + a_{AB} b_A b_B}{2(b_B b_{AB} + b_A b_{AB} + b_A b_B)} \tag{5}$$

The minimum size of a_{AB} in this case is shown in (6) below.

$$a_{AB} \geq \frac{1}{b_A b_B}\left(\sqrt{b_{AB}\left(b_A a_B{}^2 + b_B a_A{}^2\right)\left(b_B b_{AB} + b_A b_{AB} + b_A b_B\right)} - b_{AB}(b_A a_B + b_B a_A)\right) \tag{6}$$

If a_{AB} is lower than this the prices of A and B will be equal to the unrestricted monopoly prices $p*_A$, $p*_B$ and the price of AB will be equal to the highest of these.

Case 3) $m_1 = 0$, $m_2 > 0$

This corresponds to case 1 but requires $a_A < a_B$ which is ruled out by definition.

Since we are interested in cases where AB is important for total profitability (otherwise the two components would be rather independent of each other implying that there would be no spill-over effects from one market to another) the analysis will be concentrated on the cases when a_{AB} is equal or higher than the minimum levels (as shown in (4) and (6) above). There is also a second reason for this choice; if a_{AB} is lower than the minimum level so that the highest of $p*_A$, $p*_B$ will be used, the analysis will become trivial. Entry on the high demand market will always reduce the price of AB (which is held well above the optimal level) so that revenue from AB will always increase.

Further, in case 2 above, demand for B is restricted from below by the price $\overline{p}_{A,\,AB}$ implying that demand of B is rather high (at least higher than in case 1). Therefore, entry on A in case 2 will not be studied here. In appendix 1 the results of entry in case 2 will be shown.

2.4 Entry on the high profit market A

2.4.1 Market outcome on A

Entry occurs on the market for A which is now produced by two firms, *f1* (the incumbent who also produces B) and *f2*.

Assumptions:

- Quality level will not change.
- The new firm *f2* will also have marginal cost $c_A=0$.
- There are constant returns to scale, implying that the "fixed" cost of supplying A is divided between the firms according to their share of total production.
- Consumers always choose the lowest price – products are homogeneous

The market equilibrium that occurs in the market for component A (in this duopoly setting) will depend on which type of competition that evolve; if firms compete with prices (Bertrand like competition) or with quantities (competition a la Cournot). It will also depend on whether firms are identical or if some firm dominates the market (price or quantity leadership) and on the degree of product differentiation. The (theoretical) prediction will also be different if the market game is a one-shot game (only period is considered) or if the game is repeated infinitely. The choice of model depends on the context of the particular economic situation or industry under examination.

In appendix 1 the different models indicated above are discussed. Below we summarise the discussion.

In the case of price competition (the Bertrand model), there is a unique Nash equilibrium, where $p_1 = p_2 = c$. This means that two firms are enough to have the perfectly competitive outcome. In many cases, this is not a plausible prediction of the market outcome. In contrast, the Cournot model displays a gradual reduction in market power as the number of firm increases. However, in many cases firms seem to choose prices, not quantities. "For this reason, many economists have thought that the Cournot model gives the right answer for the wrong reason." (Mas-Colell et. al p 394).

By giving the Cournot model an alternative interpretation, and instead think of the quantity choices as a long-run choice of *capacity*, with the determination of price from the inverse demand function being a proxy for the outcome of short-run price competition given these capacity choices. We can think of this as a two-stage game where the firms first choose their capacity levels and then compete in prices. It can be shown (Kreps and Scheinkman (1983)) that under certain conditions the unique subgame perfect Nash equilibrium in this game is the *Cournot outcome*. Therefore, the Cournot quantity competition captures the long run competition through quantity choice, with price competition occurring in the short run given those levels of capacity.

The static, one-shot nature in the models above is of course rather unrealistic. In reality, there are repeated interactions between firms. When taking dynamics under consideration it is easy to se that even with price competition as in the Bertrand model, if the discount factor is high enough (meaning that firms care about the future profits), it is possible to sustain any price $p \in [c, p^*]$ (a price between marginal cost and the monopoly price level) as a subgame perfect Nash equilibrium. Therefore, a possible market outcome is a price level above the competitive level, even if firms act as Bertrand competitors.

The choice of competition model in the market for A is not the main subject in this paper. However, to be able to calculate effects in the other markets, we have to make an assumption about the outcome in terms of prices for product A. To conclude, the Cournot model seems to be most appropriate when quantities can only be adjusted slowly, especially when quantity is interpreted as capacity. In the following, it is therefore assumed that the new price of A is according to Cournot competition.

Under the assumption of Cournot competition, the new market equilibrium for component A follows from profit maximisation under for each of the two firms. Profit maximisation for each firm, taking the decision of the other firm as given, gives the following reaction functions:

$$R(f1) = q_{Af1}(q_{Af2}) = \frac{a_A - b_A q_{Af2}}{2b_A}$$

$$R(f2) = q_{Af2}(q_{Af1}) = \frac{a_A - b_A q_{Af1}}{2b_A}$$

The resulting equilibrium quantity of A gives the following duopoly price:

$$p'_A = \frac{a_A}{3} < p^*_A$$

We are interested in how this will affect prices and profits on products B and AB. There are two main possibilities, depending on whether or not the new duopoly price on A will be lower than the pre-entry price (which is lower than the unrestricted monopoly price).

2.4.2 Case 1.1 $p'_A \geq \overline{p}_{A,\,AB}$

The new duopoly price of A, p'_A, is higher than or equal to the pre-entry price, $p'_A \geq \overline{p}_{A,\,AB}$. The incumbent, *f1*, will continue to charge $\overline{p}_{A,\,AB}$ which has to be chosen also by the entrant, *f2*. The only thing that happens is that revenue from A is shared between the two firms.

Table 1: Post-entry prices case 1.1

Product	Price *f1*	Price *f2*
A	$\overline{p}_{A,\,AB}$	$\overline{p}_{A,\,AB}$
B	p^*_B	
AB	$\overline{p}_{A,\,AB}$	$\overline{p}_{A,\,AB} + p^*_B$

The incumbent firm, *f1*, will charge the lowest price of AB is $\overline{p}_{A,\,AB}$, profits on B and AB are unchanged.

2.4.3 Case 1.2 $p'_A < \overline{p}_{A,\,AB}$

The duopoly price of A, p'_A, is lower than the pre-entry price. In this case both firms will charge p'_A. To analyse the changes on B and AB we start by stating the profit maximisation problem.

$$\underset{p_B,\, p_{AB}}{Max}\, p\,(B, AB) = p_B \cdot q_B(p_B) + p_{AB} \cdot q_{AB}(p_{AB}) - F_B \tag{7}$$

$$s.t.\quad p_{AB} \geq p'_A$$
$$p_{AB} \geq p_B$$
$$p_{AB} \leq p'_A + p_B$$

$$L = p_B\left(\frac{a_B - p_B}{b_B}\right) + p_{AB}\left(\frac{a_{AB} - p_{AB}}{b_{AB}}\right) - l_1(p_{AB} - p'_A) - l_2(p_{AB} - p_B) - l_3(p'_A + p_B - p_{AB}) - F_B \quad (8)$$

Depending on the values that the $l:s$ can take, a number of profit maximising price combinations of B and AB emerge.

To get an overview, we start by showing all combinations.

Case 1.2.1 $l_1 > 0, l_2 = 0, \; l_3 = 0$

$l_1 > 0$ implies that $p'_A > p^*_{AB}$ and the price of AB will be equal to p'_A

$l_2 = 0$ implies that $p^*_B < p'_A = p_{AB}$ and the price of B will be equal to p^*_B

$l_3 = 0$ implies that $p'A < p'A + p^*B$

Case 1.2.2 $l_1 = 0, l_2 = 0 \quad l_3 = 0$

$l_1 = 0$ implies that $p^*_{AB} \geq p'_A$ and the price of AB is equal to p^*_{AB}

$l_2 = 0$ implies that $p^*_{AB} \geq p^*_B$ and the price of B will be equal to p^*_B

$l_3 = 0$ implies that $p^*AB < p'A + p^*B$

Case 1.2.3 $l_1 = 0, l_2 = 0 \quad l_3 > 0$

$l_1 = 0$ implies that $p^*_{AB} \geq p'_A$ and the price of AB is equal to p^*_{AB}

$l_2 = 0$ implies that $p^*_{AB} \geq p^*_B$ and the price of B will be equal to p^*_B

$l_3 > 0$ implies that $p^*AB > p'A + p^*B$

Case 1.2.4 $l_1 = 0, l_2 > 0 \quad l_3 = 0$

$l_1 = 0$ implies that $p^*_{AB} \geq p'_A$ and the price of AB is equal to p^*_{AB}

$l_2 > 0$ implies that $p^*_{AB} < p^*_B$ and therefore the price of B and AB will have to be adjusted

$l_3 = 0$ since $\overline{p}_{B,\,AB} < p'_A + \overline{p}_{B,\,AB}$

Case 1.2.5 $l_1 > 0$, $l_2 > 0$ $l_3 = 0$

$l_1 > 0$ implies that $p'_A > p^*_{AB}$ and the price of AB is equal to p'_A

$l_2 > 0$ implies that $p^*_B > p'_A = p_{AB}$ and the price of B will also be equal to p'_A

$l_3 = 0$ since $p'_A < p'_A + p'_A$

Case 1.2.1 $l_1 > 0, l_2 = 0,\ l_3 = 0$

Post-entry profit maximising prices in case 1.2.1 are shown in table 2 below.

Table 2: Post-entry prices in Case 1.2.1

Product	Price *f1*	Price *f2*
A	p'_A	p'_A
B	p^*_B	
AB	p'_A	$p'_A + p^*_B$

Firm *f1* offer the lowest price for AB. There are no possibilities for the entrant *f2* to profitable compete for consumers of AB by lower the price of A, since this will always be followed by *f1*. *f2* knows that *f1* has to consider both restriction 1 and 2 in the pricing of AB implying that it will not be possible to undercut the price of *f1* . The lowest possible price for A is the break-even level p^0_A (where revenue just covers fixed costs). But the price of AB offered by *f2* will still be higher than the price by *f1*. Since both firms are aware of the consequences of such a price-war, the most probable outcome is the prices shown in table 4 above.

The change in revenue from B and AB for *f1* is accordingly:

$$\Delta TR_{B,\,AB} = \frac{9b_{AB}^2\,(a_A - a_{AB})^2 - (b_A + b_{AB})^2(2a_A - 3a_{AB})^2}{36b_{AB}(b_A + b_{AB})^2} \tag{9}$$

$$\frac{\partial^2 \Delta TR}{\partial a_{AB}{}^2} = \frac{3b_{AB}{}^2 - 3(b_A + b_{AB})^2}{b_{AB}(b_A + b_{AB})^2} < 0 \tag{10}$$

$\Delta TR_{B,\,AB}$ is strictly concave in a_{AB}. If it can be shown that $\Delta TR_{B,\,AB}$ is positive for both the lowest possible value of a_{AB} (*min* a_{AB}) and highest possible value (*max* a_{AB}) then it will be positive for all values in the relevant range.

$max\ a_{AB} = \frac{2}{3} a_A$ (since $p'_A > p^*_{AB}$), which gives $\Delta TR_{B,\,AB} = \frac{b_{AB}\left(\frac{a_A}{3}\right)^2}{4(b_A + b_{AB})^2} > 0$

$min\ a_{AB}$ depends on the relation between b_A and b_{AB}.

- if $b_A \geq \frac{5}{4} b_{AB}$ then $min\ a_{AB} = \frac{a_A(2b_A - b_{AB})}{3b_A}$
- if $b_A < \frac{5}{4} b_{AB}$ then $min\ a_{AB} = \frac{a_A}{b_A}\left(\sqrt{(b_A + b_{AB})b_{AB}} - b_{AB}\right)$

In the first case, when $b_A \geq \frac{5}{4} b_{AB}$, $\Delta TR_{B,\,AB} = 0$

Differentiating the expression for $\Delta TR_{B,\,AB}$ with respect to b_A gives

$$\frac{\partial \Delta TR_{B,\,AB}}{\partial b_A} = -\frac{b_{AB}(a_A - a_{AB})^2}{2(b_A + b_{AB})^3} < 0$$

and therefore the change in revenue is positive for lower values of b_A.

To summarise, in case 1.2.1. total revenue in market for B and AB is unchanged or increased. The intuition is clear; since the pre-entry price of AB is forced to be greater than the revenue-maximising price, a price reduction will increase revenue. Under the restriction $p'_A \geq p^*_{AB}$ it will never be the case that the new price will be lower than the optimal level.

Case 1.2.2 $l_1 = 0$, $l_2 = 0$ $l_3 = 0$

This implies that no restriction is binding. Prices are summarised in table 3 below.

Table 3: Prices after entry in case 1.2.2

Product	Price *f1*	Price *f2*
A	p'_A	p'_A
B	p^*_B	
AB	p^*_{AB}	$p'_A + p^*_B$

The price of AB from *f1* is lower than or equal to the price from *f2*. If it happens to be the case that $p*_{AB} = p'_A + p*_B$ and if consumers only care about price it will be profitable for *f1* to charge a price just slightly below $p*_{AB}$ to get the whole quantity if AB. By the same reasoning as in case 1.2.1 above, it can be seen that *f2* will never be able to match the price of AB from *f1*. The lowest possible price of A is p^0_A. If $p*_{AB} > p^0_A + p*_B$ then *f1* will charge a price of AB equal to $p^0_A + p*_B - e$ just slightly below the lowest possible price by *f2* ($p^0_A + p*_B$). Since both firms are aware of the consequences of such a price-war the most probable prices are as in table 5 above.

The change in revenue from B and AB for firm *f1* is therefore equal to:

$$\Delta TR_{B,\,AB} = \frac{b_{AB}(a_A - a_{AB})^2}{4(b_A + b_{AB})^2} \geq 0 \tag{11}$$

Case 1.2.3 $l_1 = 0$**,** $l_2 = 0$**,** $l_3 > 0$

The implication of this case is that demand for B is very low relative to demand for both A and AB. Firm *f1* cannot influence the price of A and therefore the prices of B and AB have to be adjusted to fulfil the requirement that $p_{AB} \leq p_A + p_B$.

The profit maximising prices of B and AB are in this case:

$$\hat{p}_B = \frac{a_B b_{AB} + a_{AB} b_B - 2b_B p'_A}{2(b_B + b_{AB})} \tag{12}$$

$$\hat{p}_{AB} = \frac{a_B b_{AB} + a_{AB} b_B + 2b_{AB} p'_A}{2(b_B + b_{AB})} \tag{13}$$

Table 4: Post-entry prices in case 1.2.3

Product	Prices *f2*	Prices *f2*
A	p'_A	p'_A

B	$\widehat{p}_B$	
AB	$\widehat{p}_{AB}$	$p'_A + \widehat{p}_B$

But then the price of AB charged by the two firms will be equal, since per definition $\widehat{p}_{AB} = p'_A + \widehat{p}_B$. If consumers only care about price it will be profitable for *f1* to charge a price of AB slightly below the above price $\widehat{p}_{AB}$ to get all consumers of AB.

Also in this case it is clear that *f2* can never successfully compete for consumers of AB. A reduction in the price of A will immediately induce changes in both $\widehat{p}_B$ (increase) and $\widehat{p}_{AB}$ (decrease) and *f1* will always be able to charge a price of AB slightly below the price of *f2*.

Therefore the prices shown in table 6 above maximises profits for both firms. The change in revenue from B and AB for *f1* is (almost) equal to:

$$\Delta TR_{B,\,AB} = \frac{9(a_A - a_{AB})^2 b_B(b_B + b_{AB}) - (b_A + b_{AB})^2(2a_A + 3a_B - 3a_{AB})^2}{36(b_B + b_{AB})(b_A + b_{AB})^2} \tag{14}$$

$$\frac{\partial \Delta TR}{\partial a_{AB}} = = \frac{-3(a_A - a_{AB})b_B(b_B + b_{AB}) + (b_A + b_{AB})^2(2a_A + 3a_B - 3a_{AB})}{6(b_B + b_{AB})(b_A + b_{AB})^2} \tag{11}$$

$$\frac{\partial^2 \Delta TR}{\partial a_{AB}{}^2} = = \frac{b_B(b_B + b_{AB}) - (b_A + b_{AB})^2}{2(b_B + b_{AB})(b_A + b_{AB})^2} < 0 \text{ if } (b_A + b_{AB})^2 > b_B(b_B + b_{AB}) \tag{15}$$

The first order partial derivative evaluated at *max* a_{AB} and *min* a_{AB} shows that the function is downward sloping in the relevant range:

$$\textit{max } a_{AB} = a_A \text{ and } \frac{\partial \Delta TR}{\partial a_{AB}} = \frac{(-a_A + 3a_B)}{6(b_B + b_{AB})} \le 0 \text{ since } a_B \le \frac{3a_{AB} - 2a_A}{3} \text{ and therefore } a_A - 3a_B \ge 0$$

$$\textit{min } a_{AB} = \frac{2a_A + 3a_B}{3} \text{ and } \frac{\partial \Delta TR}{\partial a_{AB}} = \frac{-(a_A - 3a_B)b_B}{6(b_A + b_{AB})^2} \le 0$$

It remains to see if the change in revenue is positive or negative. At *max* a_{AB} the change in revenue is obviously negative since this implies that the pre-entry prices are equal to p^*_B and p^*_{AB}. In terms of the parameters the change in revenue is equal to

$$\Delta TR_{B,\,AB} = -\frac{(-a_A + 3a_B)^2}{36(b_B + b_{AB})} < 0\,.$$

At *min* a_{AB} the change in total revenue at minimum a_{AB} is equal to

$$\Delta TR_{B,\,AB} = \frac{(a_A - 3a_B)^2 b_B}{36(b_A + b_{AB})^2} > 0$$

To summarise, the change in revenue is positive for small values of a_{AB} and negative for higher values of a_{AB}. $\Delta TR_{B,\,AB}(a_{AB})$ is downward sloping within the relevant range and crosses the horizontal axis at value between *min* and *max* a_{AB} max and min. This occurs at

$$a_{AB} = \frac{(2a_A + 3a_B)(b_A + b_{AB})^2 - 3a_A b_B (b_B + b_{AB}) + \sqrt{b_B(b_B + b_{AB})}\,(b_A + b_{AB})(a_A - 3a_B)}{3\left((b_A + b_{AB})^2 - b_B(b_B + b_{AB})\right)}$$

Case 1.2.4 $\quad l_1 = 0,\ l_2 > 0,\ l_3 = 0$

$l_1 = 0$ implies that $p^*_{AB} \ \geq\ p'_A$ and the price of AB is equal to p^*_{AB}

$l_2 > 0$ implies that $p^*_{AB} \ <\ p^*_B$ and therefore the price of B and AB will have to be adjusted in the same manner as the pre-entry price of A and AB to $\overline{p}_{B,\,AB}$, see (16) below.

$l_1 = 0$ implies that $p'_A < \overline{p}_{B,\,AB}$ so that no further adjustment of prices is required.

The profit maximising price for B and AB is equal to:

$$\overline{p}_{B,\,AB} = \frac{a_B b_{AB} + a_{AB} b_B}{2(b_B + b_{AB})} \tag{16}$$

Table 5: Post-entry prices in case 1.2.4

Product	Prices *f1*	Prices *f2*
A	p'_A	p'_A
B	$\overline{p}_{B,\,AB}$	

AB	$\overline{p}_{B,\ AB}$	$p'_A + \overline{p}_{B,\ AB}$

The lowest price of AB is offered by *f1*. The change in revenue for *f1* from B and AB is equal to:

$$\Delta TR_{B\ +\ AB} = \frac{(a_A - a_{AB})^2 b_{AB}(b_B + b_{AB}) - (a_B - a_{AB})^2 (b_A + b_{AB})^2}{4(b_B + b_{AB})(b_A + b_{AB})^2} \qquad (17)$$

$$\frac{\partial^2 \Delta TR_{B,\ AB}}{\partial a_{AB}{}^2} = \frac{b_{AB}(b_B + b_{AB}) - (b_A + b_{AB})^2}{2(b_B + b_{AB})(b_A + b_{AB})^2} \qquad (18)$$

The sign of the second order derivate depends on the relation between $(b_A + b_{AB})^2$ and $b_{AB}(b_B + b_{AB})$. We start however by examining the sign of $\Delta TR_{B,\ AB}$ at the extreme points.

$max\ a_{AB} = a_B$ at which $\Delta TR_{B,\ AB} = \dfrac{(a_A - a_B)^2 b_{AB}}{4(b_A + b_{AB})^2} > 0$

$min\ a_{AB} = \dfrac{2}{3} a_A$

$$\Delta TR_{B\ +\ AB} = \frac{\left(\dfrac{a_A}{3}\right)^2 b_{AB}(b_B + b_{AB}) - \left(a_B - \dfrac{2}{3} a_A\right)^2 (b_A + b_{AB})^2}{4(b_B + b_{AB})(b_A + b_{AB})^2} \qquad (17)$$

The sign of $\Delta TR_{B\ +\ AB}$ will in this case depend of the value of a_B which lies within the following range:

$$\frac{2}{3} a_A < a_B < \frac{a_A b_{AB} + a_{AB} b_A}{b_A + b_{AB}}$$

$min\ a_B$ gives $\Delta TR_{B\ +\ AB} = \dfrac{a_A{}^2 b_{AB}}{36(b_A + b_{AB})^2} > 0$

$max\ a_B$ gives $\Delta TR_{B\ +\ AB} = \dfrac{a_A{}^2 b_B b_{AB}}{36(b_B + b_{AB})(b_A + b_{AB})^2} > 0$

The change in revenue is therefore positive for both maximum and minimum values of a_{AB}. In the case that $\Delta TR_{B,\ AB}(a_{AB})$ is concave the change in revenue is positive for all values in the relevant range. If $\Delta TR_{B,\,AB}(a_{AB})$ is convex the function will reach the lowest value at $\overline{\overline{a}}_{AB} = \frac{a_A b_{AB}(b_B + b_{AB}) - a_B(b_A + b_{AB})^2}{b_{AB}(b_B + b_{AB}) - (b_A + b_{AB})^2}$. This minimum point is however outside the relevant range since $\overline{\overline{a}}_{AB} > max\ a_{AB} = a_B$.

To summarise, it can be concluded that regardless of the shape of the $\Delta TR_{B,\,AB}$ function, the change in revenue is positive for all values of a_{AB} within the relevant range. for all

Case 1.2.5 $l_1 > 0,\ l_2 > 0\ \ l_3 = 0$

$l_1 > 0$ implies that $p'_A > p^*_{AB}$ and the price of AB is equal to p'_A

$l_2 > 0$ implies that $p^*_B > p'_A = p_{AB}$ and the price of B will also be equal to p'_A

Table 6: Post-entry prices in case 1.2.5

Product	Prices *f1*	Prices *f2*
A	p'_A	p'_A
B	p'_A	
AB	p'_A	$2p'_A$

$$\Delta TR_{B,\,AB} = \frac{9b_B b_{AB}{}^2(a_A - a_{AB})^2 - (b_A + b_{AB})^2\left(b_{AB}(2a_A - 3a_B)^2 + b_B((2a_A - 3a_{AB})^2\right)}{36 b_B b_{AB}(b_A + b_{AB})^2} \qquad (19)$$

Since this case involves changes in the price of both B and AB the expression for $\Delta TR_{B,AB}$ involves ..

$$\frac{\partial^2 \Delta TR_{B,\,AB}}{\partial a_{AB}{}^2} = \frac{-b_A(b_A + 2b_{AB})}{2b_{AB}(b_A + b_{AB})^2} < 0 \qquad (20)$$

$$\frac{\partial^2 \Delta TR_{B,\,AB}}{\partial a_B{}^2} = -\frac{1}{2b_B} < 0 \qquad (20)$$

	max	*min*
a_{AB}	$\frac{2}{3}a_A$	$b_A \geq \frac{5}{4}b_{AB}:\ a_{AB} > \frac{a_A(2b_A - b_{AB})}{3b_A}$ $b_A < \frac{5}{4}b_{AB}:\ a_{AB} \geq \frac{a_A}{b_A}\left(\sqrt{(b_A + b_{AB})b_{AB}} - b_{AB}\right)$
a_B	$\frac{2}{3}a_A$	$\frac{a_A b_{AB} + a_{AB} b_A}{b_A + b_{AB}}$

We have to examine the change in revenue for all four possible combinations.

- $max\ a_{AB}$ and $min\ a_B$

$$\Delta TR = \frac{b_{AB}a_A^{\ 2}}{36(b_A + b_{AB})^2} > 0$$

- $max\ a_{AB}$ and $max\ a_B$

$$\Delta TR = \frac{b_{AB}a_A^{\ 2}(b_B - b_{AB})}{36b_B(b_A + b_{AB})^2} \begin{array}{l} > 0 \ if\ \ b_B > b_{AB} \\ = 0 \ if\ \ b_B = b_{AB} \\ < 0 \ if\ \ b_B < b_{AB} \end{array}$$

The sign therefore depends on the relative size of b_B and b_{AB}.

- $min\ a_{AB}$

If $b_A \geq \frac{5}{4}b_{AB}$ the minimum size of a_{AB} is equal to $\frac{a_A(2b_A - b_{AB})}{3b_A}$ which gives

$$\Delta TR = \frac{-(2a_A - 3a_B)^2}{36b_B}$$

At this minimum size of a_{AB} both $max\ a_B$ and $min\ a_B$ is equal to $\frac{2}{3}a_A$ and therefore $\Delta TR = 0$.

By differentiating the expression (xx) for ΔTR with respect to b_A:

$$\frac{\partial \Delta TR}{\partial b_A} = -\frac{b_{AB}(a_A - a_{AB})^2}{2(b_A + b_{AB})^3} < 0$$

It can therefore be seen that the change in revenue will be positive for smaller values of b_A.

To summarise, when demand for AB is low (a_{AB} is low), the change in revenue on B and AB will be ≥ 0. When demand for AB is high and demand for B is low the change in revenue will be positive. But when demand for both B and AB are high the change in revenue is indeterminate and will depend on the relation between b_B and b_{AB}.

1.2.1	$\Delta TR_{B,\ AB} \geq 0$	
1.2.2	$\Delta TR_{B,\ AB} \geq 0$	
1.2.3	$\Delta TR_{B,\ AB} < 0$	$max\ a_{AB}$
	$\Delta TR_{B,\ AB} > 0$	$min\ a_{AB}$
1.2.4	$\Delta TR_{B,\ AB} \geq 0$	
1.2.5	$\Delta TR_{B,\ AB} < 0$	$max\ a_{AB} + max\ a_B, b_B < b_{AB}$
	$\Delta TR_{B,\ AB} \geq 0$	$min\ a_{AB},\ max\ a_{AB} + min\ a_B,\ max\ a_{AB} + max\ a_B, b_B \geq b_{AB}$

2.6 Summary general case

In a world with a single profit maximising producer and the simple network described above, the interdependences between the products will lead to prices different from the unrestricted profit maximising prices. In the case analysed in this section, the interdependences will force the price of the composite product to be set above the (unrestricted) profit maximising level. At the same time, the price of the high demand product will be set below its (unrestricted) profit maximising level.

When entry occurs on the high demand market the expected price reduction will therefore be less than with independent products. If the competitive price on the high demand market is lower than the pre-entry price, this will have implications on the low demand markets. This is due to the fact that a lower price on the high demand market will weaken the restriction on the price for the composite product and this can therefore be set closer to the optimal level. In most cases revenue on the low demand markets will therefore increase. The only exceptions are when demand for the low demand products (the composite AB and the low demand component B) are rather high. If this finding can be generalised to the market for railway passenger services then the “cream-skimming-argument” will not hold. Liberalising entry may instead increase revenue on low demand markets!

3. Railway passenger services

The results from the analysis of the general case in the preceding section do to a large extent depend on the assumptions made in section 2.4. In order to see if the results from the preceding section can be applied to railway passenger services, the relevance of the assumptions will be briefly discussed.

3.1 The products

The components A and B are two connecting train routes, A covering the distance a to b, B the distance b to c and AB is therefore a to c, see figure 3 below. To fix ideas, A can be thought of as Gothenburg-Stockholm and B as Stockholm-Gävle.

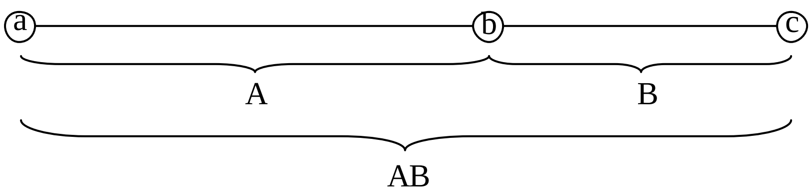


Figure 3: The products

One important difference between the railway passenger case and the general case is fact that different units of for example A are not a homogenous. Instead, each departure, D_A, is somewhat different from the others. They will differ at least with respect to departure/arrival time and direction. To form the composite product AB we cannot use any A and B, they have to be linked in time and space, maybe only one of the As and one of the Bs is functioning together. The degree to which operators coordinates A and B to make the combined trips AB possible depends on the size of the different markets and on the cost of coordination. If demand for A and B are large, it will be profitable to optimise supply (timetables) in each market separately. If demand for AB is large and/or important to total profitability it will be profitable to coordinate timetables.

3.2 Prices

The price restrictions presented in section 2.2 ought to suit the railway passenger services case very well. A quick look at some actual prices charged by SJ shows that the prices of combined trips are never larger than the sum of separate prices (restriction 2). In some cases the price of a combined trip is equal to the price of one of the components (restriction 1).

Table 9: Examples of p_A, p_B and p_{AB} (www.resor.sj.se)

AB	A	B	p_A	p_B	$p_A + p_B$	p_{AB}
Göteborg-Gävle	Göteborg-Stockholm	Stockholm-Gävle	1041	494	1535	1012
Västerås-Halmstad	Västerås-Göteborg	Göteborg-Halmstad	442	190	632	523
Linköping- Borlänge	Linköping -Stockholm	Stockholm-Borlänge	532	233	765	556
Göteborg- Avesta	Göteborg-Stockholm	Stockholm-Avsta-	1041	176	1217	884

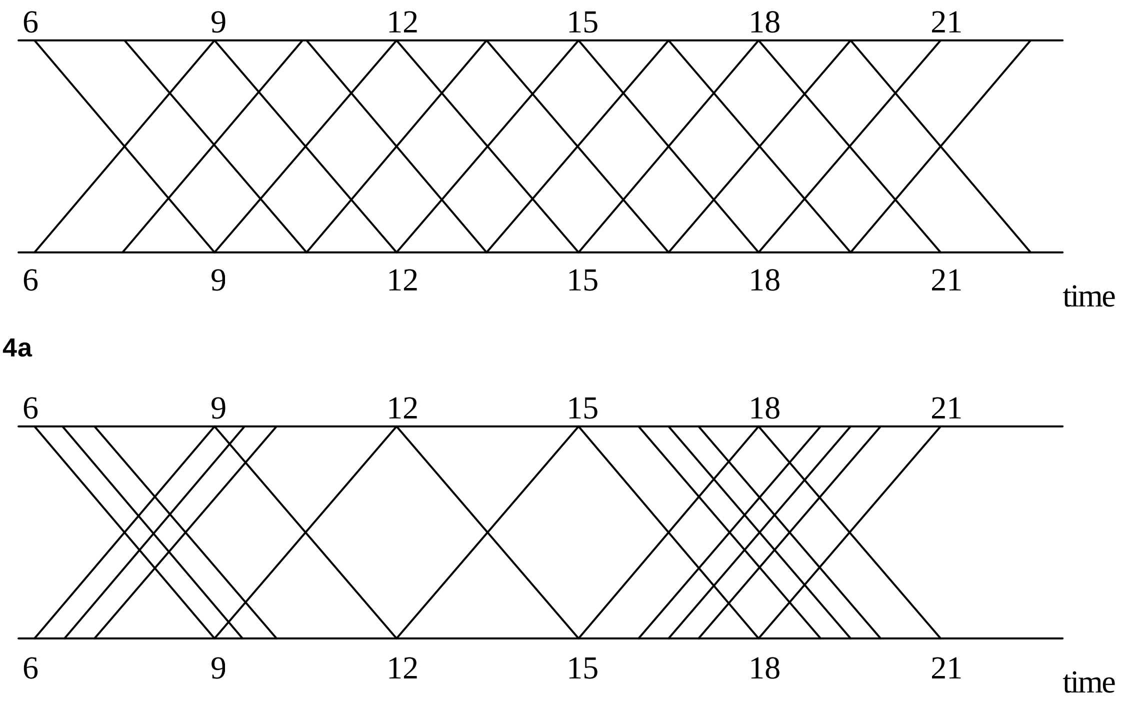
3.3 The assumptions

3.3.1 Quality and operating cost

Demand does also depend on the choice of quality. In the context of railway passenger services the concept of quality has may dimensions and most are interdependent. The most obvious determinants of quality are frequency, travelling time and reliability but there are also others like type of vehicle (comfort). Quality is therefore to a large extent determined by the design of timetables in which frequency and each departure's exact location in time as well as travelling time are specified. To each possible design of timetables belongs some degree of reliability.

The implication of the assumption of no quality change is that the overall time-table will be unchanged after entry. This may seem implausible, but on high density routes (where "our" trains will have to be coordinated with freight trains as well as local trains), capacity constraints will normally imply that the scope for increases in frequency, changes in departure times and/or travelling time will be very limited.

Operating cost will also vary with quality. It is not only the number of departures, distance and transport time that determines operating costs. The exact location in time of different departures will to a large extent determine both cost and revenues. This is shown by an example in figure 4a and 4b below. Total number of departures is the same in both figures and equal to 10 in each direction. In figure 4a this is effectuated according to minimization of production cost. In figure 4b the design is according to demand condition – maximising number of passengers - (very high frequency in peaks) and the cost will in this latter case be much higher. The optimal time-table will probably lie somewhere between these two extremes.



4b

Figure 4: Time-tables designed according to a) minimising operating costs and b) maximising number of passengers. In both cases there are 10 departures in each direction.

Introducing competition into a formerly system of a single producer can therefore, besides the expected influences on prices, have influences also on the choice quality level and therefore on demand as well as on costs which may imply additional influences on prices. But as was discussed above, the scope for changes in quality (time-tables) may be limited by capacity restriction.

Once the decision about quality is made, most of the operating costs can be regarded as fixed since they do not - in the short run - vary with the number of consumers, *q*. To be specific; the costs for supplying a specific frequency (number of departures) with a certain train size can be regarded as fixed. Increases in the number of passengers may imply increased supply of seats but in the short run this can only be accomplished by increasing number of seats in the existing trains (since time-tables are fixed in the short run).

The assumption of constant and small marginal cost is therefore approximately applicable to the railway passenger services case. The assumption about constant returns to scale is though more problematic to justify. It is easy to think of reasons for the existence of economies of scale in railway passenger services – efficient utilisation of vehicles, personnel and maintenance facilities in a large company. But there are also signs of the reverse – smaller companies' having a cost advantage. In U.K. smaller operators have been amongst the more innovative in the new railway structure (Nash, 2002). In Sweden it has been observed that the incumbent SJ on repeated occasions seems to have large cost disadvantages compared to new entrants (Alexandersson et. al. 2000).

3.2.2 Consumers only care about price

The assumption that consumers only care about price requires that the product only differs in this respect between the firms. If one unit of AB is made up by combining any A with any B (all A:s and all B:s are homogeneous) it could be expected that consumers always choose to buy AB from the firm with the lowest price (firm *f1* in the general analysis above). But when AB is a combined trip, made up of one unit of A and B, there are other dimensions since different units of A respectively B are not homogeneous; they differ both with respect to time and direction.

Below a simple example is used to show the way the combined trips are created.

$A = (A_{1,1},\ldots, A_{1,n}; A_{2,1},\ldots, A_{2,m})$

There are *n* departures in direction 1 and *m* departures in directions 2

$B = (B_{1,1},\ldots, B_{1,i}; B_{2,1}, \ldots, B_{2,j})$

There are i departures in direction 1 and j departures in directions 2

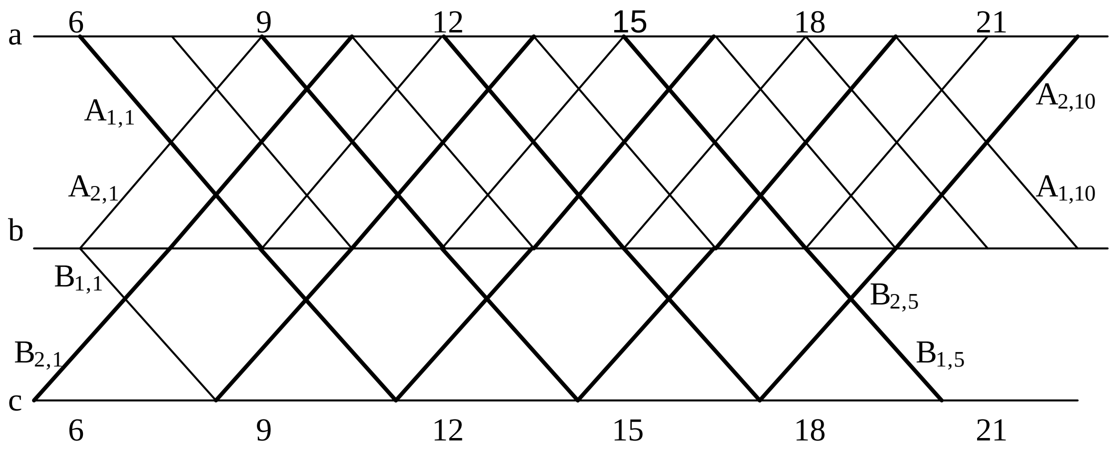
Often $i=j$ and $m=n$ but it is not necessarily so.

$AB = (AB_{1,1},\ldots, AB_{1,k}; AB_{2,1},\ldots, B_{2,l})$

Number of departures in direction 1 = $k \leq \min(n, i)$

Number of departures in direction 2 = $l \leq \min(m, j)$

Below this is shown by the use of graphical time-tables.



$AB_{1,1} = A_{1,1} + B_{1,2}$
$AB_{1,2} = A_{1,3} + B_{1,3}$
$AB_{1,3} = A_{1,5} + B_{1,4}$
$AB_{1,4} = A_{1,7} + B_{1,5}$

$AB_{2,1} = B_{2,1} + A_{2,2}$
$AB_{2,2} = B_{2,2} + A_{2,4}$
$AB_{2,3} = B_{2,3} + A_{2,6}$
$AB_{2,4} = B_{2,4} + A_{2,8}$
$AB_{2,5} = B_{2,5} + A_{2,10}$

A: $n = m = 10, n+m = 20$

B: $i = j = 5, i+j = 10$

AB: $k = 4, l = 5, k+l = 9$

In the above example there is a high degree of coordination between A and B. Since coordination is not without costs, the degree of coordination will be dependent on the gains, which here is the revenue from AB.

In the general case discussed in section 2 above, it was shown that the incumbent was able to offer the lowest price for the composite product AB after entry in every single case. When it comes to the question of railway passenger services we see that the lower price of the combined trip AB is available only from *f1* which, after entry, only supplies half of A. If the lower price comes together with reduced coordination, implying increased travelling time, it is no longer clear that passengers always prefer the lowest price. In such a case, it is very well possible that revenue from the low profitability routes will decrease which eventually leads to a reduced network (only A).

4. Summary and conclusion

In a network consisting of components and composite goods, and where there is demand for the components as well as for the composite goods, the interdependences lead to restrictions on prices generated by the demand side. These restrictions imply that optimal prices will be different from prices of independent products. In the special case analysed in this paper the interdependences forces the monopoly price of the composite product to be set above the unrestricted monopoly price. At the same time the monopoly price of the high demand component will be lower than the unrestricted monopoly price.

Competition on the high demand market may weaken the price restriction and allow the price of the composite product to be set closer to the unrestricted revenue maximising level. Under certain assumptions this implies increased revenue on low profitable markets.

When we try to implement the above finding to the specific context of railway passenger services it becomes obvious that they are not directly comparable. The most obvious difference is the non-homogeneity of different units of the goods, where direction as well as location in time are important attributes. To have the positive effect in form of lower prices (and no quality reduction) a large degree of coordination is required for the firm that supplies both high demand and low demand products. If coordination after entry is reduced entry may instead lead to higher prices and/or reduced quality for transfer trips.

2.5 Entry on the profitable market, case 2

The only thing that is different in case 2 from case 1 above is that $p^*_B > \overline{p}_{A,AB}$ ($m_1 > 0$ and $m_2 > 0$). The profit maximising price and the minim size of a_{AB} in case 2 are shown in section 2.3 (equations (5) and (6)). Below these are replicated.

$$p_A = p_B = p_{AB} = \overline{p}_{A,\,B,\,AB} = \frac{a_A b_B b_{AB} + a_B b_A b_{AB} + a_{AB} b_A b_B}{2(b_B b_{AB} + b_A b_{AB} + b_A b_B)} \tag{5}$$

$$a_{AB} \geq \frac{1}{b_A b_B}\left(\sqrt{b_{AB}\left(b_A a_B{}^2 + b_B a_A{}^2\right)\left(b_B b_{AB} + b_A b_{AB} + b_A b_B\right)} - b_{AB}(b_A a_B + b_B a_A)\right) \tag{6}$$

Entry occurs on A which now is operated by two firms, *f1* (the incumbent who also operates B) and *f2*. The same assumptions as in case 1 are used.

After entry on A the duopoly price is equal to $p'_A = \frac{a_A}{3} < p^*_A$

- **Case 2.1** $p'_A > \overline{p}_{A,\,B,\,AB}$

The new duopoly price of A, p'_A, is higher than or equal to the pre-entry price, $\overline{p}_{A,\,B,\,AB}$. The incumbent, *f1*, will continue to charge $\overline{p}_{A,\,B,\,AB}$ which has to be chosen also by the entrant, *f2*. The only thing that happens is that revenue from A is shared between the two firms.

- **Case 2.2** $p'_A < \overline{p}_{A,\,B,\,AB}$

$$\underset{p_B,\,p_{AB}}{Max}\,p\,(B, AB) = p_B \cdot q_B(p_B) + p_{AB} \cdot q_{AB}(p_{AB}) - F_B$$

$s.t.$ $p_{AB} \geq p'_A$

$p_{AB} \geq p_B$

$p_{AB} \leq p'_A + p_B$

$$L = p_B\left(\frac{a_B - p_B}{b_B}\right) + p_{AB}\left(\frac{a_{AB} - p_{AB}}{b_{AB}}\right) - g_1(p_{AB} - p'_A) - g_2(p_{AB} - p_B)$$
$$- g_3(p'_A + p_B - p_{AB}) \qquad (21)$$

However, since $p*_B > \overline{p}_{A,\,AB}$ we know that $p*_B > p*_{AB}$. This means that $g_2 = 0$ is not possible and therefore the number of alternatives are reduced to two

Case 2.2.1 $g_1 = 0$ **,** $g_2 > 0$ **,**$g_3 = 0$

$g_1 = 0$ implies that $p'_A \leq p*_{AB}$
$g_2 > 0$ implies that $p*_B > p*_{AB}$
$g_1 = 0$ implies that $p'_A \leq \overline{p}_{B,\,AB}$

$$p_B = p_{AB} = \overline{p}_{B,\,AB} = \frac{a_B b_{AB} + a_{AB} b_B}{2(b_B + b_{AB})}$$

Table 7: Post-entry prices in case 2.2.1

Product	Prices *f1*	Prices *f2*
A	p'_A	p'_A
B	$\overline{p}_{B,\ AB}$	
AB	$\overline{p}_{B,\ AB}$	$p'_A + \overline{p}_{B,\ AB}$

Firm *f1* will offer the lowest price for AB. Total revenue for *f1* (from B and AB) will be changed by the following:

$$\Delta TR_{B,\ AB} = \frac{b_B b_{AB}\left(a_B b_{AB} + a_{AB} b_B - a_A (b_B + b_{AB})\right)^2}{4(b_B + b_{AB})(b_A b_B + b_A b_{AB} + b_B b_{AB})^2} > 0 \tag{22}$$

Case 2.2.2 $g_1 > 0$ **,** $g_2 > 0$ **,**$g_3 = 0$

$g_1 > 0$ implies that $p'_A > p^*_{AB}$
$g_2 > 0$ implies that $p^*_B > p'_A$

$p_B = p_{AB} = p'_A$

Table 8: Post entry prices in case 2.2.2

Product	Prices *f1*	Prices *f2*
A	p'_A	p'_A
B	p'_A	
AB	p'_A	$2\ p'_A$

Also in this case firm f1 will always be able to offer the lowest price of AB. If *f2* reduces the price of A in order to compete for consumers of AB, then *f1* will choose the set the price of AB equal to p^*_B. The only possibility for *f2* to have the same price as *f1* is of course to set $p_A = 0$ which is not a probable outcome.

The change in revenue from B and AB for *f1* is equal to:

$$\Delta TR_{B,\,AB} = \frac{\begin{pmatrix} a_A b_B^2 b_{AB}^{\;2}\left(5a_A(b_B + b_{AB}) - 6(a_B b_{AB} + a_{AB} b_B)\right) - \\ b_A\left(b_A b_B + b_A b_{AB} + 2b_B b_{AB}\right)\left(2a_A(b_B + b_{AB}) - 3(a_B b_{AB} + a_{AB} b_B)\right)^2 \end{pmatrix}}{36 b_B b_{AB}\left(b_A b_B + b_A b_{AB} + b_B b_{AB}\right)^2} \tag{23}$$

The expression (23) for $\Delta TR_{B,\,AB}$ is rather messy. The intuition for the direction of change in revenue is however straightforward. Since the pre-entry price $\overline{p}_{A,\,B,\,AB}$ is lower than $p^*{}_B$ and higher than $p^*{}_{AB}$ revenue from B will decrease and revneu from AB will increase.

$$\frac{\partial \Delta TR_{B,\,AB}}{\partial a_B} = \frac{b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})\left(2a_A(b_B + b_{AB}) - 3(a_B b_{AB} + a_{AB} b_B)\right) - a_A b_B^{\;2} b_{AB}^{\;2}}{6b_B(b_A b_B + b_A b_{AB} + b_B b_{AB})^2}$$

$$\frac{\partial \Delta TR_{B,\,AB}}{\partial a_{AB}} = \frac{b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})\left(2a_A(b_B + b_{AB}) - 3(a_B b_{AB} + a_{AB} b_B)\right) - a_A b_B^{\;2} b_{AB}^{\;2}}{6b_{AB}(b_A b_B + b_A b_{AB} + b_B b_{AB})^2}$$

$$\frac{\partial^2 \Delta TR_{B,\,AB}}{\partial a_B^{\;2}} = \frac{-b_{AB} b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})}{2b_B(b_A b_B + b_A b_{AB} + b_B b_{AB})} < 0$$

$$\frac{\partial^2 \Delta TR_{B,\,AB}}{\partial a_{AB}^{\;2}} = \frac{-b_B b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})}{2b_{AB}(b_A b_B + b_A b_{AB} + b_B b_{AB})} < 0$$

$$\frac{\partial^2 \Delta TR_{B,\,AB}}{\partial a_B \partial a_{AB}^{\;2}} = \frac{-b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})}{2(b_A b_B + b_A b_{AB} + b_B b_{AB})} < 0$$

The function is strictly concave in both .

$max\ a_B = a_A$

$$min\ a_B = \frac{a_A b_{AB} + a_{AB} b_A}{b_A + b_{AB}}$$

$$max\ a_{AB} = \frac{2}{3} a_A$$

The minimum value of a_{AB} will be the highest of the following. Which one is highest be

$$\min\ a_{AB} = \frac{1}{b_A b_B}\left(\sqrt{b_{AB}\left(b_A a_B{}^2 + b_B a_A{}^2\right)\left(b_B b_{AB} + b_A b_{AB} + b_A b_B\right)} - b_{AB}(b_A a_B + b_B a_A)\right) \quad (6)$$

$$\min\ a_{AB} \geq \frac{a_A b_B(2b_A - b_{AB}) + b_A b_{AB}(2a_A - 3a_B)}{3b_A b_B} \quad (6)$$

Start with max aAB+max aB

$max\ a_{AB}$

$max\ a_B$

$$\Delta TR = \frac{a_A{}^2 b_{AB}(b_A + b_{AB})(b_B(b_B - b_{AB}) - b_{AB}(b_A + b_B))}{36 b_B (b_A b_B + b_A b_{AB} + b_B b_{AB})^2}$$

The sign of $\Delta TR_{B,\ AB}$ will therefore depend on the relative size of the slope parameters. If

$$b_B \geq \frac{b_A + b_{AB} + \sqrt{4b_A b_{AB} + (b_A + b_{AB})^2}}{2} \text{ then } \Delta TR \geq 0$$

$max\ a_{AB}$

$min\ a_B$

$$\Delta TR = \frac{a_A{}^2 b_{AB}(b_B - b_{AB})}{36 b_B (b_A + b_B)}$$

which obviously depend on the relative size of b_B and b_{AB}.

$min\ a_{AB}$

$min\ a_B$

The minimum size of a_{AB} will be

1) $a_{AB} \geq \dfrac{a_A(2b_A - b_{AB})}{3b_A}$

2) $a_{AB} \geq \dfrac{a_A(\sqrt{b_{AB}(b_A + b_{AB})} - b_{AB}}{b_A}$

When $b_A \geq \dfrac{5}{4} b_{AB}$ the minimum size of a_{AB} is according to 1).

In this case $\Delta TR_{B,\ AB} = 0$

When $b_A < \dfrac{5}{4} b_{AB}$ the minimum size of a_{AB} is according to 2). By differentiating the expression for $\Delta TR_{B,\ AB}$ with respect to b_A and evaluate the sign it can be determined whether the change in revenue will be positive or negative for this value of miminim a_{AB}.

$$\frac{\partial \Delta TR_{B,\ AB}}{\partial b_A} = \frac{-b_B b_{AB}\left(3a_A(b_B + b_{AB}) - 3(a_B + a_{AB} b_B)\right)^2}{18(b_A b_B + b_A b_{AB} + b_B b_{AB})^2} < 0$$

Lower values of b_A will therefore imply increased change in revenue and therefore $\Delta TR_{B,\ AB} > 0$.

In this case with $min\ a_B$ and $min\ a_{AB}$ $\Delta TR_{B,\ AB} \geq 0$

$min\ a_{AB}$
$max\ a_B$

At max aB the minim values of aAB becomes

1) $a_{AB} > \dfrac{a_A\left(2b_A b_B - b_{AB}(b_A + b_B)\right)}{3b_A b_B}$

2) $a_{AB} \geq \dfrac{a_A\left(\sqrt{b_{AB}(b_A + b_B)(b_A b_B + b_A b_{AB} + b_B b_{AB})} - b_{AB}(b_A + b_B)\right)}{b_A b_B}$

The break-even point is where $b_{AB} = \dfrac{4b_A b_B}{5(b_A + b_B)}$

When $b_{AB} \le \frac{4b_A b_B}{5(b_A + b_B)}$ $min\ a_{AB}$ is equal to 1) above. And in this case $\Delta TR_{B,\ AB} = 0$

Which minimum value that is largest will depend on the relation between the slope parameters. If $b_B \ge \frac{5b_A b_{AB}}{4b_A - 5b_{AB}}$ then aAB min = the second value. In this case $\Delta TR = 0$.

When b_{AB} is equal to the break-even size the minimum value of 1 and 2 is the same. When b_{AB} is higher than the break-even size the value of $min\ a_{AB}$ is higher. $\frac{\partial min\ a_{AB}}{\partial b_{AB}} > 0$. By differentiating the expression for $\Delta TR_{B,\ AB}$ with respect to a_{AB} and evaluating this at break-even size (minaB, minaAB) it can be seen that . if this is positive, it can be concluded that the change in revenue will be positive . $\frac{\partial \Delta TR_{B,\ AB}}{\partial a_{AB}} = \frac{a_A b_B}{6(b_A b_B + b_A b_{AB} + b_B b_{AB})} > 0$

Therefore, $\Delta TR_{B,\ AB} > 0$

To summarise, in this last case, when demand for AB is low the change in revenue will be non-negative. When demand for AB is high the sign of $\Delta TR_{B,\ AB}$ will depend on the slope-parameters.

than the break-even size the value of minimum aAB

By differentiating the expression for $\Delta TR_{B,\ AB}$

When minimum value is determined by the first value the sign of ΔTR again depend on the relative sizes of the slope parameters.In general, low values of bB yields negative change in revenue while high values yiled positive change in revenue.

The sign of $\Delta TR_{B,\ AB}$ will be indeterminable, conditional .

Ass a general rule the higher demand is for B and AB the greater is the loss in revenue since the pre-entry price will be high.
It will also depend on the relative size of bB. The

At min aB and max aAB

$$\Delta TR = \frac{aA^2 bAB^2 (bB - bAB)\left((bB + bAB) bA (bAbB + bAbAB + 2bBbAB) + bB^2 bAB^2\right)}{36(bA + bAB)^2 bBbAB(bAbB + bAbAB + bBbAB)}$$

if bB0bAB delta TR=0
bB>bAB delta TR>0

max a_B

$$\Delta TR = \frac{\begin{array}{l} a_A b_B{}^2 b_{AB}{}^2 (b_B(5a_A - 6a_{AB}) - a_A b_{AB}) - \\ b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})(b_B(2a_A - 3a_{AB}) - a_A b_{AB})^2 \end{array}}{36 b_B b_{AB} (nnn)^2}$$

The sign will depend on the size of aAB (higher aAB more negative change in revneu)) as well as on the size of bB relative bA and bB (higher bB more positive change in revenue)
Low aAB and high bB positive change in revenue
High
Max aAB

	bB	
	Low	high
aAB		
low	-	+
high	-	+

min a_B

$$\Delta TR = \frac{\begin{array}{l} a_A b_B{}^2 b_{AB}{}^2 (bA + bAB)((5a_A - 6a_{AB})(bAbB + bAbAB + bBbAB) - a_A b_{AB}{}^2) - \\ b_A(b_A b_B + b_A b_{AB} + 2b_B b_{AB})((2a_A - 3a_{AB})(bAbB + bAbAB + bBbAB) - a_A b_{AB}{}^2)^2 \end{array}}{36(bA + bAB)^2 b_B b_{AB} (nnn)^2}$$

	bB	
	Low	high
aAB		
low	+	+
high	-	+

This case is not representative for our main purpose since it requires demand for B to be very high or close to demand for A.

By inserting the value of *max* a_B in the first order partial derivative (24) it can be shown that the slope is negative; $\frac{\partial \Delta TR_{B,\,AB}}{\partial a_B} = -\frac{\left(a_A b_B b_{AB}{}^2\right)}{6\left(b_A b_B + b_A b_{AB} + b_B b_{AB}\right)} < 0$

With regard to *min* a_B this will depend on the size of a_{AB} in the following way:

$$a_{AB} \le \frac{a_A(2b_A - b_{AB})}{3b_A} \rightarrow \quad a_B \ge \frac{2a_A b_A(b_B + b_{AB}) - a_A b_B b_{AB} - 3a_{AB} b_A b_B}{3b_A b_{AB}}$$

$$a_{AB} > \frac{a_A(2b_A - b_{AB})}{3b_A} \rightarrow \quad a_B > \frac{a_A b_{AB} + a_{AB} b_A}{b_A + b_{AB}}$$

When $a_B = \frac{2a_A b_A(b_B + b_{AB}) - a_A b_B b_{AB} - 3a_{AB} b_A b_B}{3b_A b_{AB}}$ then $\Delta TR_{B,\,AB} = 0$ and the slope is $\frac{a_A b_{AB}}{6} > 0$. Higher values of a_B implies therefore $\Delta TR_{B,\,AB} > 0$.

In the other case, evaluation of $\Delta TR_{B,\,AB}$ at $a_{AB} = \frac{a_A(2b_A - b_{AB})}{3b_A}$ and $a_B = \frac{a_A b_{AB} + a_{AB} b_A}{b_A + b_{AB}}$ gives $\Delta TR_{B,\,AB} = 0$. Since $\partial a_B / \partial a_{AB} > 0$ and since the slope of the $\Delta TR_{B,\,AB}$ function is $\frac{a_A b_{AB}}{6} > 0$ the change in revenue will be positive for higher values of a_B and a_{AB}.

Appendix 1

Competiion model

Static models of oligopoly

Static models means that only one period is considered. Competition between firms in a market characterised as oligopoly is inherently a setting of strategic interaction. For this reason, it is appropriate to use game theoretical tools in the analysis.

Price competition

Suppose there are two profit-maximising firm (a duopoly). The firms are simultaneously choosing their prices (the price is the strategic decision). The demand function is $Q(p)$ – the products are homogenous. $Q(p)$ is strictly decreasing in p and there exists a $\bar{p} < \infty$ such that $Q(p) = 0$ for all $p > \bar{p}$. With regard to the cost function, this is assumed to be: $c(Q) = cQi$, and $c > 0$

In this setting (Bertrand competition), there is a unique Nash equilibrium, where $p_1 = p_2 = c$. This means that two firms are enough to have the perfectly competitive outcome. In many cases, this is not a plausible prediction of the market outcome. Therefore, the analysis will be altered in the following ways:

- quantity competition
- capacity constraints
- product differentiations

Quantity competition

Here it is assumed that both firms simultaneously decide on quantities. Given these quantities, the price adjusts to clear the market. With quantities as the strategic decision, it is more useful to use the inverse demand function: $p(Q_1+Q_2) = Q^{-1}(p)$. It is assumed that $p'(Q) < 0$ (demand slopes downward), and that $p(0) > c$.

To find the (pure strategy) Nash equilibrium, each firms maximises profits, taking the quantity decision of the others as given. This gives the following result:

$$p'(Q_1+Q_2)\left(\frac{Q_1+Q_2}{2}\right)+p(Q_1+Q_2)=c$$

In equilibrium, under the above assumptions regarding costs and demand conditions, the market price is greater than c and smaller than the monopoly price.

It is often useful to make assumptions about the specific form of the inverse demand function (because it makes it easier to derive and interpret the market outcome). This will be done below for the case of quantity competition.

Inverse demand function: $p(Q)=a-bQ$
Cost function: $C(Q_i)=cQ_i$

The negative sign in front of b is the same as $p'(Q)<0$. The assumption that $p(0)>c$ means that $a>c$. The equilibrium prices and quantities, following from profit maximisation is shown below.

$$Q^n=\sum_{i=1}^{n}q_i=\frac{n(a-c)}{(n+1)b}$$

$$P^n=\frac{(a+nc)}{n+1}$$

Where n is the number of competitors in the industry. If $n = 1$ we get the monopoly outcome, $n = 2$ is the duopoly and so on.

If competition is introduced into the rail passenger network, we have seen that there will be vertical effects (which increase prices) as well as horizontal effects (which reduce prices due to increased competition). The magnitude of the horizontal effect will thus depend on the degree of competition that already exists (from other modes).

This can be interpreted as an oligopolistic market with imperfect substitutes.

In contrast with the Bertrand model, the Cournot mode (quantity competition) displays a gradual reduction in market power as the number of firm increases. However, in many cases firms seem to choose prices, not quantities. “For this reason, many economists have thought that the Cournot model gives the right answer for the wrong reason.” Mas-Colell et.al s 394.

Capacity constraints

To get an alternative interpretation of the Cournot model, we can instead think of the quantity choices as a long-run choice of *capacity,* with the determination of price from the inverse demand function being a proxy for the outcome of short-run price competition given these capacity choices. We can think of this as a two-stage game where the firms first choose their capacity levels and then compete in prices. It can be shown (Kreps and Scheinkman (1983)) that under certain conditions the unique subgame perfect Nash equilibrium in this game is the *Cournot outcome*. Therefore, the Cournot quantity competition captures the long run competition through quantity choice, with price competition occurring in the short run given those levels of capacity.

Product differentiation

In the Bertrand model above, each firm faced an infinitely elastic demand curve. Often, however, consumers perceive differences among the product from different firms. When product differentiation exists, each firm will possess some market power. As was discussed above, rail passenger services will differ according to some characteristics, at least on departure and/or arrival time. There is therefore an inevitable product differentiation inherent in this market.

Product differentiation is often analysed by spatial models of product differentiation, because each firm is identified with an address in product space.

Dynamic models

The static, one-shot nature in the models above is of course rather unrealistic. In reality, there are repeated interactions between firms. Here we consider two identical firms that compete for sales repeatedly, with competition in each period *t* descried by the Bertrand model. There is a discount factor $d < 1$ and each firm attempts to maximise the discounted value of profits. Such a game is a dynamic game of a special kind: repeated game.

The firms play a game in which both will choose a specific price level above the competitive level as long as the other does the same. If one deviates, they will forever charge a price equal to marginal cost. If the discount factor is high enough (meaning that firms care about the future profits) it is possible to sustain any price $p \in [c, p*]$ (a price between marginal cost and the monopoly price level) as a subgame perfect Nash equilibrium. A price above the competitive level is sustainable if and only if the present value of future losses of deviating is large enough relative to the gain from deviation

Therefore, a possible market outcome is a price level above the competitive level, even if firms act as Bertrand competitors.

Stackelberg leadership

In the above analysis it is assumed that firms are symmetric. This is not a realistic assumption with regard to the question of introducing competition into the rail industry. The new firms that possibly will enter will almost surely be much smaller than the incumbent, SJ.

This situation can be analysed according the model of Stackelberg leadership. Firm 1 is quantity leader and will choose its quantity q_1 first. The other firms will thereafter choose their quantities. This game is solved by backward induction.

Assumption

Inverse demand function: $P(Q) = a - bQ$

Cost function: $C_i(Q_i) = cQ_i$

$$Q = \sum_{i=1}^{i} q_i$$

For the case of n firms the equilibrium quantities and price are as follows:

Quantity of leading firm = $q_1 = \dfrac{a-c}{2b}$

Quantity of the follower firms $q_j = \dfrac{a-c}{2nb}$, $j = 2,\ldots,n$

Total quantity $\displaystyle\sum_{i=1}^{n} q_i = \frac{a-c}{2b} + \frac{(n-1)(a-c)}{2nb} = \frac{(2n-1)(a-c)}{2nb}$

Equilibrium price $p = \dfrac{a+(2n-1)c}{2n}$

It follows that this form of competition will lead to higher total quantity and lower equilibrium price than the standard Cournot game.

The choice of model depends on the context of the particular economic situation or industry under examination.

The Cournot model seems to be most appropriate when quantities can only be adjusted slowly, especially when quantity is interpreted as capacity.