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by

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Estimation of Commodity by Commodity IO-Matrices*

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Abstract

In this paper we derive a method for the estimation of symmetric input output tables (SIOTs), that makes it possible to use the commodity technology assumption even when make- and use tables are rectangular. The method also solves the problem of negative coefficients. In the empirical part we derive annual SIOTs in order to evaluate the differences between SIOTs calculated with different methods and the change in technological coefficients over time. To deal with the shortage of annual SIOTs, many empirical researchers use an assumption of constant technological coefficients over time and thus it is important to check whether this is an appropriate assumption or not.

Our results, based on data from Sweden, show that the mean deviation in technical coefficients for different technology assumptions is rather large. However, in a factor content of trade application the impact of different technology assumptions does not seem to be very important. The size of the changes in the technical coefficients over time is found to be rather large and in the application an assumption of constant technical coefficients over time seems to be inappropriate. Thus our main conclusion is that it is important to calculate SIOTs annually.

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1 Introduction

There are two kinds of input-output (IO) models. In the SNA make-use model, industries are allowed to produce more than one commodity.[1] In contrast, in the famous Leontief model (or the symmetric IO-table "SIOT") each industry produces only one commodity and each commodity is produced by only one industry. Furthermore, the SIOT can be divided into two subgroups. They can either be defined as industry-by-industry or commodity-by-commodity, and this paper deals only with commodity-by-commodity SIOTs. In such tables all use of commodities as intermediate inputs is distributed to the production of a specific commodity. If an industry produces more than one output the inputs must be distributed to specific outputs.

In many countries make and use tables are produced annually, while the SIOTs are produced less frequently. In the case of Sweden, the make and use tables have been produced on a yearly basis since 1993, but the SIOTs have only been produced for 1995 and 2000 during the same period. The 1995 table was published in 2003 and the 2000 table was published in 2004 in accordance with a requirement from the EU. For some research purposes, annual SIOTs are needed. One example is studies concerned with the development of the factor content of trade over time where SIOTs are used to capture the intermediate flow of factors of production. If annual official SIOTs are not available, the researcher has to choose between calculating them from make- and use tables, or using an assumption of constant technical coefficients over time, calculated from the official SIOT.

Is an assumption of constant technical coefficients over time appropriate or is it worthwhile to calculate annual SIOTs? More specifically, is the five year interval used by Statistics Sweden sufficient to capture the dynamics of the industry structure? Furthermore, if constant coefficients are assumed, will the most realistic assumption be constant cost shares or constant quantities per unit of output? One purpose of the empirical part of this paper is to answer these questions by evaluating the size of the changes in the technical coefficients that have occurred over time, based on data from Sweden. To evaluate the changes in technical coefficients over time we need to calculate annual SIOTs. However, there are several methods for calculating SIOTs. Consequently, the second purpose of the empirical part is to evaluate the difference between SIOTs computed with different methods. Furthermore, in the theoretical part of the paper we make some developments of the methods of calculating SIOTs. These developments concern the possibility to mix

[1] This model was introduced by the United Nations in its 1968 System of National Accounts.

different assumptions, the ability to avoid negative technical coefficients and the ability to use rectangular make- and use tables when using the commodity technology assumption and the possibility to use an official SIOT in the calculation of the subsequent years.

The paper is divided into six chapters, including this introduction. In the next we derive and describe two of the most common techniques in compiling SIOTs from make- and use tables. In chapter 3 we describe the method used in the paper. Results and sensitivity analyses are presented and discussed in chapter 4. In chapter 5 we apply some of our estimated time series of SIOTs in a factor content of trade framework, and finally, in chapter 6, we conclude and discuss some further developments of the proposed method.

2 Methods of IO-compilation

When the United Nations introduced the SNA make-use model they also proposed two alternative technology assumptions to be used in compiling SIOTs. These were the industry technology assumption (ITA) and the commodity technology assumption (CTA). There has, over the years, been a dispute concerning which assumption to use.[2] Since ITA is inconsistent with economic theory, many economists prefer CTA. A drawback of CTA is however that it often produces negative coefficients.[3]

In order to derive the methods of IO-compilation, we define a variable b_{ijkt} as the quantity of commodity i that is used for producing one unit of commodity j in industry k at year t. Those b-coefficients may be called industry-specific technical coefficients.[4] With these coefficients it is possible to define two relations between make-, use- and SIOTs. The quantity of commodity i used for producing commodity j must be equal to the sum of the use of commodity i that is distributed to output j in all industries producing commodity j. This gives the following equation:

$$z_{ijt} \equiv \sum_{k=1}^{K} b_{ijkt} v_{jkt}, \qquad (1)$$

where z_{ijt} is the total quantity of commodity i that is used for producing commodity j in the whole economy at year t and v_{jkt} is the quantity of

[2] For a review of the relevant literature see Guo, Lawson & Planting (2002) or chapter 2 in Rueda Cantuche J (2004) and references therein.

[3] See for example chapter 11.3 in Eurostat (2002).

[4] This variable should not be confused with the "ordinary" technological coefficients that are defined for the whole economy. In this paper those are denoted a_{ij}. See appendix A for a list of variables used in this paper and their definitions.

commodity j that is produced in industry k at year t.

The quantity of commodity i used in industry k must be equal to the sum of the use of commodity i for all commodities produced in industry k. This gives the following equation:

$$u_{ikt} \equiv \sum_{j=1}^{J} b_{ijkt} v_{jkt}, \quad (2)$$

where u_{ikt} is the total quantity of commodity i that is used in industry k at year t.

The implication of identity (1) is that if the b-coefficients are known, the SIOT can be calculated from the make table. But those coefficients are often difficult to calculate even at the firm level, and even more so for an economist outside the firm. The make- and use tables together with identity (2) give us some more information, but the b-coefficients can not be derived from the make- and use tables as equation (2) is a system of $I \times K \times T$ equations in $I \times J \times K \times T$ unknowns.

In order to be able to solve this system of equations we need some further assumptions. The two main principles, ITA and CTA, will be described in the following subsections.

2.1 The Industry Technology Assumption (ITA)

According to the Industry Technology Assumption (ITA), the same industry uses the same mix of inputs for all its outputs, i.e. the b-coefficients in a specific industry are all equal. This means that the b-coefficients are all equal to their mean, i.e.

$$b_{ijkt} = \bar{b}_{ikt}, \quad (3)$$

where $\bar{b}_{ikt}$ is the mean of all b-coefficients in industry k for input i.

By using equation (3), the number of unknowns in equation (2) is equal to the number of equations. Using this assumption, it is always possible to calculate a SIOT from make- and use tables. The ITA is commonly used in practice but often criticized.[5]

2.2 The Commodity Technology assumption (CTA)

According to the Commodity Technology Assumption (CTA), the same mix of inputs is used for producing a specific product in all industries that produce that specific product, i.e.

[5] See for example Almon (2000) and Jansen & ten Raa (1990).

$$b_{ijkt} = \bar{b}_{ijt} = a_{ijt} \quad , \tag{4}$$

where $\bar{b}_{ijt}$ is the mean of the industry specific technical coefficients. If these are all equal, they also have to be equal to the technical coefficients in the whole economy a_{ijt}. The a_{ijt} can also be defined as z_{ijt} divided by output at basic prices (the column sum in the IO-matrix).

By using equation (4), the number of unknowns in equation (2) is equal to the number of equations only if the number of industries is equal to the number of products, i.e. if the make- and use tables are symmetric. If this is the case, then it is possible to calculate an IO matrix under the CTA assumption.

The CTA may be a little bit more realistic than the ITA even if it is obvious that some commodities are produced using different technologies. One example is electricity produced from hydropower, which naturally does not use the same inputs as electricity produced in nuclear- or coal power plants. Another drawback of this method is that it can produce negative technical coefficients.

In the literature, ITA and CTA are commonly defined using matrix expressions. In appendix B1 we show that these matrix expressions can be derived from our equalities (3) and (4).[6]

3 The method proposed

In this chapter we will describe two different variants of the method proposed. In the first, the SIOTs are calculated using only information from each year separately. In the second we also use a base year SIOT in the compilation. For every subsequent year the base year SIOT will have less and less influence on the compiled SIOTs.

3.1 Estimation of year by year SIOTs

In this subsection we derive a method for estimating year by year SIOTs. The ITA and CTA can be mixed if we relax the assumption of equality in equations (3) and (4). Instead we set up the problem as a problem of minimizing the variance of the b-coefficients. Thus, we can set up a minimization

[6] For a more detailed description of CTA and ITA, as well as methods used in constructing industry-by-industry tables, see chapter 11 in Eurostat (2002).

problem with a weighted sum of these variances as the objective function[7]. The relative importance we want to put on each assumption will determine the weights in the objective function. Those b-coefficients must be consistent with make- and use tables, therefore equation (2) is used as a constraint. The solution to the following i number of minimization problems will be a set of b-coefficients which is then used in equation (1) to calculate the SIOT.

$$\text{Min } \sum_{j=1}^{J_k} \frac{1}{K_{ij}} \left[\sum_{k=1}^{K_{ij}} \left(\mu b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 - \frac{1}{K_{ij}} \left(\sum_{k=1}^{K_{ij}} \mu b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 \right] +$$

$$\sum_{k=1}^{K_{ij}} \frac{1}{J_k} \left[\sum_{j=1}^{J_k} \left(\omega b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 - \frac{1}{J_k} \left(\sum_{j=1}^{J_k} \omega b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 \right] \quad (5)$$

S.T.
$b_{ijk} \geq 0,$
$u_{ik} = \sum_{j=1}^{J} b_{ijk} v_{jk}$

where K_{ij} is the total number of industries producing commodity j and using commodity i, J_k the number of commodities produced in industry k, μ and ω are weights , p_{ik} is the price of commodity i at purchaser prices when purchased by industry k and p_j is producer price index for commodity j at basic prices. The first term in equation (5), referred to as the CTA term in what follows, minimizes the variance of the b-coefficients for a specific output. The second term, referred to as the ITA term, minimizes the variance of the b-coefficients in a specific industry. The weights μ and ω determine the relative importance of the commodity- and the industry technology assumption respectively. On one hand, a value of 0 for μ and a positive value for ω will give us the ITA-model. On the other hand, a value of 0 for ω and a positive value for μ will give us a version of the CTA-model that always produces non-negative coefficients and our method can therefore be seen as an alternative to the solutions of the negative coefficients problem proposed in the previous literature.[8] One other interesting property of this method is the possibility to use rectangular make- and use tables in the calculations.

Up to now we have not discussed the difference between constant cost shares and constant quantity of inputs per unit of output. In minimization

[7]As can be seen in equation (5) below, we do not exactly use the variance as we have K_j and J_k instead of $K_j - 1$ and $J_k - 1$ in the denominator. This is due to programming reasons in order to avoid division by zero.

[8]For a relevant review of this literature, see chapter 4 in Rueda Cantuche (2004).

problem (5) the variables are deflated to constant prices. Then ϵ will determine if we impose CTA and ITA as constant cost shares or as constant quantities. If we impose $\epsilon = 0$ the variance minimized will be technical coefficients in constant prices or quantities. If we impose $\epsilon = 1$ instead we will minimize the variance in cost shares (quantities multiplied by relative prices).

It turns out that, in the case of CTA, $\epsilon = 1$ can be regarded as if firms minimize cost according to a Cobb-Douglas production function and thus have the same cost shares. The interpretation of this is that an industry facing a high price for a specific input due to taxes and trade margins will use less of that input than an industry facing a low price. In the same way, $\epsilon = 0$ can be regarded as if firms minimize costs according to a Leontief production function, meaning that all firms use the same quantity per unit produced regardless of the input prices.

Of course, we are not restricted to using just zero or one as elasticities. We can use any number which we regard as a realistic elasticity of substitution between the specific input and a composite of all other inputs. If we use econometrically estimated elasticities of substitution, we have turned the merely accounting principle of CTA to a method consistent with economic theory. In appendix B2 we present an effort to estimate such elasticities for the Swedish economy based on our data.

3.2 Using information from base year SIOT

In this subsection we use the same method as in the former subsection, but we add a component to our minimization problem, which uses information from an official SIOT (a base year table). If we have a base year SIOT that we believe is very reliable, for example based on detailed survey data where firms have distributed their costs to specific outputs, we can use this information even for the years to come.

To do this, we use an assumption of small changes over time in the technical coefficients to create a third term in the objective function. In this term, which will be referred to as the prediction term, we minimize the squared deviation between the b-coefficients from year t and year $t-1$. To be able to do this we need an estimator of the industry-specific technical coefficients from a base year. By solving minimization problem (5) under the restriction that the b-coefficients shall be consistent with a base year SIOT, we can get a possible set of such estimates. This gives us the following i number of minimization problems for the base year:

$$\text{Min} \sum_{j=1}^{J_k} \frac{1}{K_{ij}} \left[\sum_{k=1}^{K_{ij}} \left(\mu b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 - \frac{1}{K_{ij}} \left(\sum_{k=1}^{K_{ij}} \mu b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 \right] +$$

$$\sum_{k=1}^{K_{ij}} \frac{1}{J_k} \left[\sum_{j=1}^{J_k} \left(\omega b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 - \frac{1}{J_k} \left(\sum_{j=1}^{J_k} \omega b_{ijk} \left(\frac{p_{ik}}{p_j} \right)^{\epsilon_i} \right)^2 \right] \tag{6}$$

S.T.
$b_{ijk} \geq 0,$
$z_{ij} = \sum_{k=1}^{K} b_{ijk} v_{jk}$
$u_{ik} = \sum_{j=1}^{J} b_{ijk} v_{jk}$

The b-coefficients from the solution of this minimization problem can then be used to calculate the b-coefficients for the subsequent years. For those years we use the following minimization problem where the assumption of slow changes over time is imposed. In the minimization problem we include the prediction term that minimizes the squared sum of deviations in b-coefficients from year t to year $t-1$.

$$\text{Min} \sum_{j=1}^{J_{kt}} \frac{1}{K_{ijt}} \left[\sum_{k=1}^{K_{ijt}} \left(\mu b_{ijkt} \left(\frac{p_{ikt}}{p_{jt}} \right)^{\epsilon_i} \right)^2 - \frac{1}{K_{ijt}} \left(\sum_{k=1}^{K_{ijt}} \mu b_{ijkt} \left(\frac{p_{ikt}}{p_{jt}} \right)^{\epsilon_i} \right)^2 \right] +$$

$$\sum_{k=1}^{K_{ijt}} \frac{1}{J_{kt}} \left[\sum_{j=1}^{J_{kt}} \left(\omega b_{ijkt} \left(\frac{p_{ikt}}{p_{jt}} \right)^{\epsilon_i} \right)^2 - \frac{1}{J_{kt}} \left(\sum_{j=1}^{J_{kt}} \omega b_{ijkt} \left(\frac{p_{ikt}}{p_{jt}} \right)^{\epsilon_i} \right)^2 \right] +$$

$$\sum_{j=1}^{J_{kt}} \sum_{k=1}^{K_{ijt}} \left(\delta b_{ijkt} \left(\frac{p_{ikt}}{p_{jt}} \right)^{\epsilon_i} - \delta b_{ijkt-1} \left(\frac{p_{ikt-1}}{p_{jt-1}} \right)^{\epsilon_i} \right) \tag{7}$$

S.T.
$b_{jkt} \geq 0,$
$u_{kt} = \sum_{j=1}^{J} b_{jkt} v_{jkt}$

A high value of δ will give a time series of SIOTs with small changes in the b-coefficients over time, and vice versa with a small value. A high value of δ will also give us a series of SIOTs where we regard the base year SIOT to be very reliable and we want this information to greatly influence the SIOTs for subsequent years. For every passing year, the impact from the base year SIOT will give less and less influence on the result. If we put a very small value on δ we will get almost the same results as if we had computed the SIOTs year by year. $\delta = 0$ give us of course an identical problem as the yearly computation problem. μ and ω will as before determine the relative importance of CTA and ITA.

In the minimization problem, equation (7), we have imposed the same elasticity of substitution in all three parts of the equation. It can be argued that the elasticity in the CTA term will be larger than the elasticity in the prediction term, since the elasticity in the prediction term can be regarded as a short-run elasticity reflecting the substitution possibilities from one year to the other. The elasticity in the CTA term will, at least if differences in taxes and trade margins are relatively constant over time, be a long-run elasticity where the capital stock is adjusted to the relative prices.

How do we interpret the elasticity in the ITA term? As the industry technology assumption is not very compatible with economic theory, it is not easy to evaluate the meaning of ϵ in this term. The main reason for imposing the price quota and the same elasticity here is just to formulate the problem in a consistent way. If we want to impose CTA in cost shares it seems more consistent to also impose ITA in cost shares. As an economist, one may prefer to place a very low value on μ as it is very difficult to motivate why ITA should be used.[9]

4 Results

In the following section we present our empirical results. This section is divided into 5 subsections where in subsection 4.1 we calculate the difference between the official 1995 and 2000 SIOTs, in 4.2 we compare our year by year compilations with the official SIOTs, in 4.3 we use information from the official 1995 SIOT and check whether this improves the estimations of our 2000 SIOTs in comparison with the official 2000 SIOT, in 4.4 we make some further investigations of the impact of different elasticities, and in 4.5 we check the yearly development in a sample of our models.

[9] ITA is the assumption used by Statistics Sweden to compile the official IO-tables for 1995 and 2000.

4.1 Changes in technical coefficients over time

Is it worthwhile to update SIOTs annually or is an assumption of constant technical coefficients over time appropriate? To answer this question we calculated technical coefficients from the official SIOTs of 1995 and 2000 and compared them with each other.[10] We also calculated what the cost shares would have been if the quantity of different inputs per unit of outputs had not changed. To do that, we recalculated the official SIOT from 1995 into 2000 prices and computed technical coefficients from that table. The results of a comparison between every combination of those three matrices are shown in table 4.1.

Table 4.1 Comparison between the official SIOTs from 1995 and 2000.

	TK 95	TK 95 in 2000 prices
TK 95 in 2000 prices	12.46 1377 (46.36)	
TK 2000	25.65 1881 (63.33)	24.54 1864 (62.76)

Notes: The first row in each cell shows the mean deviation of technological coefficients between the compared models. The second row shows the number of technological coefficients with a deviation larger than 10 % with their percentage share within parentheses.

The total change in technical coefficients between the two official SIOTs is seen in the third row of the second column. The mean difference is about 25 percent and 63 percent of the cells have a deviation of more than 10 percent.[11] The implication of this is that the assumption of constant cost shares over time does not seem very appropriate.

What about an assumption of constant quantity of inputs per unit of output? From the third row of the third column we see that if we compare the technical coefficients from the resource use in the official SIOT from 1995, expressed in the price level of 2000, we get almost as bad results as before. Thus, this is not a very appropriate assumption either.

From the second row of the second column we see that if we only impose the price changes and compare the differences in a_{ij}, they are only 12 percent. This means that the changes in relative prices over time are about 12 percent for a mean of all possible combination of commodities. If one regards these as small changes, it may not be very important to keep track of the impacts

[10] Both official IO-tables are compiled in accordance with the European System of Accounts - ESA 1995.

[11] The mean difference is calculated as $\frac{1}{N}\sum_{i=1}^{N}\frac{|a_i-b_i|}{((a_i+b_i)/2)}$, where a_i and b_i represent the same element in the two different matrices that are being compared, and N is the total number of elements in any of the matrices, since they are of equal size.

of price changes.

Our conclusion is that the change over time in technical coefficients is so large that it is worthwhile computing yearly SIOTs.

4.2 Year by year compilations

In this subsection we compute SIOTs according to equation (5). We do not use information from the official SIOT from 1995 but only the yearly make- and use tables. In table 4.2 below we compare SIOTs for 1995 computed year by year using different assumptions with the official 1995 SIOT. In column three we show the mean percentage deviation between technical coefficients. In column four we show the number of technical coefficients that deviates more than 10% from the official 1995 SIOT with their percentage share within parenthesis.

Table 4.2 SIOTs for 1995 computed year by year using different assumptions compared with the official 1995 SIOT.

Year	Model	Percentage deviation of technological coefficients	# cells with a deviation >10%
1995	ITA (1)	9.17	379 (12.76)
1995	ITA (E)	9.31	389 (13.34)
1995	ITA (0)	9.78	424 (14.28)
1995	CTA (1)	33.41	1297 (43.67)
1995	CTA (E)	34.84	1353 (45.56)
1995	CTA (0)	32.94	1307 (44.01)
1995	Mixed (1)	25.93	1188 (40.00)
1995	Mixed (E)	26.34	1217 (40.98)
1995	Mixed (0)	26.51	1209 (40.71)

Notes: In the model column the type of model used are shown together with the elasticities within parenthesis. E denotes econometrically estimated elasticities. The mixed models are calculated with equal weights on ITA and CTA. In the last column the percentage share of technological coefficients with a deviation of more than 10% are shown within parentheses. All models are calculated using equation (5).

When using the ITA in cost shares, i.e. model ITA(1), we are closest to replicating the official 1995 SIOT. This should not come as a surprise since Statistics Sweden use the ITA when producing their table. As also can be seen, the model with ITA in quantity shares, i.e. model ITA(0), is also a good replica of the official table. When we use CTA we can see that the deviation is rather large (around 33 percent) and when we use both ITA and

CTA with equal weights, the deviation is around 26 percent.

Below we show the same comparisons as above for the year 2000.

Table 4.3 SIOTs for 2000 computed year by year using different assumptions compared with the official 2000 SIOT.

Year	Model	Percentage deviation of technological coefficients	# cells with a deviation >10%
2000	ITA	5.74	235 (7.91)
2000	CTA	35.62	1328 (44.71)
2000	Mixed	28.06	1275 (42.93)

Notes: All three models have an elasticity equal to 1. The mixed model is calculated with equal weights on ITA and CTA. In the last column the percentage share of technological coefficients with a deviation of more than 10% are shown within parentheses. All models are calculated using equation (5).

This table shows the same pattern as in the table for 1995. One interesting result shown in the two tables above is that the ITA and CTA give rather different results in the calculations. One conclusion is that SIOTs calculated with either ITA or CTA will probably give different results when used in empirical studies. Another conclusion is that if one prefers the CTA one should not use the Swedish official SIOTs which are calculated using ITA.

4.3 Using the base year official SIOT

In this subsection we use the official 1995 SIOT to compute b-coefficients, which are then used in the computations of SIOTs for subsequent years according to equations (6) and (7).

Table 4.4 SIOTs for 1995 computed using information from the official 1995 SIOT compared with the official 1995 SIOT

Year	Model	Percentage deviation of technological coefficients	# cells with a deviation >10%
1995	ITA	0.87	43 (1.45)
1995	CTA	0.97	44 (1.48)
1995	MIX	0.97	45 (1.52)

Notes: All three models have an elasticity equal to 1. The mixed model is calculated with equal weights on ITA and CTA. In the last column the percentage share of technological coefficients with a deviation of more than 10% are shown within parentheses. All models are calculated using equation (6).

Since we use the official 1995 SIOT as a constraint in the minimization problem, we get small deviations from the official SIOT for all models.[12] In

[12] The differences we get are due to the fact that we have to use an interval for the constraint, i.e. that the b-coefficients should be consistent with the official IO-table from

the table below, we compare our calculations of SIOTs for 2000 with the official 2000 SIOT.

Table 4.5 SIOTs for 2000 computed using information from the official 1995 SIOT compared with the official 2000 SIOT.

Year	Model	Percentage deviation of technological coefficients	# cells with a deviation >10%
2000	ITA	7.20	482 (16.23)
2000	CTA	23.46	1134 (38.18)
2000	MIX	18.97	971 (32.69)

Notes: All three models have an elasticity equal to 1. The mixed model is calculated with equal weights on ITA and CTA. In the last column the percentage share of technological coefficients with a deviation of more than 10% are shown within parentheses. All models are calculated using equation (7).

The pattern in the table is similar to the year-by-year computations in the former subsection. With ITA, the deviation has increased giving that using information from the official 1995 SIOT does not improve the results if our aim is to replicate the official 2000 SIOT. When using CTA we find that the deviation from the official tables is smaller when using information from the official 1995 SIOT. One reason for this reduction in deviation can be the fact that the official 1995 SIOT is computed with ITA, so the computations will be influenced by ITA indirectly.

4.4 The impact of different elasticities

Our results in the previous subsections were not sensitive to the choice of elasticities. However the fact that two SIOTs have almost the same mean deviations from the official SIOTs does not imply that they are equal since they can differ in different ways. If we regard CTA to be the preferred technique, the deviation from the official SIOT based on ITA may not be very important. So, in the following table we do a comparison between different SIOTs from 1995 based on the mix of ITA and CTA calculated with different assumptions on the elasticity, i.e. $\epsilon_i = 1$ constant cost shares, $\epsilon_i = 0$ constant quantity per unit of output or $\epsilon_i =$ econometrically estimated elasticities.

1995, in order to find a feasible solution.

Table 4.6 Impact of different elasticities for the mixed model in 1995.

	ε = 0	ε = 1
ε = 1	4.06 329 (11.08)	
Econ. estimated ε	2.90 192 (6.46)	2.38 166 (5.59)

Notes: The first row in each cell shows the mean deviation of technological coefficients between the compared models. The second row shows the number of technological coefficients with a deviation larger than 10 % with their percentage share within parentheses. All models are calculated using equation (5).

The impacts of different elasticities of substitution do not seem to be very important, indicating that differences in purchaser prices between different industries are small.

4.5 Further investigation of the development over time

In section 4.1 we saw that the deviations between 1995 and 2000 were rather large, but how much do they differ from one year to the next? In table 4.7 we compare the deviations between one year and the next year for a sample of the models. As a comparison, we also show the year-by-year change in the make- and use tables.

Table 4.7 Comparison of the deviation between subsequent years for a sample of calculated models.

Model	95-96	96-97	97-98	98-99	99-00	95-00
Make	5.58 330 (10.52)	5.80 346 (11.03)	5.85 361 (11.51)	5.92 356 (11.35)	6.63 400 (12.76)	10.48 457 (14.57)
Use	9.07 951 (30.33)	8.27 864 (27.55)	8.57 884 (28.19)	9.56 974 (31.06)	9.91 1033 (32.94)	20.07 1689 (53.86)
CTA	18.74 1298 (43.70)	21.30 1331 (44.81)	20.39 1337 (45.02)	21.77 1430 (48.15)	23.22 1501 (50.54)	32.81 1841 (61.99)
ITA	14.51 1192 (40.13)	14.64 1218 (41.01)	15.76 1323 (44.55)	15.61 1372 (46.20)	17.92 1440 (48.48)	27.95 1948 (65.59)
Mixed (eq.5)	14.99 1267 (42.66)	17.10 1407 (47.37)	17.62 1405 (47.31)	18.97 1520 (51.18)	20.94 1569 (52.83)	30.65 1976 (66.53)
Mixed (eq.7)	18.19 1341 (45.15)	15.84 1309 (44.07)	16.99 1363 (45.89)	19.08 1451 (48.86)	21.59 1577 (53.10)	32.84 1956 (65.86)

Notes: All four models have an econometrically estimated elasticity. The model "mixed (eq.5)" is calculated with equal weights on ITA and CTA. The model "mixed (eq.7)" is calculated with equal weights on ITA, CTA and the prediction term. The first row in each cell shows the mean deviation of technological coefficients between the compared years. The second row shows the number of technological coefficients with a deviation larger than 10 % with their percentage share within parentheses.

In table 4.7 we see that the deviation over time in the SIOTs are larger than the changes over time in the use- and make matrices.[13] Furthermore,

[13] If one believes in the Leontief assumption, this may come as a surprise. In this case the use of inputs will be determined from output quantities. The SIOT will be stable

we see that the deviations from one year to the other are almost as large as the deviation over the whole period, indicating that there are a lot of changes back and forth rather than an ongoing trend. This may be a further argument for using annually computed IO-tables. In the table we also see that the ITA seems to produce more stable SIOTs than the CTA does.

5 Applications

In applied research, different parts of the SIOT may be more or less important. Maybe it is the case that just a few of the b-coefficients were changing due to different assumptions about technology, but these coefficients may be of great importance in a specific analysis. One example is coefficients of energy inputs in an environmental analysis. It may also be the case that large proportions of the coefficients are influenced by the technological assumption, when the most important coefficients are not. In general we will expect that coefficients in industries with a large share of complementary products to be more affected by the technology assumption than coefficients in industries that only produce their main commodity.[14]

In order to investigate one aspect of the significance of different technology assumptions, we apply some of the estimated SIOTs into a factor content of trade framework.[15] In this specific application we use information from the whole intermediate part of the SIOT, except sectors 11, 12 and 95, and not a subset of it.[16] According to the Heckscher-Ohlin-Vanek (HOV) theorem we can think of trade as the international exchange of the services of factors of traded goods. The HOV-theorem shows that, if trade is balanced, countries will have an embodied net export of factors in which they have an abundant relative endowment and a net import of factors in which they have a scarce relative endowment, where abundance and scarcity are defined in terms of a factor-price-weighted average of all resources. To show the different impacts

over time and changes in the make matrix will impose changes in the use matrix, as firms have to change their use of inputs when they change their mix of outputs. If we, on the other hand, think that firms have rather stable cost shares over time, the use table will be rather stable. Then the changes in output, i.e. changes in the make matrix, will impose changes in the use of inputs per unit of output, meaning that changes in the make matrix will impose larger changes in the SIOT than in the use matrix.

[14] See Guo, Lawson & Planting (2002).

[15] The data used in this application is described in appendix A. The theoretical model is further described and also used empirically in Widell (2004).

[16] The reason for excluding ISIC rev.3 sectors 12 (extraction of uranium and thorium ores) and 95 (private households with employed personnel) is that there is no activity in those sectors, which make the IO-table uninvertable. Sector 11 (oil and natural gas extraction) is excluded due to non-representative factor input requirements.

of differently specified SIOTs, we calculate the human capital content of trade in high skilled labor in Sweden for the period 1995-2000. The following equation will be used in the calculation,

$$z_{ft} = \frac{\sum_{i=1}^{I} x_{it} a_{ift}}{\sum_{i=1}^{I} m_{it} a_{ift}}, \tag{8}$$

where x_{it} and m_{it} are the share of the ith industry in the total exports and imports at time t respectively, and a_{ift} the total use of factor f per unit of production from the ith industry at time t.[17] The factor used in the calculations is skilled labor, which is measured as labor with at least a post secondary education. The z-measure has a simple interpretation, i.e. the average[18] requirements of a factor f per unit of exchange[19] of exports, compared to the average requirements of the imports. This gives us information about the difference in export- and import structure with respect to a particular factor's intensity in products and services, regardless of the trade balance.

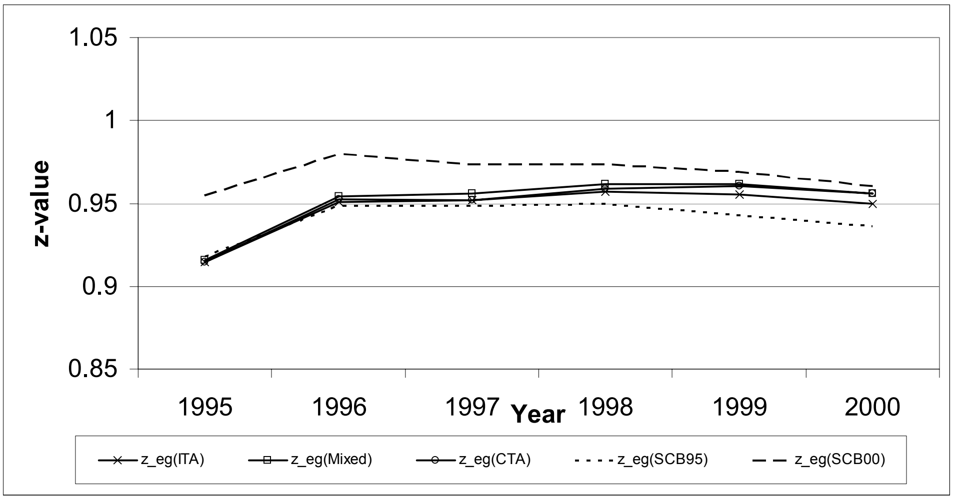
In figure 5.1 below we compare five different z_{ft}-curves calculated using identical trade- and factor input requirements data but using different SIOTs.

[17] The a_{ift} variable in equation (8) is an element in the total factor input requirements matrix, i.e. the direct inputs multiplied by the Leontief inverse, and they are not to be confused with the a_{ij}, i.e. the technical coefficients.

[18] Weighted by trade shares.

[19] In thousands of Swedish kronor.

Figure 5.1 Factor content of skilled labor in Swedish trade for the period 1995-2000 using different SIOTs.



Notes: All curves are calculated according to equation (8). The technological coefficients based on ITA, CTA and Mixed are calculated according to equation (5). The curves z_eg(SCB95) and z_eg(SCB00) are based on technological coefficients calculated from the official 1995 and 2000 SIOTs respectively.

When examining figure 5.1, in what follows, we avoid interpreting the development of z_{ft} over time in economic terms.[20] The intention here is instead to evaluate the performance of our annually compiled SIOTs and compare them with the assumption of constant cost-shares over time, i.e. the curves calculated with official SIOTs. As we can see, all three of the z_{ft}-curves compiled with annual SIOTs behave similarly to each other. However, in 1995 all curves compiled from SIOTs calculated by data from 1995 give similar results, but the curve compiled from the official SIOT from 2000 differ. In year 2000 we have the other way around, since the calculations based on the official 1995 SIOT differ from the curves compiled by data from 2000. In this application, it seems more important to compile SIOTs annually, than which method to use in the compilation of annual SIOTs.

Surprisingly, the curve based on CTA, i.e. z_eg (CTA), is situated closer to the official curve in 2000 then the ITA-curve. Since ITA is the method used by Statistics Sweden in constructing a SIOT, this is not in accordance with our *a priori* expectations. Remember from table 4.3 that the technical coefficients of the ITA model have a mean deviation from the official SIOT of only 5.74% while the CTA model differed by 35.62%. According to table 4.1, the official SIOT from 1995 differs by 25.65% percent from the official SIOT from 2000. So, despite the fact that the CTA has the largest difference in technical coefficients from the official SIOT from 2000, the z_{ft}-measure based on CTA is the one that comes closest to the z_{ft}-measure based on

[20]For an economic interpretation see Widell (2004).

the official SIOT from 2000. Conclusions about the sensitivity of technology assumptions are quite different if we do the evaluation from this application or if we only compare technical coefficients.

6 Conclusions and further developments

When we compare the technical coefficients from SIOTs using different technology weights we find rather large deviations indicating that it is important which technology assumption to use. We also find rather large deviations in the technical coefficients over time, indicating that it is important to compute annual SIOTs. However, only studying the mean deviation in technical coefficients may not be the best way to evaluate differences, as different a_{ij} may be more or less important in different applications.

For the application chosen in this study, it seems that the impact from different compilation methods of the SIOTs has less influence on the results compared to the use of annual SIOTs versus constant SIOTs. This result indicates that it is more important to produce SIOTs annually than what compilation method to choose. This is of course only one application and it will be interesting to see whether this also holds for other applications.

Our results discussed earlier in this study show that the inclusion of the prediction term in equation (7) did not improve the prediction of the 2000 official SIOT. Since a pure ITA calculation comes rather close the official SIOTs, these SIOTs probably do not include much more information. Because of this, it does not make any sense to hold on to the information from the base year SIOT over the years. However, if more detailed survey data over the distribution of costs to specific outputs at plant level were available in specific intervals and included in the derivations of the SIOTs, our prediction term might make more sense.

The impact of different elasticities in our CTA term was not very large. However, our price data are uncertain as they are based on trade margins, which are regarded as uncertain.[21] Moreover, even if the data on trade margins are reliable, it may be the case that price changes between industries do not only occur in the trading sector. It can also be the case that different purchasers face different prices from the producers. If price changes over time are larger than price differences between industries, the treatment of prices is probably most important in the prediction term. Since this term turned out to be unimportant, it may not be important to include prices at all in the computations.

[21] See p. 226 in SOU 2002:118.

The method proposed in this study uses only information from make- and use tables and price indices. However, if more information is available, it is easy to incorporate this into our model. For example, the availability of survey data over some specific subset of the b-coefficients can be included in the minimizing problems as constraints or, if the information is uncertain, as another set of weighted squared deviations in the objective function. The weights for the CTA can be indexed over j if we have information about what products that are produced with a homogenous technology in different industries and what products that are not. In the same way, the weights for the ITA can be indexed over k if we have information about what industries that uses a homogenous mix of inputs for their different products and which industries that do not.

Another possible development is to simultaneously estimate SIOTs for different but similar countries, where an assumption of common production functions will be appropriate. In this case, it will be possible to minimize the deviation of the b-coefficients for different countries adjusted for the country specific relative price. Even if the method is proposed for simple approximation purposes, with the developments described in this section, the method may also be useful for statistical offices.

References

[1] Almon, C, [2000], "Product-to-Product Tables via Product-Technology with No Negative Flows", *Economic Systems Research*, 12, No. 1, pp. 27-43.

[2] Eurostat (2002), *The ESA 95 Input-Output Manual: Compilation and Analysis*, Eurostat (version August 2002).

[3] Guo, J, Lawson, A & Planting, M, [2002] "From Make-Use to Symmetric I-O Tables: An Assessment of Alternative Technology Assumptions", *WP2002-03*, Bureau of Economic Analysis, U.S Department of Commerce, USA.

[4] Jansen, P & ten Raa, T, [1990], "The Choice of Model in the Construction of Input-Output Coefficients Matrices", *International Economic Review*, 31, No. 1, pp. 213-227.

[5] Rueda Cantuche, J, [2004], *Stochastic Input-Output Analysis of the Andalusian Economy*, PhD dissertation, Dep. of Economics and Business Administration, Pablo De Olavide University at Seville, Spain.

[6] SOU (2002:118), " Utveckling och förbättring av den ekonomiska statistiken" by the Commission on the Review of Economic Statistics (in Swedish), Stockholm, Fritzes förlag.

[7] Widell, L, [2004], "On Measurements of the Factor Content of Trade: The Case of Sweden", mimeo, Örebro University.

Appendix

A Definitions and data

The estimation of commodity by commodity IO-matrices in this study are based on data collected from Statistics Sweden. This data contains the following: make matrices at current basic prices, use matrices at current purchaser prices, use matrices at current basic prices, IO-matrices at current basic prices and producer price index.

b_{ijkt}: The quantity of commodity i that is used for producing one unit of commodity j in industry k at year t. N.b! The coefficients b_{ijk} are not always defined, only if commodity j is produced in industry k with the help of commodity input i.

v_{jkt} $V_{[j\times k]}$: The quantity of commodity j that is produced in industry k at year t. (The elements in the make matrix divided by producer price index).

z_{ijt} $Z_{[i\times j]}$: The total quantity of commodity i that is used for producing commodity j in the whole economy at year t. (The elements of the intermediate part of the IO-matrix divided by producer price index).

u_{ikt} $U_{[i\times k]}$: The total quantity of commodity i that is used in industry k at year t. (The elements in the use matrix at current basic prices divided by producer price index).

a_{ijt} $A_{[i\times j]}$: The quantity of commodity i that is used for producing one unit of commodity j in the whole economy at year t. (The technical coefficients computed as z_{ijt} divided by the column sum).

p_j: Price of commodity j at basic prices. (Producer price index).

p_{ik}: Price of commodity i at purchaser prices when purchased from industry k. (They are calculated by element-by-element division of the use table at purchaser prices with u_{ikt}).

q_j: Total quantity of commodity j. (Total output of j or total use of i).

κ_j: Number of industries that are producing commodity j.

τ_k: Number of commodities produced by industry k.

Application part:

Data on imports and exports are taken from the foreign trade statistics, data on factor inputs are taken from the database RAMS (register based labor force statistics) and the financial statistics, all maintained by Statistics Sweden.

z_{ft}: the average requirements of a factor f per unit of exchange of exports, compared to the average requirements of the imports.

a_{ift}: the total use of factor f per unit of production from the ith industry at time t.

x_{it}: the share of the ith industry in total exports at time t.

m_{it}: the share of the ith industry in total imports at time t.

B Derivations

B.1 ITA and CTA in matrix algebra

The matrix expression $\mathbf{Z} = \mathbf{UM}$, commonly used in the literature to define the industry technology assumption, can be derived from equation (3). By substituting $\bar{b}_{ik}$ into equation (2) we get:

$$u_{ik} = \bar{b}_{ik} \sum_{j=1}^{J} v_{jk}.$$

Rearranging this gives:

$$\bar{b}_{ik} = \frac{u_{ik}}{\sum_{j=1}^{J} v_{jk}}.$$

Substituting this into equation (1) gives:

$$z_{ij} = \sum_{k=1}^{K} \bar{b}_{ik} v_{jk} = \sum_{k=1}^{K} \frac{u_{ik}}{\sum_{j=1}^{J} v_{jk}} v_{jk} = \sum_{k=1}^{K} u_{ik} \frac{v_{jk}}{\sum_{j=1}^{J} v_{jk}},$$

which can be written in matrix notation as:

$$\mathbf{Z} = \mathbf{UM},$$

where $\mathbf{M}$ is a matrix calculated by dividing each element in the make table by the row total.

Even the commodity technology assumption can be derived using matrix algebra.

Using the fact that $\bar{b}_{ik} = a_{ij}$ and substituting this into equation (2) gives:

$$u_{ik} = \sum_{j=1}^{J} \bar{b}_{ij} v_{jk} = \sum_{j=1}^{J} a_{ij} v_{jk}$$

In matrix notation the i times k number of equations can be written as:

$$\mathbf{U} = \mathbf{A}\mathbf{V}^{'}$$

The solution to the equation system is:

$$\mathbf{A} = \mathbf{U}\left(\mathbf{V}^{'}\right)^{-1}.$$

This expression gives us the matrix of technical coefficients. The SIOT can then be calculated from the **A**- matrix by element-by-element multiplication with total output of the commodities.

B.2 Econometric estimations of elasticities

In this section we will present our regressions of the elasticities that are used in this paper for the models with econometrically estimated elasticities.

B.2.1 Derivation of the regression equation

The prediction term in minimization problem (7) will, if the squared deviations are minimized to zero, be equivalent to:

$$\left(\frac{p_{i,k,t}}{p_{j,t}}\right)^{\varepsilon_i} b_{i,j,k,t} = \left(\frac{p_{i,k,t-1}}{p_{j,t-1}}\right)^{\varepsilon_i} b_{i,j,k,t-1}, \qquad (9)$$

where ε_i is the short run elasticity of substitution between input i and a composite of all other inputs. Taking logs of both sides of equation (9):

$$\varepsilon_i \ln\left(\frac{p_{i,k,t}}{p_{j,t}}\right) + \ln b_{i,j,k,t} = \varepsilon_i \ln\left(\frac{p_{i,k,t-1}}{p_{j,t-1}}\right) + \ln b_{i,j,k,t-1},$$

and rearranging:

$$\ln b_{i,j,k,t-1} - \ln b_{i,j,k,t} = \varepsilon_i \left[\ln\left(\frac{p_{i,k,t}}{p_{j,t}}\right) - \ln\left(\frac{p_{i,k,t-1}}{p_{j,t-1}}\right)\right]. \qquad (10)$$

Assuming that ε_i is constant over k, j and t, then ε_i can be estimated by OLS from the i number of equations with $k \times (t-1)$ degrees of freedom, by the following equation:

$$\ln \frac{u_{i,k,t-1}}{\sum_{j=1}^{J} v_{j,k,t-1}} - \ln \frac{u_{i,k,t}}{\sum_{j=1}^{J} v_{j,k,t}} = \beta_i \left[\ln \left(\frac{p_{i,k,t}}{p_{k,t}} \right) - \ln \left(\frac{p_{i,k,t-1}}{p_{k,t-1}} \right) \right], \qquad (11)$$

where $\beta_i = -\varepsilon_i$ and p_k is a weighted average of the prices of the commodities produced in industry k.[22] This estimation will probably be biased upwards as the substitution possibilities in an industry are probably higher than in the production of a specific commodity as the industry can also change the mix of outputs.

If the purpose is to estimate the "true" technology this regression is not preferable as we do not include other explanatory variables. For example, the temperature will probably influence the result of the input shares of energy inputs (sectors 23 and 40) and will thus be included if we want to estimate the true production function. In the prediction term this problem may not be very important as we are only interested in the correlation between price changes and changes of the cost shares. If a cold winter results in high prices and high cost shares of electricity and district heating, we will catch this correlation between prices and cost shares even if we are far from the true production function. But this is, of course, not the case for the elasticity in the CTA term as there is no reason to believe that price changes between industries are correlated with these kinds of variables in the same way as price changes over time.

Now, what about the use of these estimates in the CTA term? Here we will have at least three different biases. Firstly, the CTA term will be a long-run elasticity making the elasticity estimates probably biased downwards. Secondly, we still have the problem that the substitution possibilities on the industry level are probably higher than the substitution possibilities for the production of a specific commodity which will give a bias upwards. Thirdly, we have the problem of omitted variables that are probably not correlated with price changes between industries. We have to choose between those estimates and simple assumptions. In this paper, however, we use both and compare the results. One may argue that there will still be a high correlation between those short-run elasticities and the true long-run elasticities, so

[22] Note that the assumption of ε_i to be constant over k, j and t is not an assumption of the b_{ijk} to be constant but only that they have the same relation to the series of relative prices.

that using these estimates will be preferable compared to only assuming all elasticities to be equal to zero or one. An input with large short-run substitution possibilities may also have larger long-run substitution possibilities than an input with small short-run substitution possibilities. For a further development of our method of IO-compilation, it will be interesting to derive elasticities for the CTA-term from micro data using more sophisticated regression methods.

B.2.2 OLS estimates for the elasticities

In the table below, we present the estimated elasticities for the different outputs in the second column with their standard deviation in the third. T-tests are computed both for the assumption of Leontief technology and for the assumption of Cobb-Douglas technology. T-values are presented in column 4 (Leontief Technology) and in column 6 (Cobb- Douglas Technology). The hypothesis of Leontief technology can be rejected if we have a low probability in column 5. The hypothesis of Cobb- Douglas technology can be rejected if we have a low probability in column 7.

As negative elasticities are not compatible with the model and simply make no sense, an elasticity of zero is used instead for those inputs. An elasticity of zero is also used for those inputs where the number of industries using the inputs is so small that we do not find it meaningful to estimate elasticities due to the small sample. It can be argued that we should use en elasticity of zero in all cases where the OLS-estimates do not differ significantly from zero. However, any OLS-estimate at all is still a better linear estimator than the use of zero. Thus one can argue that the OLS-estimate is the “best guess” even if the confidence interval around it will include zero.

Figure B1 Econometrically estimated elasticities

input	epsilon	std	leont-t	leont-p	C-D-t	C-D-p
01	1.2940	0.3105	4.1677	0.0001	0.9469	0.3453
02	0.7083	0.4210	1.6823	0.0972	-0.6928	0.4909
05	1.0795	0.2967	3.6385	0.0007	0.2681	0.7898
10	0.1215	0.2665	0.4560	0.6500	-3.2961	0.0016
11	0.4175	0.1532	2.7242	0.0198	-3.8011	0.0029
14	0.6854	0.1896	3.6151	0.0004	-1.6592	0.0988
15	-0.0328	0.2920	-0.1125	0.9106	-3.5373	0.0006
17	0.7134	0.1957	3.6450	0.0003	-1.4646	0.1445
18	1.2004	0.2191	5.4798	0.0000	0.9147	0.3617
19	0.1442	0.2912	0.4952	0.6214	-2.9387	0.0039
20	0.6685	0.1269	5.2669	0.0000	-2.6120	0.0096
21	0.8791	0.1083	8.1175	0.0000	-1.1159	0.2655
22	0.8496	0.1860	4.5670	0.0000	-0.8085	0.4196
23	0.6511	0.0409	15.9158	0.0000	-8.5303	0.0000
24	0.4514	0.1568	2.8798	0.0043	-3.4995	0.0005
25	0.5034	0.1335	3.7722	0.0002	-3.7208	0.0002
26	0.8260	0.2180	3.7891	0.0002	-0.7983	0.4255
27	0.0464	0.1133	0.4089	0.6830	-8.4141	0.0000
28	0.5229	0.1280	4.0867	0.0001	-3.7286	0.0002
29	0.6226	0.1787	3.4844	0.0006	-2.1119	0.0356
30	1.0834	0.2920	3.7106	0.0003	0.2856	0.7754
31	0.7343	0.1351	5.4357	0.0000	-1.9665	0.0503
32	0.6454	0.1665	3.8774	0.0001	-2.1301	0.0344
33	0.7390	0.1528	4.8369	0.0000	-1.7085	0.0893
34	0.8550	0.2101	4.0690	0.0001	-0.6898	0.4913
35	-0.2268	0.1571	-1.4435	0.1502	-7.8072	0.0000
36	0.4826	0.1845	2.6155	0.0096	-2.8036	0.0055
37	1.7458	0.4525	3.8578	0.0004	1.6480	0.1062
40	0.2977	0.0923	3.2248	0.0014	-7.6063	0.0000
41	0.3233	0.1430	2.2614	0.0248	-4.7335	0.0000
45	0.2786	0.2185	1.2753	0.2033	-3.3018	0.0011
50	0.4343	0.1706	2.5453	0.0115	-3.3150	0.0010
55	0.4418	0.1763	2.5064	0.0128	-3.1662	0.0017
60	0.4035	0.1082	3.7283	0.0002	-5.5109	0.0000
61	1.7290	0.3718	4.6504	0.0000	1.9607	0.0515
62	0.4384	0.1598	2.7429	0.0065	-3.5133	0.0005
63	0.7301	0.1498	4.8740	0.0000	-1.8021	0.0728
64	0.4691	0.0882	5.3174	0.0000	-6.0175	0.0000
65	0.5718	0.1096	5.2186	0.0000	-3.9087	0.0001
66	0.0015	0.1776	0.0082	0.9935	-5.6227	0.0000
67	0.8425	0.1389	6.0640	0.0000	-1.1336	0.2581
70	0.7230	0.2375	3.0438	0.0026	-1.1660	0.2447
71	0.2566	0.1517	1.6912	0.0919	-4.8998	0.0000
72	0.0555	0.1333	0.4166	0.6773	-7.0878	0.0000
73	0.5704	0.2403	2.3739	0.0184	-1.7882	0.0750
74	-0.0568	0.1364	-0.4164	0.6774	-7.7486	0.0000
75	0.0850	0.1295	0.6560	0.5124	-7.0661	0.0000
80	-1.1032	0.3858	-2.8597	0.0048	-5.4519	0.0000
85	0.2847	0.2532	1.1243	0.2619	-2.8252	0.0051
90	-0.3592	0.1954	-1.8388	0.0674	-6.9573	0.0000
91	0.4946	0.1380	3.5855	0.0004	-3.6633	0.0003
92	-0.3147	0.3630	-0.8669	0.3886	-3.6217	0.0005
93	0.4127	0.2978	1.3860	0.1673	-1.9721	0.0500

Notes: All elasticities are estimated with equation (10). Column 4 and 5 are t- and p-values from testing the hypothesis of epsilon equals zero. Column 6 and 7 are t- and p-values from testing the hypothesis of epsilon equals one.