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Risk-adjusted long term social rates of discount for transportation infrastructure investment*

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Abstract

We modify a method recently suggested by Martin Weitzman (2012) for determining a risk-adjusted social discount rate (SDR) term structure consistent with both the (augmented) Ramsey rule and the consumption-based CAPM. Using this approach we estimate SDR for transportation infrastructure investments based on an analysis of correlations between transportation work, split on road and rail, and passenger travel and freight transport, and GDP in Sweden 1950-2011. We show that this can be estimated from two time-series following a random walk with drift, even if they are not co-integrated. Based on current estimates of the risk-free rate and the equity risk premium, we estimate the relevant SDR to be 5-6 percent, possibly somewhat lower for investment in railroads for passenger travel, and only slowly declining within the investment horizon. This is higher than the current rates used in, for instance, Sweden, Germany and the UK.

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Introduction

The single most important economic parameter in economic appraisal of transportation infrastructure investments is the rate of discount. However, while most countries derive this rate within a common theoretical framework, national recommendations vary astonishingly much, from 1 to 15 percent (Harrison, 2010). Also, these rates seldom take account of neither the risk within a project, nor the uncertainty of future economic growth. In this paper, based on an idea suggested by Martin Weitzman (2012), we estimate empirically risk-adjusted social discount rate (SDR) term structures for transportation infrastructure in Sweden.

The two most used theoretical constructs for analyzing the SDR is the Ramsey equation and the consumption Capital Asset Pricing Model (CAPM). The Ramsey rule sets the SDR to the sum of a utility discounting term and a consumption smoothing term, while the CAPM gives the equilibrium rate of return requirement for a risky asset as the sum of the rate of return on a riskless asset and a term compensating for systematic risk, i.e., the risk that cannot be diversified away. While the Ramsey equation is often seen as the natural candidate for analysis of public investments, it is derived within a deterministic framework without consideration of project or macroeconomic risk. Recently it has been shown how this equation can be augmented to account for the uncertainty of overall consumption growth. This uncertainty gives rise to a third term that reflects a precautionary, or insurance-like, aspect of investments as a means for hedging against macroeconomic risk. This research also shows that under a variety of assumptions the SDR term structure is falling, at least in the very long run. Still, the augmented Ramsey equation does not, as the CAPM, account for the systematic project risk. Even more recently however, in response to a challenge from a committee under the Norwegian Ministry of Finance, Martin Weitzman has suggested a way to close the gap between the consumption-based CAPM and the Ramsey rule (Weitzman 2012, Hagen et al. 2012).[1] Using this approach we here derive risk-adjusted SDR for evaluation of investments in transportation

[1] Until 2012, Norway was one of the few OECD countries that used risk-adjusted social disocunt rates, motivated by the CAPM. The new recommendation in 2012 is inspired by Weitzman´s suggested approach.

infrastructure, based on Swedish data. Our contribution here is two-folded. First, we make the first empirical application of this method, which also provides some input to discussions on the choice of SDR for transportation-infrastructure investments. Second, we demonstrate how the Weitzman approach can be implemented to data with various time-series properties.

The benefits of transport infrastructure investments to a large extent depend on traffic volumes, which are likely to be correlated with GDP. Using annual data for GDP and traffic volumes, split on road and railroad, freight and personal travel, respectively, for the period from 1950-2011 we estimate what Weitzman (2012) calls "real project beta" (henceforth "MWbeta") showing expected-value normalized correlations between traffic volumes and GDP. We show that this parameter can be estimated when the traffic volume variable and GDP both follow a random walk with drift, even when the variables are not co-integrated. Moreover we find that for all four measures of traffic volume the value of the parameter is close to one. Therefore transportation infrastructure investment benefits to a large extent replicate the variation in GDP. This conclusion is further strengthened if in addition to this correlation between GDP and volumes, also the willingness to pay for travel time reductions is correlated with GDP. Therefore, although Weitzman's approach generally yields a declining term structure that eventually falls down to the riskless rate level, we find that over the typical time horizon of a transportation infrastructure investment, the relevant SDR remains close to the level of the rate of return required on non-diversifiable wealth. We therefore do not find support for the recent reduction of SDR rates in for instance UK, Sweden and Norway.

In the next section, we give a theoretical background and describe briefly the approach suggested by Weitzman. In section 3 we extend his analysis by showing how the "real project beta" can be estimated from two variables that both follow a random walk with drift even if they are not co-integrated. In section 4, a simple model of the social value of a transportation infrastructure investment is set up. Section 5 describes and analyzes data, and estimates "real project beta" for four types of infrastructure investments. In section 6 we use these results to compute the SDR from

Weitzman's equations and relevant riskless and risk return rates in Sweden. These results are discussed in Section 7 and Section 8 concludes.

2. Theory

2.1 The Ramsey rule and the augmented Ramsey rule

The Ramsey rule was originally derived by Frank Ramsey (1928) who asked how much a nation should save. To answer this he set up a model where social welfare is expressed as the sum of a discounted infinite stream of instantaneous utility $u(c_t)$ from consumption c_t. Maximizing welfare results in a first order condition of the following form (Blanchard and Fischer, Ch. 2):

$$f'(k_t) = \delta + n - \left[\frac{c_t u''(c_t)}{u'(c_t)}\right]\left(\frac{dc_t/dt}{c_t}\right) \qquad (2.1)$$

where $f'(k_t)$ is the marginal productivity of capital, hence the rate of return on a marginal investment, δ is the utility discount rate and n is the population growth rate. The last term in this equation is the consumption smoothing term, accounting for the decrease of the marginal utility of consumption (captured by the second derivative of the utility function) as consumption increases over time. We introduce $r = f'(k_t)$ to represent the SDR, i.e., the rate of return requirement on a marginal public investment; iso-elastic utility with the elasticity of marginal utility represented by the parameter $-\gamma$; and g denoting a constant rate of growth of consumption per capita. The Ramsey equation can then be written in the more familiar form:

$$r = \delta + \gamma g \quad (2.1')$$

This condition for optimization of social welfare thus tells that SDR should equal the utility discount rate and the consumption smoothing term. The first term is the pure time preference, due to unequal weighting of different generations or impatience within a generation, or both. The second term takes care of the fact that an investment is a transfer from the poor (current generation) to the rich (subsequent generations), and adjusts the discount rate in proportion to the expected relative decrease of the marginal utility of consumption due to growth in consumption per capita.[2]

In the Malthusian economy characterizing the neolithic era up to around year 1500, the long term growth of consumption per capita was more or less zero (Bairoch 1993). After the industrial takeoff, an increasing share of the global population is living in nations with positive growth, and it may seem plausible to assume that consumption per capita is growing exponentially at a constant rate. However, infinite exponential growth is not very plausible and if the growth rate eventually declines, it follows from the Ramsey rule that so will the SDR (Sterner 1994). However, more fundamentally, growth rates are divergent across nations and over time. They should therefore not be apprehended as deterministic parameters but as stochastic factors.

Gollier (2008) derives an augmented Ramsey rule for stochastic consumption growth. Assuming that consumption per capita follows an arithmetic Brownian motion process with trend μ and i.i.d. normally distributed stochastic terms with standard deviation (volatility) σ, he comes up with the following version of this rule:

$$r = \delta + \gamma\mu - 0.5\gamma(\gamma + 1)\sigma^2. \qquad (2.2)$$

The third term on the right-hand side is the precautionary effect on SDR. It reduces the SDR and thus to some extent promotes investment (in a safe asset) as a means for hedging against volatile consumption. However, Gollier shows that, at least in the case of the U.S., the standard deviation of GDP growth is small so the reduction of SDR is of second-order importance. Given the uncertainty surrounding the levels of the two terms in the ordinary Ramsey rule, it can perhaps be ignored.

[2] Zuber and Asheim (forthcoming) discusses social discounting in a case where all future generations are not expected to be richer than the present one.

However, if growth rates shocks are persistent, because of serial correlation in error terms or Markow regime-switches (for instance representing the possibility of catastrophic recessions), the precautionary term will not be constant but grow over time. This gives rise to a substantially declining SDR, which means that the precautionary effect will be large when discounting benefits in the far future (Gollier 2008, 2011). As explained by Gollier and Weitzman (2010) a similar effect arises when future social discount rates are uncertain (see also Weitzman 1998, 2001, Newell and Pizer 2003, Hepburn et al. 2009, Freeman and Groom 2012).

2.2 Discounting of risky projects

2.2.1 CAPM

The CAPM developed by Sharpe (1964), Lintner (1965) and Mossin (1966) tells that, in equilibrium, an investor requires a return r_i on a risky asset that is equal to the return r^f on a riskless asset plus a risk premium that is equal to the equity premium $r^e - r^f$ on a fully diversified market portfolio times β_i , the latter factor being the correlation (slope coefficient in a linear regression) of the return on the specific asset with market portfolio return (i.e. the systematic risk). Thus:

$$r_i = r^f + \beta_i \left(r^e - r^f\right) \quad (2.3)$$

where $\beta_i = \frac{Cov(r_i, r^e)}{\sigma_e^2}$.

CAPM is a standard “workhorse” for analyzing financial assets pricing, although there is much evidence that prices of financial assets are influenced by other factors than the beta. However, for use as a guide to the determination of the (risk-adjusted) SDR, CAPM has two major limitations. One is that the model is not fully dynamic (as the Ramsey setting) but only holds for one period; the second that total national wealth encompasses much more than corporate stocks, including non-tradable assets like human capital and infrastructure. The consumption-based CAPM therefore extends the CAPM by focusing on the correlation between the yield from a specific asset and overall economic activity (consumption).

Another important problem is how to relate the Ramsey rule to CAPM. Using a range of plausible values for the parameters of the augmented Ramsey equation and the standard formula for the equity premium,

$$r^m - r^f = \gamma\sigma^2 \text{ , (2.4)}$$

one gets substantially higher risk-free rates, and substantially lower equity premiums, than those actually observed on markets. This is called the riskless rate/equity premium puzzles (Mehra and Prescott 1985, Mehra 2008).

2.2.2 Weitzman´s dynamic model

In a recent paper Martin Weitzman (2012)[3] offers a possible resolution to these puzzles that builds on an idea originally suggested by Barro (2006). Weitzman argues that risk-averse investors expect that low-probability large loss events are somewhat more likely than what is implied by the normal distribution. He shows that more thick tails will both reduce the implied risk-free rate and increase the equity premium. In a ”reverse-engineering” approach he postulates that low-probablity tails are thick enough to reconcile the Ramsey equation with the empirically observed levels of the risk-free rate and equity premium.

Going from this he uses a standard macro finance model (the so-called fruit-tree model) to derive the optimal risk-adjusted discount rate schedule to be applied to a risky project's time-dependent pay-offs . The model assumes, as we did above, constant relative risk aversion and iso-elastic utility from consumption.

The instantaneous net benefit of a single marginal investment project B_t is assumed to be a linear combination of contemporary consumption, C_t , standardized with the expected value, and a

[3] This paper was initially written in response to discussions held at a workshop on the social rate of discount in Bergen May 25-26, 2012, organized by a committee under the Norwegian Ministry of Finance. The committee presented a final report in October, 2012 (NOU 2012:16) on principles for cost-benefit analysis in Norway. The committee suggested SDR levels that were inspired by Weitzman's approach (but without empirical estimation of the "real project beta").

project-specific random variable I_t that is uncorrelated with consumption (which therefore can be made deterministic by diversification over a pool of projects). More specific, the net benefit at time t is

$$B_t = b_t\left((1-\beta_t)I_t + \beta_t \frac{C_t}{E(C_t)}\right), \quad (2.5)$$

where β_t is the proportion of the pay-offs at time *t* that is correlated with aggregate consumption, and therefore is non-diversifiable, while (1- β_t) is stochastically independent of the aggregate economy. The latter component is normalized by setting $E(I_t) = 1$ for all *t*. That implies that expected net benefits at time *t*, are given by $E(B_t) = b_t$.

Weitzman (2012) calls the coefficient β_t the “real project beta”. We will use “MWbeta” to distinguish it from the “regular” CAPM beta. MWbeta is defined as “the *fraction* of expected payoffs that on average is due to the uncertain macro-economy” (Weitzman 2012, p. 15, italics in original). Introducing the rate of return on a risk free asset r^f and risky equity r^e, respectively, Weitzman shows that the discount rate for a project with MWbeta β_t will be

$$SDR_t = -\frac{1}{t}\left[\ln(1-\beta_t)\, e^{-r^f} + \beta_t e^{-r^e t}\right]. \quad (2.6)$$

The MWbeta is used as weight in computing a weighted average of the riskless and risky *discount factors.* Weitzman shows that in the limit as t -> 0, or more precisely when the number of periods is two, the risk adjusted rate of discount is

$$r_0^{\beta_0} = (1-\beta_0)r^f + \beta_0 r^e, \quad (2.7)$$

which is similar to the CAPM equation, but with a different definition of the beta. Thus, in this case the risk-adjusted rate of discount is a (beta-weighted) weighted average of the riskless and risky rates. However, as t increases, the risk-adjusted rate will approach the risk-free rate.

Using this framework Weitzman shows that the term structure of a risk-adjusted social rate of discount will be falling, just as previously has been shown for social discount rates for discounting certainty equivalent net benefits. The basic economic intuition is once again related to insurance against uncertain future prospects of the overall economy. The more the net benefits of a specific project are uncorrelated with the macroeconomic development, the larger will the precautionary effect be. The reason for the declining term structure is however not persistence of growth rate shocks, as these are assumed to be i.i.d. Instead, as in Weitzman (1998, 2001) it emerges out of the computation of a weighted average of the two discount rates using their respective capital value (present value) as weights, which over time gives a stronger relative weight to the riskless rate.

Weitzman notices that this analytical framework may be difficult to apply to the computation of the SDR for a specific public investment as there are no frequent market data for such projects. However, usually an equal level of the SDR is used for all public investments, at least within a category. Here we will therefore estimate SDRs for public investments in transportation infrastructure, assuming that MWbeta is constant for all future periods[4].

3. Analysis and modification of MWbeta

3.1 Defining MWbeta

Consider two random variables, y_t and x_t, and that there is a linear relationship between these two variables, that is:

$$y_t = \mu_0 + \mu_1 x_t + u_t \text{, with } \quad u_t \sim i.i.d.(0, \sigma^2) \qquad (3.1)$$

Note that Weitzman uses a different notation for these two variables, with $B_t \equiv y_t$ and $C_t \equiv x_t$, see Weitzman (2012, eq. 37). Moreover in his analysis, he adjusts the model (3.1) by introduction of first a new random variable:

[4] As we will discuss below, we think that this assumption should be challenged in future research.

$$A_t \equiv \mu + u_t \quad , \tag{3.2}$$

and in the next step defining the MWbeta as :

$$\beta = \frac{\mu_1 E(x_t)}{E(A_t) + \mu_1 E(x_t)}, \tag{3.3}$$

with $\mu = \dfrac{Cov(y_t, x_t)}{Var(x_t)}$.

Using this definition (3.3) in equation (3.1) we have what Weitzman calls the "*weighted average decomposition of variation equation*":

$$\frac{y_t}{E(y_t)} = (1-\beta) A_t + \beta \frac{x_t}{E(x_t)_t} \tag{3.4}$$

Further, using the definition (3.2) in equation (3.4) and for $u_t \sim i.i.d.(0, \sigma^2)$ it is easy to see that we can transform equation (3.1) to the following:

$$\frac{y_t}{E(y_t)} = (1-\beta) + \beta \frac{x_t}{E(x_t)} + \frac{(1-\beta)}{\mu_0} u_t \tag{3.5}$$

With this equation (3.5) we could use the new "mean standardized" variables to directly estimate the MWbeta.

3.2. What values can the estimated MWbeta get?

Weitzman defines the "real project beta" as a "fraction", which seems to imply that the value is between zero and one. However, this is not necessarily so, as we now show.

1. Consider that the linear relationship between the two variables is *positive,* then based on equation (3.3) for

   a) $E(A_t) > 0$, that is, with $\mu_0 > 0$, we have $\beta < 1$

b) $E(A_t) = 0$, and $\mu_0 = 0$, we have $\beta = 1$

c) $E(A_t) < 0$, that means the $\mu_0 < 0$, we have $\beta > 1$

2. If instead the linear relationship between the two variables is *negative,* repetition of the previous analysis based on equation (3.3) gives for

d) $E(A_t) > 0$, and $\mu_0 > 0$, we have $\beta > 1$

e) $E(A_t) = 0$, that means the $\mu_0 = 0$, we have $\beta = 1$

f) $E(A_t) < 0$, that is, the $\mu_0 < 0$, we have $\beta < 1$

As a result of these simple observations and remembering that the two variables have a linear relationship, MWbeta is positive but can exceed unity. Therefore, we need to modify the equation (3.5) to be restricted for $0 < \beta \leq 1$.

3.3 A simple modification

Consider again equation (3.5) with a modification of y_t, x_t so that we have the following equation:

$$y_t^* = (1-\beta) + \beta x_t^* + u_t \tag{3.6}$$

It is well known that the OLS estimation of the constant term is:

$$(1-\hat{\beta}) = \bar{y}^* - \hat{\beta}\bar{x}^* \tag{3.7}$$

From this, is not difficult to see that the modified variables have to have a "mean" equal to one to meet the restriction (3.7), i.e.,

$$\bar{y}^* = \bar{x}^* = 1 \tag{3.8}$$

Now, let us look on another simple relationship in the regression between two variables:

$$\hat{\beta} = r_{x^*,y^*} \frac{s_{y^*}}{s_{x^*}} \qquad (3.9)$$

Based on that relationship, for our MWbeta estimation to meet the restriction $0 < \beta \leq 1$ it must be that:

$$\left(r_{x^*,y^*}\right)\left(s_{x^*}\right) \geq \left(s_{y^*}\right) \qquad (3.10)$$

From this result, we see that for $\left(s_{x^*}\right) = \left(s_{y^*}\right)$ the correlation coefficient is equal to MWbeta.

The simplest way to meet this restriction while maintaining the same relationship between the two variables is to use the following simple transformation:

$$y_t^* = \left[\left(y_t - \bar{y}\right) / s_{y_t}\right] + 1, \quad x_t^* = \left[\left(x_t - \bar{x}\right) / s_{x_t}\right] + 1, \qquad (3.11)$$

In this way both variables have the same means and the same standard deviations equal to one. And in that case the correlation of these two variables should be a good estimate of the MWbeta. But we do not need to make all these data transformations, as MWbeta is just the correlation coefficient in a static regression system.

However, because below we will use time series in regression (3.6) the results become more complicated. In particular, when the times series have a unit root then the estimation of MWbeta has to take care of that aspect too. In the next section we will therefore see how we can extend the analysis of two series that follow a random walk with drift.

3.4 Analysis of a linear relationship between two random walk series with drift

In what follows we give an intuitive explanation of why we can use a regression between two time-series that both follow a random walk with drift, i.e., having a linear trend, even if they are not co-integrated. A complete analysis, based on multivariate VAR models, can be found in Johansen (1996) and Johansen (2010).

Consider again the two random variables y_t and x_t that follow a random walk with drift:

$y_t = \alpha_y + y_{t-1} + e_{yt}$, $e_{yt} \sim i.i.d.(0, \sigma^2)$ and $x_t = \alpha_x + x_{t-1} + e_{xt}$,$e_{xt} \sim i.i.d.(0, \sigma^2)$

and make the transformation :

$$y_t^* = \left[\left(y_t - y_1 \right) / s_{y_t} \right], \quad x_t^* = \left[\left(x_t - x_1 \right) / s_{x_t} \right],$$

With this transformation, both series have the same start value, equal to 0. Also, both have a sample standard deviation equal to one.

Case 1: Co-integration

In the case that the two series are co-integrated, they can be written in an Error Correction Model form:

$$\Delta y_t = \alpha \left(\beta_y y_{t-1} + \beta_x x_{t-1} \right) + a_y + \varepsilon_t$$

Here, the estimation of MWbeta will not be affected by the fact that the two variables follow a random walk with drift. The MWbeta estimated from equation (3.6) converges to:

$$\hat{\beta} \xrightarrow{P} -\frac{\beta_y}{\beta_x} \equiv \beta_{coin}, t \to \infty \tag{3.12}$$

Thus, $\beta_{coin} \equiv \beta$, which makes the two series stationary, that is $\left(y_{t-1}^* - \beta_{coin} x_{t-1}^*\right) \to I(0)$.

Hence, the estimated MWbeta is equal to the estimated “co-integrating beta”.

Case 2: No Co-integration:

Consider again the two modified standardized random variables y_t^* and x_t^* that follow a random walk with drift:

$y_t^* = \alpha_y^* t + \sum_1^T e_{yt}$ and $x_t^* = \alpha_x^* t + \sum_1^T e_{xt}$ with $e_{yt} \sim i.i.d.(0, \sigma^2)$ and $e_{xt} \sim i.i.d.(0, \sigma^2)$.

Now because the linear trend dominates the process, the regression coefficient MWbeta from equation $y_t^* = (\delta) + \beta x_t^* + u_t$, converges to:

$$\hat{\beta} \xrightarrow{P} \frac{a_y^*}{a_x^*}, t \to \infty \tag{3.13}$$

This is an interesting result, because it means that we can estimate MWbeta when two unit root with drift series are not co-integrated directly as:

$$\hat{\beta} = \frac{\hat{a}_y^*}{\hat{a}_x^*}$$

Before turning to empirical estimation, we need to consider the economic problem at hand, i.e., economic evaluation of a transportation infrastructure investment. In the next section we will show a general form of the main element of such an appraisal, i.e., the present value of the discounted net stream of benefits.

4. Discounting of a transport infrastructure project

Public investments in major infrastructure projects are dynamic in nature, and decision making must account for the uncertainty. There are multiple sources of uncertainty, such as uncertainty with regard to traffic demand, deterioration and costs.

The use of the infrastructure, thus the future traffic demand, is obviously a main source of uncertainty. Another important source of uncertainty concern the future relative prices used to value project benefits. The lion´s share of benefits of a road or rail project normally emerge from improvements in travel time durations, travel time reliability, and traffic safety compared to a reference alternative (for instance a "do nothing" alternative). Recent research on the value of travel time savings (Börjesson et al. 2012, Ramjerdi et al. 2012, Abrantes and Wardman 2009) suggests that the willingness to pay for travel time reductions are closely related to income, with an elasticity close to one. Moreover, the value of traffic safety and in particular the value of a statistical life also strongly depends on income and the income elasticity may exceed unity (Hammit and Robinson 2011). Based on such results, Norway, Sweden and the UK have recently revised CBA guidelines, recommending that these economic parameters are assumed to increase over time with the growth of GDP per capita.

Flyvbjerg et al. (2003) find that there is a systematic underestimation of costs (and overestimation of benefits) for so-called mega-projects. However, for relatively standardized projects this might be less of a problem. Even if construction cost overruns are common, costs are relatively close in time compared to future traffic demand. Some countries have, partly in response to Flybjerg´s founding, introduced new procedures where construction costs are recalculated at a late stage of the investment planning process, when more of the real constraints to the project are known than in early stages. Another approach is taken in the UK where instead an “optimism bias” component is added to the calculated construction cost.

Deterioration of roads and railroads, finally, may well be described as a deterministic function dependent on age and (heavy) traffic (Lindberg 2002). Thus road and rail deterioration inherits the stochastic properties from traffic flow and is not an independent stochastic process.

To focus on these sources of uncertainty, consider an infrastructure project *I* with a known upfront cost *I* and a stream of uncertain net benefits $B_t, (t = 1, 2 \dots, T)$ held from the value of the travel-time savings achieved (i.e., the difference between the “do something” and “do nothing” alternatives); discounted with year-specific discount rates r_t. The expected net present value of this project is:

$$E(\pi_i) = \sum_{t=1}^{T} e^{-r_t t} E(B_t) - I = \sum_{t=1}^{T} e^{-r_t t} \{A + [k_i m E(y_t/pop_t)] \cdot E(z_i) E[x_t(y_t)]\} - I \text{ , (4.1)}$$

where A is a constant, representing a constant stream of “other” project benefits not related to time savings. $k_i m \frac{E(y_t)}{pop_t} = k_i E(v_t)$ is the travel-time saved by the project per traveler (k_i) times $E(v_t)$, the expected value of saving an hour of travel time, assumed to be proportional (with the proportion factor *m*) to $E(\frac{y_t}{pop_t})$, the expected GDP per capita. Further, $E[x_t(y_t)]$ is the expected national traffic volume, assumed to be a function of GDP; and $E(z_i)$ is the expected portion of national traffic that will use the specific infrastructure.

From this specification, we see that a possible covariance between project benefits and GDP emerges in two ways; first by the relation between wages (GDP/capita) and value of travel-time savings; second indirectly through correlation between income (GDP) and traffic volumes. A simplified version of this model can be constructed by assuming that that the rate of discount *r* is constant, GDP/capita grows exponentially at rate *g*, the value of time at the constant rate *wg*, and traffic volume at the constant rate *vg.* Eq. 8 can then be expressed in terms of the first year benefits $w_0 v_0 = [k_{i0} m y_0 / pop_{0t}] \cdot z_i \cdot x_0$ as

$$\pi_i = \sum_{t=1}^{T} e^{-rt}\{A + e^{(w+v)gt} w_0 v_0\} - C. \qquad (4.2)$$

In this setting[5] we can define an "effective" or" net" rate of discount as $r(net) = r - (w + v)g$. If *r* is determined by the Weitzman formula, a large covariance between benefits and GDP increases "real project beta" and therefore raises *r*. On the other hand, the difference between *r* and *r(net)* decreases with the growth rate of GDP per capita. We notice these opposite sign effects for later use in discussion of how our results for SDR can be related to current CBA practices in some countries.

In this paper we will build on the specification made in eq. (8) with two modifications. First, we look for a generic, sector-wise, SDR for evaluation of public investment in transportation infrastructure within a country. This means that project specific traffic-volume risk, i.e., the variation of the factor z_i, is assumed to be diversified away. The remaining risk thus depends on the variation in the national traffic volumes and in the value of travel time savings. Second, we will assume that the value of travel-time savings is constant (in relative prices). This is still a standard assumption in transportation infrastructure evaluations, but more importantly, it is a natural research strategy to go step-by-step and to start by analyzing this source of uncertainty. As it turns out from our results, there is no much need to go further.

5. Empirical estimation of MW beta for road and rail transportation in Sweden

5.1 Introduction

Infrastructure investments often have a long investment horizon. Some current roads have an ancient history, for instance, some roads built during the Roman Empire turned out to last for 2000 years. In comparison, the ex-ante investment horizons for road and rail investments used by Swedish

[5] A similar model is used by de Rus (2010) to analyze the costs and benefits of high-speed rail.

authorities are much shorter; 40 and 60 years, respectively. The actual life span of different components of such investments varies; for example, the average life time of road pavements is only 12 years (Haraldsson 2007).

During the life span of a specific object, the nature of the relationship between economic growth and traffic demand may change. Economic growth has both short-term and long-term effects on traffic demand. Some long-run determinants of increased traffic demand such as higher disposable incomes and improved car technology depend on GDP growth. The short-term effect of economic growth on traffic demand is a different and more complex issue. The relationship between economic growth and traffic demand in Sweden was first analyzed in SIKA (2005). The results indicate that the time series of traffic and GDP are not co-integrated and hence that traffic and GDP will not converge to a long-run equilibrium relationship after short-run deviations from each other. Therefore, GDP and traffic do not share a stochastic trend in addition to the deterministic trend exhibited by both time series.

The analysis here uses annual person-kilometer and ton-kilometer data for roads and railroads for the period 1950–2011. With obvious notation, the four variables will be called RoadP, RoadT, RailP and RailT. Ton-kilometer and person-kilometer data for road transportation is obtained from the statistics section of *Trafikanalys* (a government agency). The methods used for constructing these time series are described in SIKA (2004). Table 1 summarizes the descriptive statistics of GDP and traffic volumes seen over the entire sample period 1950–2011.

Table 1: Descriptive statistics of variables 1950 - 2011

Variable	Obs	Mean	Std. Dev.	Min	Max
GDP	*62*	*59.7*	*25.806*	*21.5*	*111.4*
RoadP	*62*	*76.095*	*35.958*	*7.242*	*118.883*
RoadT	*62*	*22.337*	*11.836*	*2.7*	*42.367*
RailP	*62*	*6.693*	*1.781*	*4*	*11.434*
RailT	*62*	*16.356*	*4.117*	*8.64*	*23.464*

Turning to growth rates, road traffic growth in terms of both passenger-kilometers and transport-kilometers was approximately *4.8* percent per annum with a mean deviation of *6* percent. In contrast, rail traffic grew considerably less, by about *0.5* percent per year in terms of passenger-kilometers and *1.7* percent in terms of ton-kilometers. The standard deviation of the growth rate of rail traffic is comparable to that of road traffic (*5.5* percent for passenger-kilometers and *6.8* percent for ton-kilometers).

5.2 Visual analysis

Figure 1 provides regression fit diagrams of the four transportation variables against GDP, showing that there is a linear relation between GDP and both RoadT and RailT. However, there is a quadratic relation between GDP and RailP. This relation is explained by the substitution from rail to car during the first half of the century, which was reversed after the OPEC I and 2 oil price hikes and huge public investment programs at the end of the century that expanded rail capacity.

Further, Figure 2 shows the four variables against the LnGDP. Now even the RoadP vs. Ln(GDP) shows a clear linear relation and the quadratic relation between GDP and RailP is clearer.

Figure 1 Regression fit plots of the four transportation variables vs. GDP, 1950-2011.

Figure 1a RoadP= a+b GDP

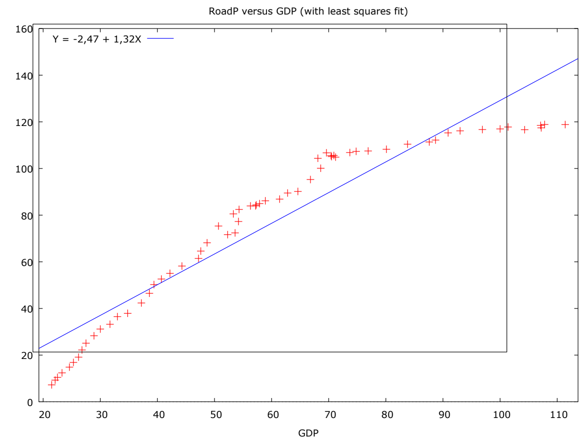


Figure 1b RoadT= a+bGDP

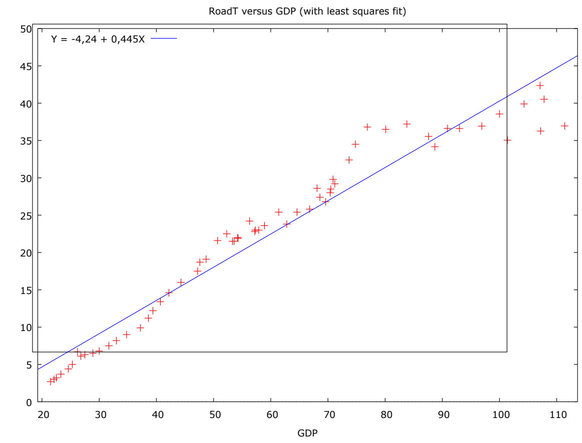


Figure 1c RailP= a+bGDP

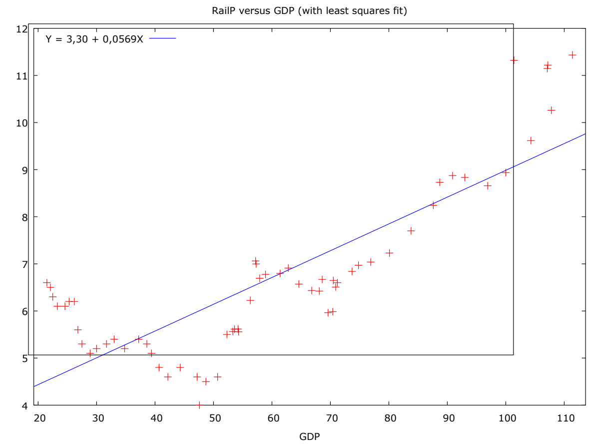


Figure 1d RailT= a+bGDP

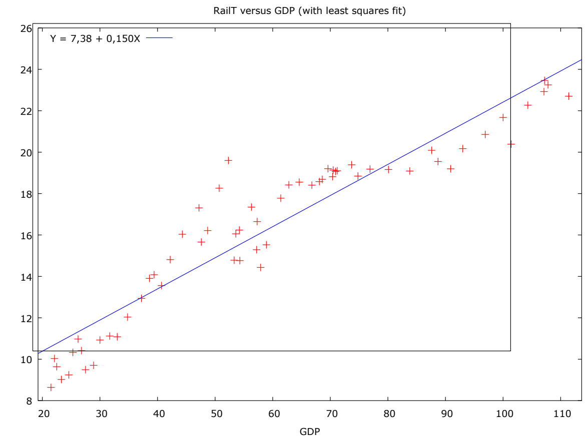


Figure 2 Regression fit plots of the four transportation variables vs. LnGDP, 1950-2011.

Figure 2a RoadP= a+b LnGDP

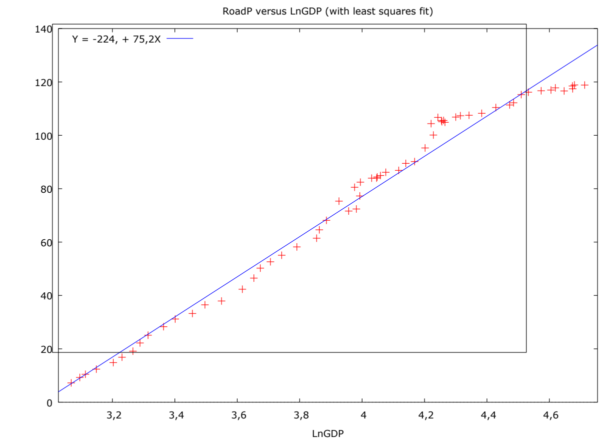


Figure 2b RoadT= a+b LnGDP

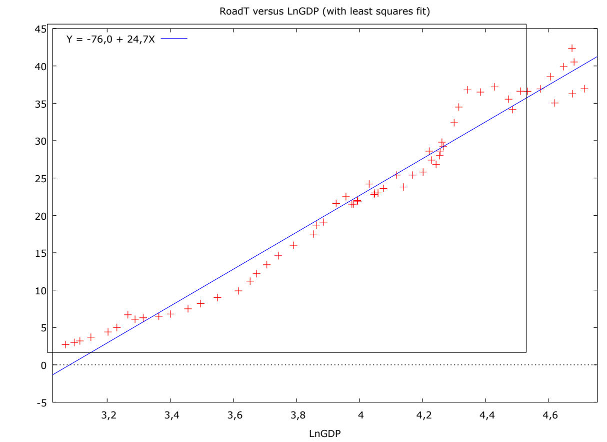


Figure 2c RailP= a+b LnGDP

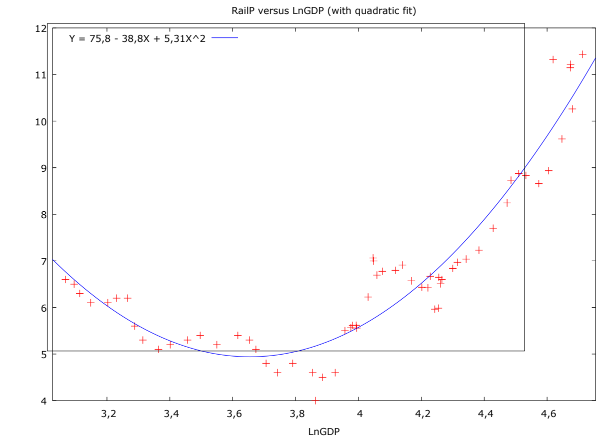
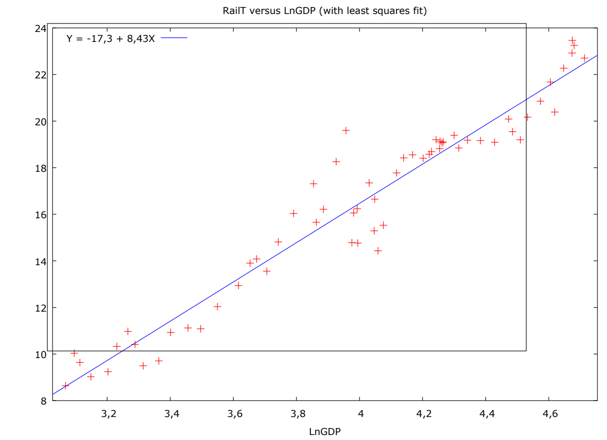


Figure 2d RailT= a+b LnGDP



Finally, Figure 3 shows the “mean standardized” transportation variables mRailP, mRailT, mRoadP, and mRoadT vs. mGDP and mLnGDP, respectively.

Figure 3. The level series transformed to mRoadP, mRoadT, and mRailT, and mRailP vs. mGDP and mLnGDP, respectively, 1950-2011.

Figure 3a mRailP

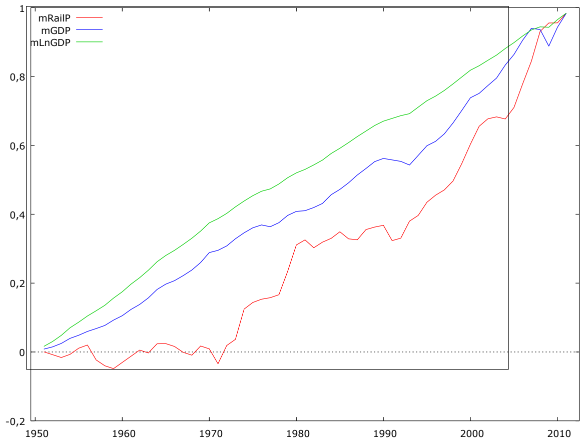


Figure 3b mRailT

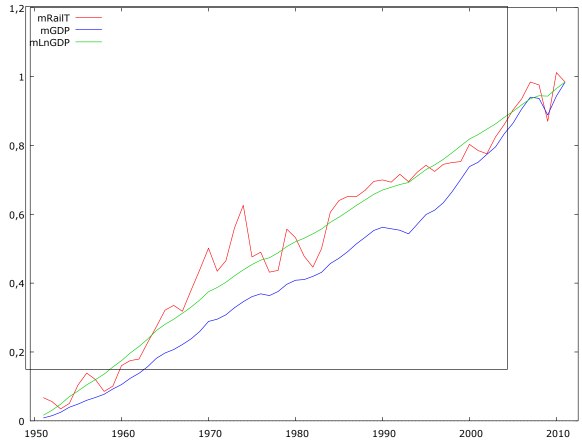


Figure 3c mRoadP

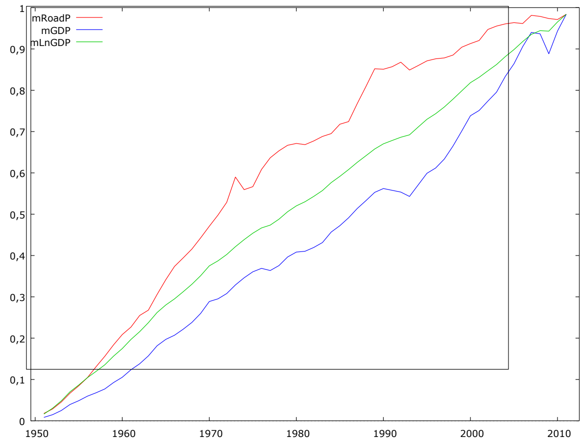
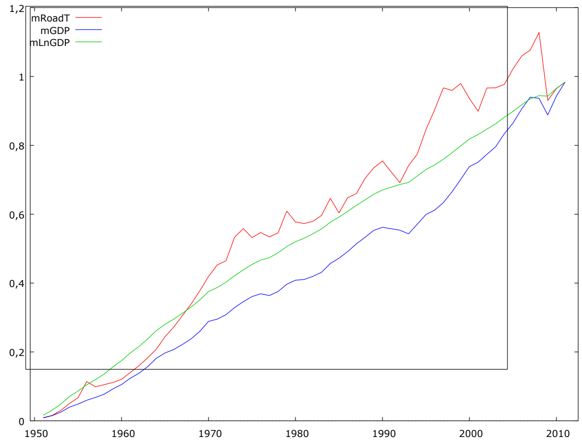


Figure 3c mRoadT



5.3 MWbeta estimates

As was shown in section 3, depending on the nature of the stochastic processes of the time-series we define MWbeta in the following ways:

We have two time series that both follow two random walk with drift: $y_t = \alpha_y + y_{t-1} + e_{yt}$, $e_{yt} \sim i.i.d.(0, \sigma^2)$ and $x_t = \alpha_x + x_{t-1} + e_{xt}$,$e_{xt} \sim i.i.d.(0, \sigma^2)$

1. If the two series are co-integrated with the co-integrating vector $\left(y_{t-1} - b_{coin} x_{t-1}\right)$, then the "beta" is:

$$\hat{\beta} = \hat{b}_{coin} \frac{s_x}{s_y}$$

2. If the two drift series are not co-integrated, then the MWbeta is:

$$\hat{\beta} = \frac{\hat{\alpha}_y}{\hat{\alpha}_x} \frac{s_x}{s_y}$$

In Table 3, we show the estimated MWbeta for the four transportation variables, regressed on GDP or LnGDP, respectively

Table 3. Estimates of MWbeta

	GDP		LnGDP	
	Correlation	$\hat{\beta} = \frac{\hat{\alpha}_y}{\hat{\alpha}_x} \frac{s_x}{s_y}$	$\hat{\beta} = \frac{\hat{\alpha}_y}{\hat{\alpha}_x} \frac{s_x}{s_y}$	$\hat{\beta} = \hat{b}_{coin} \frac{s_x}{s_y}$
RoadP*	0.9450	**0.89112**	**0.89208**	
RailP**	0.8241	**0.77929**	**0.78012**	
RoadT	0.9711	**0.83080**	**0.83169**	
RailT	0.9427	**0.98096**		**0.96643**

* The RoadP vs. LnGDP shows a linear relationship in the regression fit diagrams.
** RailP vs. LnGDP shows a quadratic relationship in the regression fit diagram, so by estimating RailP on LnGDP with a quadratic trend in the regression , the coefficient for LnGDP is 2.16783 and by adjusting with the standard deviations , see (3.9), we have MWbeta =0.57553.

The estimated "betas" range from 0.78 to 0.98, i.e., they are all close to one. In the different cases, we find:

- ***RailT*** is co-integrated with LnGDP, so we use the co-integrated-beta as our beta, which is **0.966.**This estimate is almost equal to the correlation coefficient between RailT and LnGDP: **0.968** (not show in the table), which is about what we could expect.

- The estimated MWbeta for ***RoadT*** is based on the not co-integrated unit root series with drift. We get estimates at about **0.83** both with GDP and LnGDP.
- The estimated MWbeta for ***RoadP*** is based on the not co-integrated unit root series with drift. In this case, estimates are about **0.89** both with GDP and LnGDP. As expected this is less than the correlation coefficient value of 0.945.
- Finally, ***RailP*** vs. LnGDP shows a quadratic relationship in the regression fit diagram, so by estimating RailP on LnGDP with a quadratic trend in the regression, the coefficient for LnGDP is 2.17 and by adjusting with the standard deviations, see (3.9), we have MWbeta at **0.58,** which is what we expect as we look at the time series diagram for RailP and LnGDP. This is of course much less than the correlation coefficient as the relationship is not so linear. Note that with analysis of the two series as unit root with drift the estimated "beta" is **0.78.**

6. Risk-adjusted SDR for transportation infrastructure investment in Sweden

6.1 Introduction

In this section we apply the framework of Weitzman (2012), with the above presented elaboration, to Swedish data. For that purpose we need estimates of r^f, r^e, as well as the MWbeta estimates from the previous section.

6.2 Risk-free rates and risk-equity premiums in Sweden

Real-valued risk-free rates in Sweden can be held directly from market rates of real-value Swedish government bonds. These are issued regularly with a 20-year duration. Current rates (September 2012) for bonds issued to 2028 are around 0,5 percent, while bonds with shorter remaining duration even are traded at negative rates.[6] However, this is an effect of the current Euro

[6] The five-year real-valued bonds emitted by the government of Sweden on September 5, 2012 were sold at an average rate of -0.29 percent.

crisis. Looking back a few years, rates have varied between 1.5 and 2.0 percent. A recent Norwegian government report (NOU 2012:16) estimates that the Norwegian pension funds in ”normal times” are able to find long-term real-valued riskfree funding at 2.0 percent. We will therefore here asssume that the real-valued riskless rate of discount in Sweden is 2.0 percent.

Equity premiums can be estimated ex post on historial data and ex post or ex ante from surveys to market particants. Sörensson (2010) reports the arithmetic mean of historic equity premiums (evaluated as the difference between stock return and return to long-term government bond) to 5.9 percent; the annual mean of the equity risk premium 1998-2010, based on an ex post survey, to 4.4 percent; while ex ante equity risk premium is 4.6 percent. After a discussion of various aspects on the interpretation of these results , the author arrives at the following ”personal judgement”: ”We believe that today (2010) a level close to 4.5 percent is appropriate.” (Sörensson 2011, p. 19).

However, equity corresponds to a market portfolio of corporate shares. Other societal assets include for instance private homes, human capital, etc., assets which are less tradable than financial assets. Grant and Quiggin (2003) show that idiosyncratic and uninsurable human capital risk leads individuals to require an enhanced risk premium for equity, while public ownership of equity may reduce this premium. The reason is that in cases when the outcome of a public investment is worse than was expected, government can raise taxes for additional funding. If these taxes are progressive or proportional, they will to a large extent be paid by owners and workers of successful firms. This would reduce the spread of the distribution within the population of negative shocks. However, we will ignore this possible effect, regarding it as a second-order effect, keeping in mind that a 4.5 percent equity premium assumption may be on the low side given that the long-term historical arithmetic mean is higher. Thus, our main case for the SDR will be based on the r^f and r^e pair 2.0 and 6.5. However, as a sensitivity analysis we will also show results for 2.0 and 5.0.

6.3 SDR estimates

Figure 4 below shows SDR rates at different terms derived from Weitzman´s equation, the assumed rate of return requirement on riskless and risky assets and the MWbeta values that we have estimated for the four transportation variables with our suggested apporach.

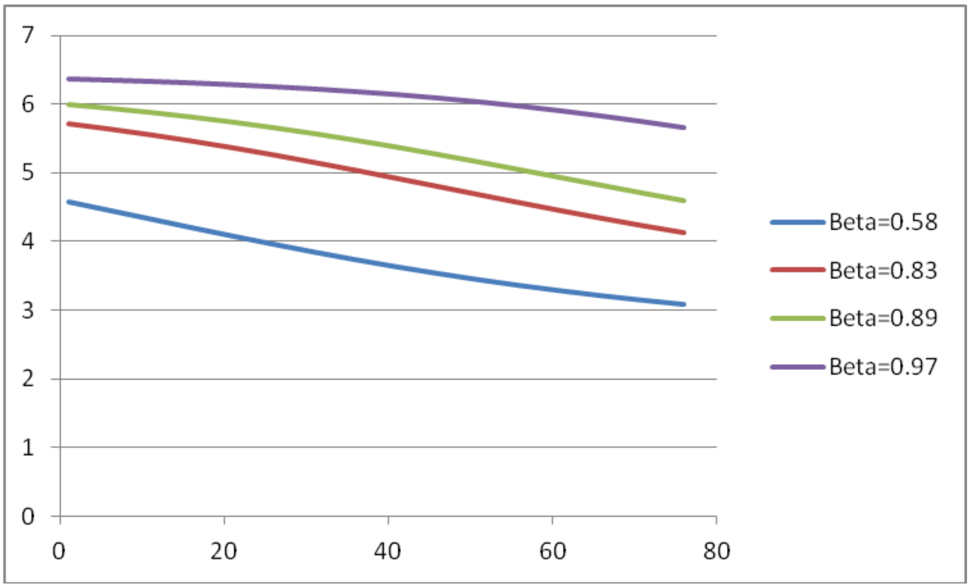
Our result implies that for a *road* project, depending on whether the main benefits emerge from freight transport (lower level) or person travel (higher level), the SDR for short-term benefits (i.e., benefits coming within a few years) should be 5.8 – 6.0. At the end of the investment horizone of such a project (i.e., benefits coming after 40 years), the relevant SDR is 5.0 – 5.4.

For a *rail freight* project, the short term rate is 6.4. By the end of the investment horizon (60 years), the SDR is down to 5.9. For a *rail passenger* project, where the MWbeta is more difficult to assess from our data, SDR declines from 4.6 in the beginning to 3.3 at the 60 years end point. However, if we instead had chosen MWbeta equal to 0.78, the SDR would have been higher, somewhat below the road freight line.

As a sensitivity analysis, Figure 5 shows corresponding results for a lower equity premium of 3.0. Road SDR then drops from 4.5-4.7 to 4.2-4.4. Rail freight transport SDR stays just below 5 (i.e., the equity rate in this case), while rail passenger travel SDR goes from 3.7 to 3.2.

Figure 4. SDR levels for 0 – 76 years at estimated MWbeta values. $r^e = 2.0, r^f = 6.5$.

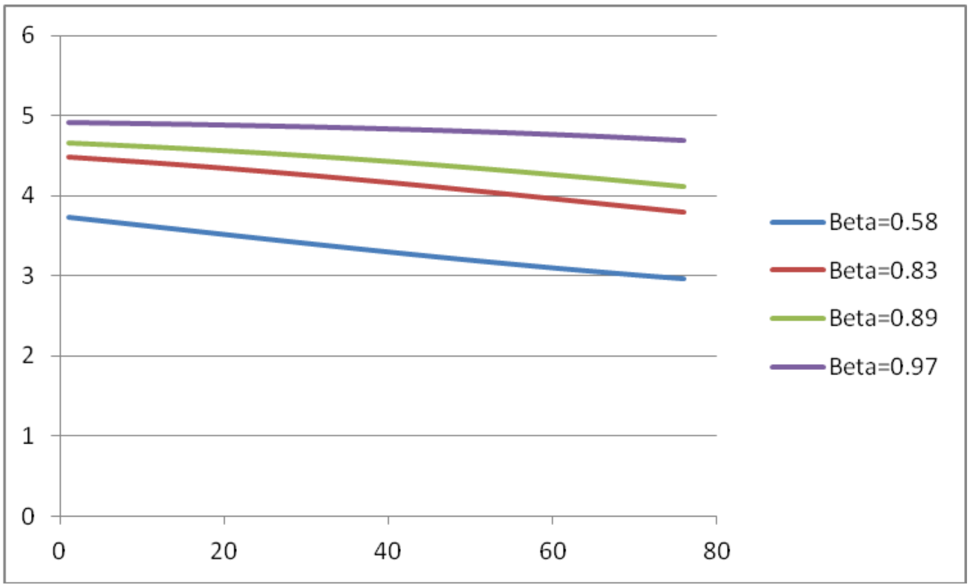
Rate of discount (%)



Years

Figure 5. SDR levels for 0 – 76 years at estimated MWbeta values. $r^e = 2.0, r^f = 5.0$.

Rate of discount (%)



Years

7. Discussion

Our results are based on an approach suggested by Weitzman (2012) that attempts to reconcile the augmented Ramsey rule with the consumption CAPM. As we have shown, the empirical interpretation of that approach is not quite straightforward since the MWbeta can take a value above unity[7], which is not consistent with Weitzman´s interpretation of MWbeta as a ”fraction”. However, we have suggested a simple transformation of variables that solves this problem. Also, we have argued that the MWbeta can be estimated even when the benefits and GDP variables both follow a random walk with drift and are not co-integrated.

We have then estimated MWbeta from the relationship between four transportation-work variables and GDP in Sweden 1950-2011. Three of these MWbeta estimates are above 0.8, while one, for rail passenger traffic, is slightly less than 0.6. It can be noted that these estimates are not much dependent on whether traffic is regressed on GDP or LnGDP. The latter estimate is however more difficult to interpret within this framework since the relationship has not been linear during the time span we study so we have modelled it with a quadratric trend. The non-leariity reflects the fall and rise of the competiveness of the rail mode, in particular for regional travel. From the 1950's rail lost to the automobile, but eventually, because of increasing road congestion and continuing urban growth, rail passenger traffic expanded again. Alternatively, we have estimated MWbeta for rail passenger travel in a random walk model with drift, and get an estimate close to 0.8.

It is interesting to compare our estimates of MWbeta with the ”regular” CAPM beta for corporate stocks. There are of course two major differences: While our MWbeta is a ”fraction”, the ”regular” CAPM beta can be outside the zero-one interval. Moreover, the ”regular” beta is defined in relation to a stock-market portfolio, while the MWbeta relates in our application to GDP.

[7] And, in fact, it does when estimated on our data without the data transformation.

However, As Weitzman (2012) shows his model converges to the ”regular” CAPM as time approaches zero, so we should perhaps expect the short term SDR for transportation investements as evaluated by Weitzman´s equation to be close to rate of return requirment on stocks that are related to the amount of activity in transportation.Unfortunately, there are not enough transport related stocks traded on the Stockholm market for estimating a meaningful ”transport CAPM beta” for Sweden. However, in the U.S. such an estimate can be held based on the iShares Dow Jones Average Index Transportation Fund. This fund includes stocks in railroad (32%), delivery services (21%), and trucking (19%), thus the emphasis is on freight transportation. It is reported to hold a beta at 0.92 in relation to SP500 (ishares 2012). Further, Kavussanos and Marcoulis (1997) estimated CAPM beta for various transportation industries using U.S. stock market data from 1984 – 1995, and found among others that beta for trucks was 0.97 and for rail 1.01. Thus, as with the model used here, this suggests that short term rate of return requirement for freight transportation investments should be close to the rate of return on a well diversified market portfolio.

We have so far not considered that the value of time savings from an infrastructure improvement may vary with wage rates, and therefore with GDP per capita. As we have observed, cost-benefit appraisals in some countries now assume that travellers´value of time will increase proportionally to GDP per capita. Clearly, accounting for such an additional effect from GDP variation on investment benefits would draw MWbeta values for personal travel even more close to one. In particular, this would imply that our estimated MWbeta for rail-passenger investments at 0.57 is too low.

Krüger (2012) proposes another beta-measure for traffic demand risk in analogy with the CAPM beta; i.e. a measure that is not like Weitzman´s MWbeta locked in to the zero-unity interval. Using basically the same data as we have used here (but with the time series ending in 2005) he decomposes the time series variation into variation at different time scales (2-4 years, 4-8 years, 8-16 years and 16-32 years) using a wavelets approach. He finds that for ton-kilometers by rail a one percent increase of GDP corresponds roughly to a two percent demand increase. For most time scales traffic demand on railroad is more GDP-sensitive than demand for road, and freight transportation is generally more GDP-sensitive than passenger transportation. He shows that traffic demand variance and GDP-sensitivity decrease with the length of time scale considered. These findings may indicate that also the kind of MWbeta we estimate in this paper is not constant over time, but possibly decays. However, we leave to further research to establish if that is the case.

Turning to the implications for the SDR, we first observe that rates, within the time span of transportation infrastructure, are only slowly declining over time, with a possible exception for passenger rail. This means that the conventional approach of using a constant SDR is not far off the mark for these kind of investments. A caveat however is, as we just observed, that we have not investigated whether the MWbeta varies with different time scales (durations); clearly a declining MWbeta would increase the negative time slope of the SDR.

Second, we notice that the relevant rates are between 5 and 6 percent, once again with a possible exception for passenger rail. This is remarkable, since a number of European countries have recently lowered SDR to or under 4 percent (UK 3.5, Sweden 3.5 and Norway 4.0). Germany has for a long time used a SDR at 3.0 percent. Our results, at least for Sweden, imply that these rates are too low.[8] This is even more so considering that the UK, Sweden and Norway also have decided to inflate values of time (and some other benefits) with the growth of GDP per capita, which further reduces the «effective» SDR.

[8] In the 2012 revision of the Swedish guidelines, the advisory group of scientific experts recommended that the rate should remain at 4 percent, but the Director General of the National Transportation Administration decided to reduce it to 3.5, probably with the U.K. level in mind.

8.Conclusions

In this paper we have estimated SDR for transportation infrastructure investments, based on Swedish time-series data from 1950-2011 for four transportation variables and GDP, and on riskless and equity premium rates valid for Sweden. We use the Weitzman (2012) approach, combined with a simple data transformation that keeps the ”real project beta” within the zero to one interval. We also show how this MWbeta can be estimated in various cases. Our estimated MWbetas in the four cases are all close to one, possibly somewhat lower for rail passenger infrastructure. This implies, for Swedish circumstances, that the SDR should be between 5 and 6 percent and decline with not more than 50 interest rate points during the time horizon of a typical road or rail infrastructure investment.

Our study is, to our knowledge, the first that makes an empirical application of Weitzman's approach. We don't expect it to be the last. As Weitzman points out in his paper various functional forms for how to combine a riskless rate with the equity rate may give different results. Likewise, there may be other ways of closing the MWbeta within the zero-unity range than the one we use here. Another caveat is that in this paper we have assumed MWbeta to be time invariant, which is not necessarily the case. Further, our approach makes use of historical data as a guide for appraisals of investments that will support transportation in the future, which also obviously is open to criticism. However, we think that by combining the two main ”workhorses” for thinking about social discount rates, i.e., the (augmented) Ramsey equation and the (consumption-based) CAPM, Weitzman's (2012) paper takes the lead into a road that we should follow.

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