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Hedging with Trees:

Tail-Hedge Discounting of Long-Term Forestry Returns

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Abstract

Tail-hedge discounting is based on decomposition of returns from long-term investments in a fraction (gamma) that is correlated with consumption and another that is not. The first part is discounted at a discount rate that includes a risk premium, the other with the risk-free rate. We estimate gamma for forestry on Swedish data for stumpage prices and GDP per capita 1909-2012. We demonstrate in three forestry cases that the result considerably changes the expected present value of long-term forestry investments.

JEL classification: D61, D63, D81, D92, Q23 Keywords: discounting, far-distant future, declining discount rates, forestry, forest economics, cost-benefit analysis.

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1. Introduction

In several places, owners of forest land plant trees although the internal rate of return on a tree plantation is substantially lower than the normal return on other investments. For instance in Sweden, 50 percent of all tree planting was made in the northern half of the country (*Norrland*), which also has about half of the country´s productive forest land (Swedish Forest Agency 2016), although only a small fraction of the forest land in this region can yield an expected return of 3 percent or more from such investments. [1] Policy makers and many forest professionals have since long defended such practices, irrespective of whether they result from deliberate forest management policies or legal reforestation requirements by various arguments, one of them being that forest investments are safe[2], so the relevant opportunity cost of capital is not the return on other productive investments but on government bonds, with a long run, so called risk-free, yield at one to two percent in real terms.[3] In this study, we investigate to what extent this claim can be corroborated, based on long historical time series for Sweden.

In recent forest economics literature, some authors have discussed implications for forestry of the precautionary savings argument in macroeconomics. This argument is based on the observation that there is uncertainty over the long-term development of the overall macro economy, including future rates of interest. It can be shown that given this uncertainty the certainty

[1] Hultkrantz (1987) estimated that the share of the forest land area with a positive soil (bare land) value was 1 and 14 percent in the northern and southern halves of Norrland, respectively.

[2] This matter was actually a main topic at the very first meeting of the Swedish Economic Association in 1887 in which a lecture was held by “merchant Sörensen” on the “future of our forests” (Sörensen 1887). He presented tables demonstrating the “astonishing” effects from discounting future revenues from young forest stands, but argued that forest resources would always be demanded and there would be fewer sellers than buyers. Prices would therefore always be set so as to cover the cost of cultivation, so there was no risk in cultivating forest resources by planting trees (pp. 17-18), in particular, he added, since planting could be made by “oldsters and under-aged people” (p. 19).

[3] For historical records on the real rate of return on «a relatively riskless» asset in the US, UK, Japan, German and France, see Mehra 2003. The inflation-adjusted return on long-term government bonds in Sweden 1901 – 2012 was 2.1 percent (geometrical average), see Waldenström (2015).

equivalent term structure of the discount rate is declining. This therefore reduces the effect from discounting on the present value of returns that come a long time after the initial investment. Hepburn and Koundouri (2007) estimated such certainty-equivalent term structures from analysis with various statistical methods of historical UK rate of interest data and show that the they lead to substantially higher discounted benefits from forest projects.[4]

However, the precautionary savings motive is an argument for raising investments in capital in general, not specifically in forest resources. While the rates of return from other assets may vary over time, this is true also for the return from forest resources. In fact, there is no point in increasing the share of forest assets in an asset portfolio if the payoff from forestry is highly correlated with returns from other assets. As the finance literature persistently insists, it is the covariant, non-diversifiable, risk that is relevant for investment decisions. Recently, Weitzman (2012, 2013) has suggested an approach for discounting future benefits at a societal level based on the idea is that the return of an investment can be (linearly) decomposed into one portion that is covariant with the non-diversifiable systematic risk of the macro-economy and another portion that is independent of it. He then argues that it would be justified to require an average rate of return from risky investments on the first portion and a return on the second that corresponds to the return on government bonds and similar assets that are considered as “safe” assets. Calculating an exponentially weighted average of these rates then yields a discount rate that is declining over time at a rate that depends on the portion of the covariant risk. Weitzman calls the proportion of the expected payoff that is correlated with the macro-economy *the real project*

[4] Amacher, Ollikainen and Koskela (2009, section 9.2.2) studies how forestry is affected by interest rate uncertainty, see also Gong and Löfgren (2003).

gamma, henceforth gamma for short. As he shows, the weighted average formula has the same form as the conventional CAPM in a two period setting, with gamma replacing the CAPM beta.

Weitzman does not show, however, how the gamma can be estimated. In Mantalos and Hultkrantz (2016) we demonstrate that such estimation is not trivial and that different estimation methods have to be used depending on the dynamic properties of the relevant time series and their correlation. We also suggest suitable approaches. In this paper we consider a series of annual stumpage prices in Sweden from 1909 – 2012 that we previously studied in Hultkrantz et al. (2014) and GDP per capita for the same period. We estimate gamma for this case and show how the result can be used to evaluate forest investments. Our perspective is that of a strategic planner taking a societal view, as for instance that of a national forest regulatory body or of an administration of public forest land.[5] While the main implications for forest management from our results are similar to the conclusions in Hepburn and Koundouri (2007), we derive them on different theoretical grounds and with a different empirical approach.

The paper is organized as follows. In the next section we briefly review the standard CAPM model for valuation of risky assets and Weitzman's "tail-hedge" discounting model. Section 3 describes how we estimated the gamma, after having made some small modifications. A more extensive description of the empirical estimation is found in the Appendix. In section 4 we demonstrate the result by evaluation of three long-term investment forestry cases. Section 5 contains a discussion and some final conclusions.

[5] In particular we will not consider the idiosyncratic risk of the useof a specific forest-management practice on a specific site.

2. Theory

In this section we review first the standard CAPM model for valuation of risky assets and then the “tail-hedge discounting” approach of Weitzman (2012, 2013).

2.1 CAPM

The Capital Asset Pricing Model, or the CAPM, is a model of asset prices (Sharpe 1964 , Lintner 1965 and Mossin 1966).[6] The model states that in equilibrium investors get a return r_i on a risky asset that is equal to the return r^f on a riskfree asset plus a risk premium that is equal to the equity premium $r^m - r^f$ on a fully diversified market portfolio times β_i , the latter factor being a measure of the non-diversifiable systematic risk, equal to the co-variance over the variance, or the slope coefficient in a linear regression of the return on the specific asset with market portfolio return. Thus:

$$r_i = r^f + \beta_i \left(r^m - r^f\right) \quad (2.1)$$

where $\beta_i = \frac{Cov(r_i, r^m)}{\sigma_m^2}$.

For analysis at a societal level, the consumption capital asset pricing model (CCAPM) was developed in the late 1970’s. It extends the CAPM by focusing on the correlation between the yield from a specific asset and overall consumption. However, in spite of its strong theoretical merits, the CCAPM is difficult to apply empirically, among others because of the diversity of the population with respect to possession of various kinds of assets (see the review

[6] Merton (1973) extended the CAPM framework to cover inter-temporal portfolio choices (ICAPM).

by Breeden at al. 2015). The "tail-hedge" discounting model suggested by Weitzman (2012, 2013) can be seen as a pragmatic short-cut approach to such difficulties.

2.2 The «tail-hedge» discounting model

Weitzman (2012, 2013) has developed a model for calculation of the social rate of discount for investments in risky assets. In Weitzman (2012) he starts with the standard so called Ramsey equation for the social rate of discount under riskfree assumptions.[7] He argues that in consideration of real-world risk, risk-averse investors expect that low-probability large loss events are somewhat more likely than what is implied by the normal distribution. He shows that with otherwise standard assumptions on the parameters of the Ramsey equation it is possible to reconcile the Ramsey equation with empirically observed levels of the risk-free rate and equity premium.

He then makes the crucial assumption that the instantaneous net benefit of a single marginal investment project B_t can be decomposed as a linear combination of contemporary consumption, C_t , standardized with the expected value, and a project-specific random variable I_t that is uncorrelated with consumption (which therefore can be made deterministic by diversification over a pool of projects). More specifically, the net benefit at time t is

$$B_t = b_t\left((1-\gamma_t)I_t + \gamma_t \frac{C_t}{E(C_t)}\right), (2.7)$$

where γ_t is the proportion of the pay-offs at time *t* that is correlated with aggregate consumption, and, therefore, is non-diversifiable, while (1- γ_t) is stochastically independent of the aggregate

[7] This is equation (2) in Hepburn and Koundouri (2007).

economy. The latter component is normalized by setting $E(I_t) = 1$ for all t. That implies that expected net benefits at time t, are given by $E(B_t) = b_t$.

Weitzman (2012, 2013) calls the coefficient γ_t the "real project gamma". We will use gamma to distinguish it from the "regular" CAPM beta. Gamma is defined as "the *fraction* of expected payoffs that on average is due to the uncertain macro-economy" (Weitzman 2012, p. 15, italics in original). Introducing the rate of return on a risk free asset r^f and risky equity r^e, respectively, Weitzman shows that the discount rate for a project with γ_t will be

$$SDR_t = -\frac{1}{t}[\ln(1-\gamma_t)\, e^{-r^f} + \gamma e^{-r^e t}]\ . \ (2.8)$$

The gamma is used for calculation of a weighted average of the riskless and risky *discount factors.* He notices that in the limit as $t \to 0$, or more precisely when the number of periods is two, the risk-adjusted rate of discount is

$$r_0^{\gamma_0} = (1-\gamma_{0)} r^f \ + \ \gamma_0\, r^m, \ (2.9)$$

which is similar to the CAPM equation, but gamma has a different definition than the CAPM beta. Thus, in this case, the risk-adjusted rate of discount is a (gamma-weighted) weighted average of the riskless and risky rates. As t increases, the risk-adjusted rate will approach the risk-free rate.

3. Estimation of gamma for forest investment from a Swedish time series

In Hultkrantz et al. (2013) we analyzed the time-series properties of stumpage prices (timber rent) in Sweden 1909-2012.[8] We found that this is **a** stationary series with a structural level break at the end of WW2. As a measure of the overall macro-economy we will here use GDP volume per capita.[9] As shown in Mantalos and Hultkrantz (2016) the appropriate statistical procedure for estimating the gamma hinges upon the time-series properties of the two variables and whether they are co-integrated.

When we have a stationary series y_t (stumpage prices) that follows an AR(1) process and another series x_t , ln(GDP), that follows a random walk with drift , by following the Box-Jenkins methodology, Box, Jenkins and Reinsel (1994), we can use the following finite distributed lags (FDL) **r**egression model **:**

$$y_t = c + \rho y_{t-1} + b\Delta x_t + u_t \qquad (3.1)$$

By moving the lagged part of the series from the right part of equation (3.1) to the left part we have:

$$\begin{aligned} y_t - \rho y_{t-1} &= c + b\Delta x_t + u_t \\ Dy_t &= c + b\Delta x_t + u_t \end{aligned} \qquad (3.2)$$

Dy_t is (2.4) is the so called “rho difference”. This can be estimated in a Cochrane-Orcutt procedure, Cochrane and Orcutt (1949).

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[8] Prices for the period 1909-1955 were collected by Streyffert (1960); for the period 1956-2002 by the Swedish Forest Agency (Statistical Yearbook of Forestry, various issues); and for 2003-2012 by the stumpage broker Rotpostmäklarna AB. Prices are deflated by the Consumer Price Index (CPI).

[9] This data comes from Edvinsson (2014).

Alternatively, by subtracting the y_{t-1} from both sides of (3.1) we get:

$y_t - y_{t-1} = c + (\rho - 1) y_{t-1} + b\Delta x_t + u_t$ or more brief

$$\Delta y_t = c + \phi y_{t-1} + b\Delta x_t + u_t \qquad (3.3)$$

where Δy_t is the first difference of the series y_t.

As shown in Mantalos and Hultkrantz (2016) for the gamma estimation to meet the restriction $0 < \gamma \le 1$ it must be that $\left(s_{x^*}\right) = \left(s_{y^*}\right)$ The simplest way to meet this restriction and equation (3.1), while maintaining the same relationship between the two variables, is to use the following simple transformation:

$$y_t^* = \left[\left(y_t - \overline{y}\right) / s_{y_t}\right] + 1, \quad x_t^* = \left[\left(x_t - \overline{x}\right) / s_{x_t}\right] + 1, \qquad (3.4)$$

In this way both variables have the same means and the same standard deviations equal to one.

We can then use the *two modified standardized* random variables Dy_t^* and Δx_t^* for estimation of gamma as:

$$Dy_t^* = c + \gamma \Delta x^* + u_t \qquad (3.5)$$

or

$$\Delta y_t^* = c + \phi y_{t-1} + \gamma \Delta x_t^* + u_t \qquad (3.6)$$

by estimating the regression with AR errors of y_t^* on Δx_t^*.

Gamma was estimated with the following procedure: We did and an ordinary regression of y_t^* on Δx_t^*, and stored the residuals. We then analyzed the time series structure of the residuals to determine if they have an AR structure. We then used Δy_t, Dy_t and Δx_t to get an idea of the number of lags that should be included in the FDL model (3.6). We found that no lags were significant so we estimated this model with zero lags. We also estimated the "Rho-difference" model (3.5) with Cochrane-Orcutt. Both approaches resulted in estimates of gamma of about 0.4. All calculations are shown in the appendix.

4. Evaluation of forestry cases

With this estimate of gamma we can calculate the risk-adjust social discount rate from equation (2.8) for any set of r^f, r^e and t. Following reasonably close to the parameters used in forest case evaluations by Hepburn and Koundouri (2007) we will assume that r^f = 2% and r^e = 6%. For demonstration we have chosen three forest investment cases, one of them being the long-horizon harvest case used by Hepburn and Koundouri, which is a 120-year cycle plantation of Scottish Oak. The other two are "typical" cases presented on a web site demonstrating a model for economic evaluation of forest management provided the Swedish National Forestry Board, one being a 74-year cycle plantation of Scots Pine in Southern Norrland, the other a 76-year cycle of Norwegian Spruce in southern Sweden (Götaland). The assumed cash flows and the corresponding present values from discounting with either the conventional re or with "tail-hedge" discount rates are shown in Table 1.

As can be seen, for all three cases the net present values (NPV) from conventional discounting are negative. The benefit-cost ratios (BCR), defined as the sum of discounted future net revenues divided by the initial outlay, are around 0.2 in all three cases. However, with the “tail-hedge” discount rate the BCR becomes 1.5 in both Swedish cases and 3 for Scottish oak in UK.

5. Discussion and conclusions

Investors regularly require an additional rate of return, a so called equity premium, on risky assets over the rate of return from a so-called risk free placement, like an AAA-rated government bond. Tail-hedge discounting is based on the assumption that returns from an investment can be linearly decomposed into one part (the gamma fraction) that is correlated (and therefore non-diversifiable) with overall consumption (here GDP per capita) and one part (one minus gamma) that is not (and there can be used for hedging the consumption risk). In discounting future returns the equity premium is then included in the discount rate for the correlated component but not for the uncorrelated component. The discount factor for total returns is an average of the discount factors for the two components and the discount factor for the correlated component (with the higher discount rate) falls faster than the discount factor for the uncorrelated component. Therefore the average rate of discount (weighted with the discount factors) will decrease over time.

As shown in Mantalos and Hultkrantz (2016) if asset returns and overall consumption follow stochastic processes that are co-integrated, then gamma is unity and the asset cannot be used as a hedge. In Mantalos et al. (2014) this is shown to be the case (or at least close to it) for road and rail infrastructure investment. However, timber is different. Based on a long-time series for stumpage prices in Sweden, from 1909 – 2012, we estimate gamma to 0.4. This means that the weighted average rate of discount is close to the risk-free discount rate already for the returns from the first thinning of a typical forest plantation in Sweden. As was demonstrated in three forestry cases this has a remarkable effect on the overall benefit-cost ratios of long-term forestry investments. It should be noted that unlike previous studies that have estimated “hyperbolic” discount-rate term structures, this result is based on the notion of relevant (co-variant) risk (within an, albeit loose, CCAPM framework).

An obvious limitation of this finding is that we have used data from one specific country only, mainly because long-time series for the relevant price (the timber rent or the net price) are very rare. However, Sweden can be characterized as an open economy during the whole period spanned by the data, possibly with exceptions during WW1 and WW2, and the Swedish forest industry has during the whole period (in fact, even much longer) been highly dependent on exports. Therefore the price series we have used to a large extent reflects developments on the world markets for forest products.

A more severe limitation is therefore that we use historical data as a guide for the future. This is a limitation that this study shares with a large part of the empirical finance literature. However, a main message of the historical data on stumpage prices is that the value of forest resources has been robust to the various phases of transformation of the economy during periods of industrialization, servification, and digitalization. Timber has proven to be a useful raw material for a wide range of products and uses. The specific products have changed over time, from for instance railroad sleepers and fuel wood to copy paper and DMT diesel, but the demand for wood has remained. Because of this basic general-purpose feature of wood (or wood fiber, bio chemicals or forests), it seems that investment in forest assets can be regarded as a useful hedge against long-term macro-economic growth risk. In fact, in comparison to conventional “risk-free” assets such as nominal government bonds, timber is a real-valued resource that also can be used as a hedge against inflation.

A more philosophical concern is how the discount rate used by the current generation of forest owners, i.e., those individuals who decides on whether to plant trees or not, is affected by the fact that a large part of the returns from these investments will be captured by future generations. This topic has been analyzed with overlapping-generation models, see Amacher et al. (2009) for a review. For example, Hultkrantz (1992) showed in a model where the next generation of forest owners harvests the trees that were planted by the current generation that the current generation should evaluate investment opportunities with the conventional net present value criterion, using a discount rate corresponding to the opportunity cost of capital, as long as the current generation puts some positive weight (even a very slight one) on the utility of future generations (i.e., there is a bequest motive).

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Table 1. Cash flows and present values (PV) in three forestry investment cases: Scots pine (Southern Norrland, land class T24); Spruce (Götaland, land class G28) and Scotish Oak (UK). r = discount rate with tail-hedge discounting, PV = present value, NPV = net present value, BCR = benefit cost ratio).

Pine, S Norrland, T24				
Age	Cash flow	PV(6%)	r	PV(r)
Year	SEK/ha	SEK/ha	%	SEK/ha
1	-8856	-8856		-8856
41	4114	377	2.9%	1249
58	11694	398	2.8%	2395
74	71128	954	2.6%	10309
NPV		**-7127**		**5098**
BCR		**0.20**		**1.58**

Spruce, Götaland, G28				
Age	Cash flow	PV(6%)	r	PV(r)
Year	SEK/ha	SEK/ha	%	SEK/ha
1	-13995	-13995		-13995
40	4916	478	2.96%	1530
50	13489	732	2.85%	3311
76	118317	1412	2.63%	16441
NPV		**-11373**		**7287**
BCR		**0.19**		**1.52**

Scottish Oak, UK				
Age	Cash flow	PV(6%)	r	PV(r)
Year	GBP/ha	GBP/ha	%	GBP/ha
1	-1100	-1100		-1100
60	4000	121	2.75%	784
80	8000	76	2.61%	1022
100	10000	29	2.50%	848
120	12000	11	2.42%	680
NPV		**-863**		**2234**
BCR		**0.22**		**3.03**

Appendix

A. Model Identification process:

1. We start by doing an ordinary regression of y_t^* on Δx_t^*, and store the residuals.

2. Analyze the time series structure of the residuals to determine if they have an AR structure.

3. If the residuals from the ordinary regression appear to have an AR structure, estimate the Δy_t (3.6), or the Dy_t (3.5)

4. We use the Δy_t, Dy_t and Δx_t to get an idea of the number of lags that we should use in our FDL Model, (3.1).

5. We estimate gamma.

B. Model Estimation process:

In the first step we estimate the residuals of y_t on, and as can be seen in Figure 1, the sample ACF and sample PACF of the residuals show clearly the pattern of an AR(1) process.

Figure 2 shows that the estimated rho is **0.94714** by using the Cochrane-Orcutt procedure while in the next step we estimate the Δy_t, Dy_t and Δx_t and make the cross-correlogram.

Figure 1 Step 1: SACF and SPACF of the residuals in the first step

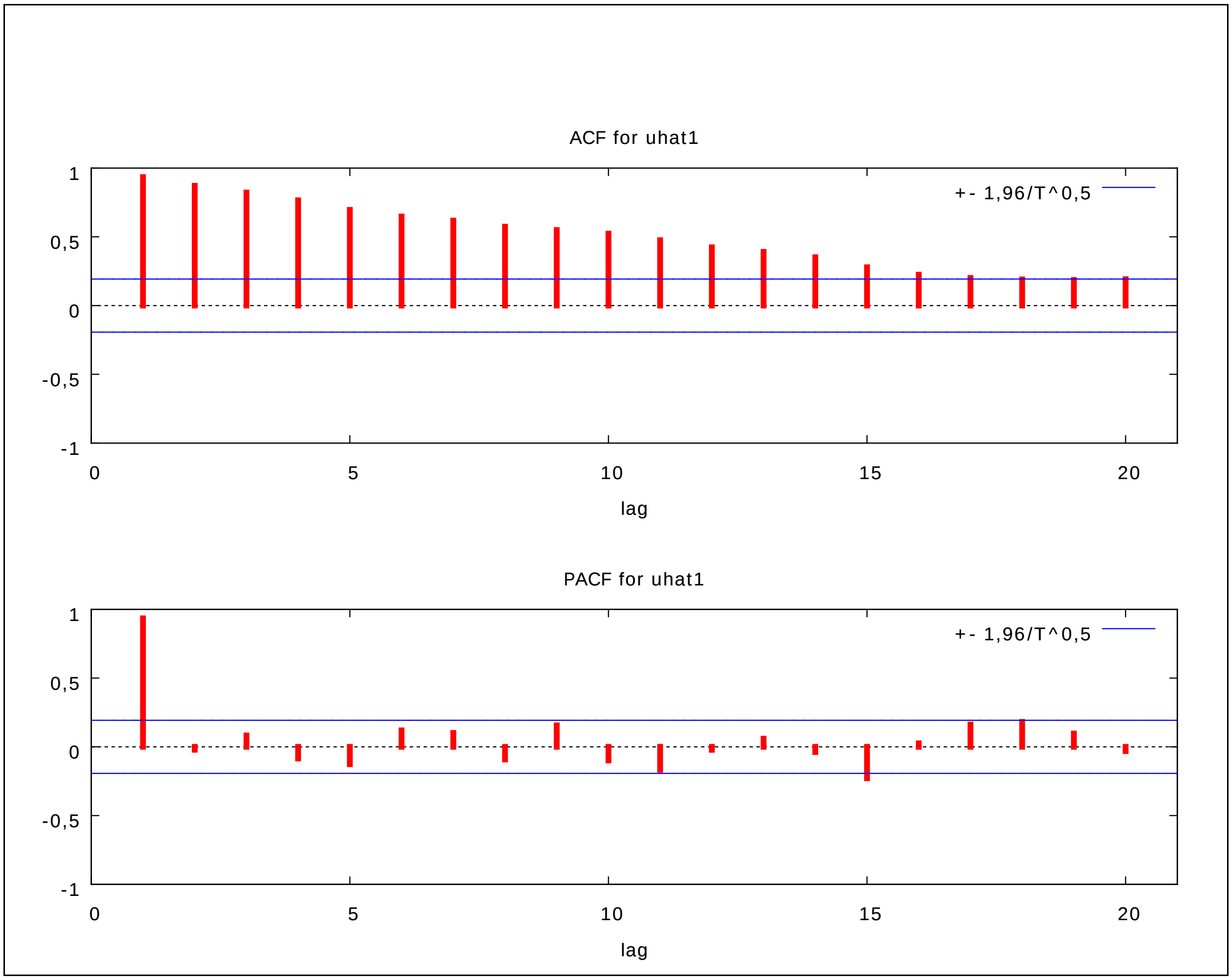


Figure 2 Step 2: Estimating the "rho" by Cochrane-Orcutt procedure

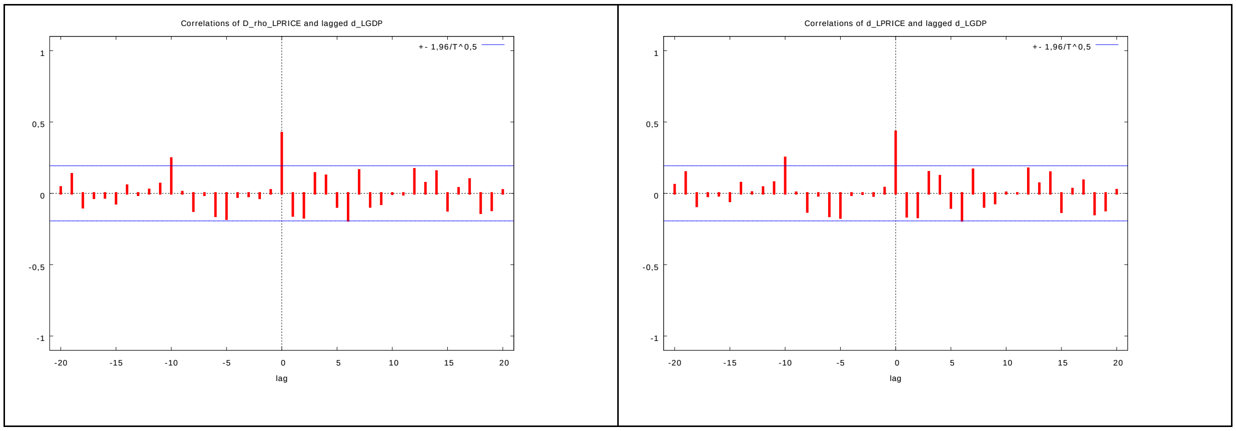
Performing iterative calculation of rho...

ITER	RHO	ESS
1	0.94185	2.45114
2	0.94669	2.45046
3	**0.94714**	2.45046

Model 2: Cochrane-Orcutt. using observations 1911-2012 (T = 102)
Dependent variable: LPRICE

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	5.79602	0.293269	19.7635	<0.00001	***
DLGDP	0.062181	0.0122651	5.0698	<0.00001	***

Figure 3 Step 3: Cross Correlations the Δy_t , Dy_t and lagged Δx_t



The correlogram in figure 3 shows identical results. The only significant lag of Δx_t is the zero lag. That means that we use only Δx_t in our regressions, 3.5 and 3.6.

We modify Δy_t , Dy_t and Δx_t as:

$$Dy_t^* = \left[\left(Dy_t - D\overline{y} \right) / s_{Dy_t} \right] + 1, \ \Delta y_t^* = \left[\left(\Delta y_t - \Delta\overline{y} \right) / s_{\Delta y_t} \right] + 1, \quad \Delta x_t^* = \left[\left(\Delta x_t - \Delta\overline{x} \right) / s_{\Delta x_t} \right] + 1,$$

Figure 4 Step 4: Estimating Gamma

Equation 3.6

Model 4a: OLS. using observations 1910-2012 (T = 103)
Dependent variable: dLPRICE1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	0.568636	0.126649	4.4899	0.00002	***
DLGDP1	**0.431364**	0.0897723	4.8051	<0.00001	***

Mean dependent var	1.000000	S.D. dependent var	1.000024
Sum squared resid	83.02509	S.E. of regression	0.906659
R-squared	0.186067	Adjusted R-squared	0.178008
F(1. 101)	23.08887	P-value(F)	5.39e-06
Log-likelihood	-135.0480	Akaike criterion	274.0960
Schwarz criterion	279.3654	Hannan-Quinn	276.2303
rho	0.008825	Durbin-Watson	1.977528

Equation 3.5

Model 4b: OLS. using observations 1910-2012 (T = 103)
Dependent variable: D_rho_LPRICE1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	0.577485	0.127235	4.5387	0.00002	***
DLGDP1	0.422515	0.0901876	4.6848	<0.00001	***

Mean dependent var	1.000000	S.D. dependent var	1.000020
Sum squared resid	83.79501	S.E. of regression	0.910853
R-squared	0.178513	Adjusted R-squared	0.170379
F(1. 101)	21.94776	P-value(F)	8.76e-06
Log-likelihood	-135.5234	Akaike criterion	275.0467
Schwarz criterion	280.3162	Hannan-Quinn	277.1810
rho	0.038893	Durbin-Watson	1.916612

Figure 4a shows the results of estimated **g**amma by using the regression 3.6 while figure 4b shows the results of estimated **g**amma by using the regression 3.5.

The difference between these two is very small 0,009 and we can without hesitation to round them off to ***0.4***.

However in both OLS regressions the residuals show a suggestion for use of MA(q) to make them white noise.

Figure 5a shows the use of model 3.6 with maximum likelihood allowing for MA(|12|,|14|,|16|) residuals. We do also the same for the model 3.5, see Figure 5b

Figure 5 Step4: Estimating the "MWbeta" with ARIMAX model

Figure 5a

Model 5a: ARMAX. using observations 1910-2012 (T = 103)
Dependent variable: dLPRICE1
Standard errors based on Hessian

	Coefficient	*Std. Error*	*z*	*p-value*	
const	0.587609	0.099677	5.8951	<0.00001	***
theta_12	-0.166448	0.0876354	-1.8993	0.05752	*
theta_14	0.216999	0.0992103	2.1873	0.02872	**
theta_16	-0.319721	0.111514	-2.8671	0.00414	***
DLGDP1	**0.417066**	0.0763813	5.4603	<0.00001	***

Mean dependent var	1.000000	S.D. dependent var	1.000024
Mean of innovations	-0.006700	S.D. of innovations	0.811883
Log-likelihood	-126.2995	Akaike criterion	264.5989
Schwarz criterion	280.4073	Hannan-Quinn	271.0018

Figure 5b

Model 5b: ARMAX. using observations 1910-2012 (T = 103)
Dependent variable: D_rho_LPRICE1
Standard errors based on Hessian

	Coefficient	*Std. Error*	*z*	*p-value*	
const	0.607006	0.102588	5.9169	<0.00001	***
theta_12	-0.141886	0.0858746	-1.6522	0.09849	*
theta_14	0.239044	0.0983899	2.4296	0.01512	**
theta_16	-0.307987	0.110411	-2.7894	0.00528	***
DLGDP1	**0.399168**	0.0765601	5.2138	<0.00001	***

Mean dependent var	1.000000	S.D. dependent var	1.000020
Mean of innovations	-0.014928	S.D. of innovations	0.821299
Log-likelihood	-127.4363	Akaike criterion	266.8725
Schwarz criterion	282.6809	Hannan-Quinn	273.2754

As we see in figure 6 the residuals after estimating the equations 3.6 and 3.5 with maximum likelihood and allowing residuals to follow MA(|12|,|14|,|16|) process are white noise.

The estimated gamma, as can be seen in figure 5, is **0.417** for model 3.6 and **0.399** for model 3.5.

To summarize our analysis we managed to estimate gamma for two time series by using the Finite Distributed Lag (FDL) model 3.1 with zero lag for the first difference of ln(GDP) in two variations for the ln(stumpage prices) the "Rho" difference and first difference. The result is that gamma is about 0.4.

Figure 6 Step 5: Sample ACF PACF for the residuals of ARIMAX models

Figure 6a SACF and SPACF (3.6) model

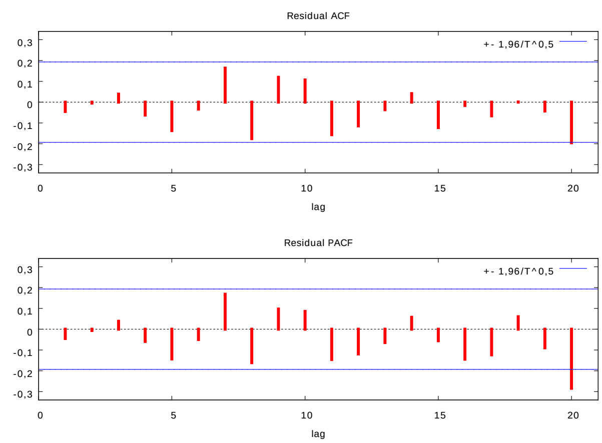


Figure 6b SACF and SPACF (3.5) model