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Estimating “Gamma” for Tail-hedge Discount Rates When Project Returns Are Co-integrated with GDP

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Abstract

Weitzman (2012, 2013) has suggested a method for calculating social discount rates for long-term investments when project returns are covariant with consumption or other macroeconomic variables, so called “tail-hedge discounting”. This method relies on a parameter called “real project gamma“ that measures the proportion of project returns that is covariant with the macroeconomic variable. We suggest two approaches for estimation of this gamma when the project returns and the macroeconomic variable are co-integrated. First we use Weitzman’s (2012) own approach, and second a simple data transformation that keeps gamma within the zero to one interval. In a Mont-Carlo study we show that the method of using a standardized series is better and robust under different data-generating processes. Both approaches are demonstrated in a Monte-Carlo experiment and applied to Swedish time-series data from 1950-2011 for annual time-series data for rail freight (a measure of returns from rail investments) and GDP.

1.Introduction

The usefulness of cost-benefit analysis as a tool for guiding decision making can be doubted when it comes to decisions that affect distant-future outcomes, since the weight given to such effects is extremely dependent on the choice of discount rate. This is a major concern to climate policy but is also important in several other policy fields. For instance, high-speed rail requires enormous upfront infrastructure investments that can only be economically justified, if at all, by benefits within a 60 years, or even longer, horizone.[1] Likewise, a forest-owner in northern Sweden, where reforestation investments after clear-cut fellings are required by law, has to wait around 120 years until a plantation of Scots pine (*Pinus Sylvestris)* can be harvested.

An unresolved question with substantial implications for the assessment of far-distant future benefits is whether they should be assessed with a discount rate that reflects the levels of the average return on (risky) investments or the, considerably lower, risk-free rate of return of a «safe» placement, for instance in government bonds. In a recent study, Weitzman (2012, 2013) has suggested a middleground solution that begins with the rate of return on risky investments in the short run and approaches the safe rate in the long run. The underlying idea is that the return of an investment can be (linearly) decomposed into one portion that is covariant with the nondiversifiable systematic risk of the macroeconomy and another portion that is independent of it. It seems then justified to require an average-risk rate of return on the first portion and the risk-free return on the second. Calculating a weighed average of the two rates with the corresponding capital values as weights gives a term structure that in the long-run

[1] The investment horizon used in appraisal of rail infrastructure investments in Sweden is 60 years.

limit reaches the low rate. Weitzman calls the proportion of the expected payoff that is correlated with the macroeconomy the *real project gamma*, henceforth gamma for short. As he shows, the weighted average formula has the same form as the conventional CAPM in a two period setting, with gamma replacing the CAPM beta.

Weitzman, however, gives no clues on how to estimate gamma. He acknowledges that this is the «most difficult stumbling block» for application of his discount rate schedule (Weitzman 2013, p. 878). This is the challenge that motivates the present study. It departures from the observation that GDP and many other economic time series follow a random walk process with drift. In the two examples of long-term investments just mentioned we may expect that returns on rail-investments have similar time-series properties, in contrast to returns on forest investments (Hultkrantz 1993, 1995; Hultkrantz, Andersson and Mantalos 2014). In this paper we extend our previous research based on these polar cases (Hultkrantz, Krüger and Mantalos 2014, Hultkrantz and Mantalos 2016) by suggesting a general approach for how to estimate gamma in the first case, i.e., when the project return and the macroeconomic variable are co-integrated.

In Section 2 we describe Weitzman´s method to determine the risk-adjusted social discount rate (SDR) from gamma. In Sections 3-5 we focus on the case of two co-integrated unit root with drift variables, beginning with a theoretical analysis in Section 3, followed in Section 4 by a Monte-Carlo simulation study that investigates the values that estimated coefficients of co-integrated coefficients get for different models, and finally with an application to real data on rail-freight volumes. Section 6 concludes on the methodological findings.

2. Economic Theory

Weitzman´s model

In the approach suggested by Weitzman (2012, 2013) the instantaneous net benefit of a single marginal investment project $B_t$is assumed to be a linear combination of contemporary consumption, C_t , standardized with the expected value, and a project-specific random variable I_t that is uncorrelated with consumption (which therefore can be made deterministic by diversification over a pool of projects). More specific, the net benefit at time t is

$$B_t = b_t\left((1-\gamma_t)I_t + \gamma_t \frac{C_t}{E(C_t)}\right), \qquad (2.1)$$

where $\gamma_t$is the proportion of the pay-offs at time *t* that is correlated with aggregate consumption, and therefore is non-diversifiable, while (1- γ_t) is stochastically independent of the aggregate economy. The latter component is normalized by setting $E(I_t) = 1$ for all *t*. That implies that expected net benefits at time *t*, are given by $E(B_t) = b_t$.

Weitzman (2013) defines the gamma, γ_t, as "the *fraction* of expected payoff that on average is due to the non-diversifiable systematic risk of the uncertain macro-economy" (Weitzman 2013, p. 876, italics in original). Introducing the rate of return on a risk free asset r^f and risky equity r^e , respectively, Weitzman shows that the discount rate for a project with gamma $\gamma_t$will be

$$SDR_t = -\frac{1}{t}[\ln(1-\gamma_t)\, e^{-r^f} + \gamma_t e^{-r^e t}]\ . \qquad (2.2)$$

Gamma is used as weight in computing a weighted average of the riskless and risky *discount factors.* Weitzman shows that in the limit as t -> 0, or more precisely when the number of periods is two, the risk adjusted rate of discount is

$$r_0^{\gamma_0} = (1-\gamma_{0)} r^f \; + \; \gamma_0 \, r^e, \qquad (2.3)$$

which is similar to the CAPM equation, but with gamma instead of the CAPM beta. Thus, in this case the risk-adjusted rate of discount is a (gamma-weighted) weighted average of the riskless and risky rates. However, as t increases, the risk-adjusted rate will approach the risk-free rate.

Using this framework Weitzman shows that the term structure of a risk-adjusted social rate of discount will be falling, just as previously has been shown for social discount rates for discounting certainty equivalent net benefits. The basic economic intuition is related to insurance against uncertain future prospects of the overall economy. The more the net benefits of a specific project are uncorrelated with the macroeconomic development, the larger will the precautionary motive be for making the investment. The reason for the declining term structure is, unlike in previous literature on precautionary motives, in this case not persistence of growth rate shocks, as these are assumed to be i.i.d. Instead, as in Weitzman (1998, 2001) it emerges out of the computation of a weighted average of the two discount rates using their respective capital value (present value) as weights, which over time gives a stronger relative weight to the riskless rate.

Weitzman notices that this analytical framework may be difficult to apply to the computation of the SDR for a specific public investment as there are no frequent market data for such projects. However, usually an equal level of the SDR is used for all public investments, at least within a category. In Hultkrantz et al. (2014) we therefore estimated SDRs for public investments in transportation infrastructure, assuming that gamma is constant for all future periods. In the next section we will now develop the approach used there further for the case of two co-integrated random-walk with drift variables.

3. Estimation of gamma with two co-integrated random-walk with drift variables

3.1 Defining gamma

Consider two random variables, y_t and x_t, and that there is a linear relationship between these two variables, that is:

$$y_t = \mu_0 + \mu_1 x_t + u_t, \text{ with } \quad u_t \sim i.i.d.(0, \sigma^2) \quad (3.1)$$

Note that Weitzman uses a different notation for these two variables, with $B_t \equiv y_t$ and $C_t \equiv x_t$, see Weitzman (2013, eq. 1). Moreover in his analysis (Weitzman 2012), he adjusts the model (3.1) by introduction of first a new random variable:

$$A_t \equiv \mu_0 + u_t, \quad (3.2)$$

and in the next step defining the gamma as :

$$\gamma = \frac{\mu_1 E(x_t)}{E(A_t) + \mu_1 E(x_t)}, \quad (3.3)$$

with $\mu_1 = \dfrac{Cov(y_t, x_t)}{Var(x_t)}$.

Using this definition (3.3) in equation (3.1) we have what Weitzman (2012) calls the "*weighted average decomposition of variation equation"*:

$$\frac{y_t}{E(y_t)} = (1-\gamma)A_t + \gamma\frac{x_t}{E(x_t)_t} \tag{3.4}$$

Further, using the definition (3.2) in equation (3.4) and for $u_t \sim i.i.d.(0,\sigma^2)$ it is easy to see that we can transform equation (3.1) to the following:

$$\frac{y_t}{E(y_t)} = (1-\gamma) + \gamma\frac{x_t}{E(x_t)} + \frac{(1-\gamma)}{\mu_0}u_t \tag{3.5}$$

With this equation (3.5) we could use the new *mean standardized* variables to directly estimate the gamma.

3.2. The effects of A_t to the estimated gamma

Weitzman defines gamma as a "fraction", which seems to imply that the value is between zero and one. However, this is not necessarily so, as we now show.

1. Consider that the linear relationship between the two variables is *positive,* then based on equation (3.3) for

   a) $E(A_t) > 0$, and, with $\mu_0 > \mu_1$, we have $\gamma < 1$

   b) $E(A_t) = 0$, and $\mu_0 = 0$, we have $\gamma = 1$

   c) $E(A_t) < 0$, that means the $\mu_0 < \mu_1$, we have $\gamma > 1$

2. If instead we still have the positive linear relationship between the two variables but

$E(A_t) < 0$, and $\mu_1 > \mu_0$, we have $\gamma < 1$

As a result of these simple observations and remembering that the two variables are assumed to have a positive linear relationship, gamma is positive but can exceed unity. Therefore, we need to modify the equation (3.5) to be restricted for $0 < \gamma \leq 1$.

But before that, let us have a look into the co-integration case.

3.3 Model with Co-integration.

Consider a bivariate co-integrated system for $\mathbf{Y}_t = (y_t, x_t)'$ with co-integrating vector $\phi = (1 - \delta)'$. By using the Phillips (1991) triangular representation we have

$$y_t = \delta x_t + u_t, \text{ where } u_t \sim I(0) \quad (3.6)$$

$$x_t = c + x_{t-1} + \upsilon_t, \text{ where } \upsilon_t \sim I(0) \quad (3.7)$$

Now the variables are $(y_t, x_t)' \sim I(1)$. If eq. (3.6) holds with a stationary error term, then the two variables are co-integrated. By using OLS to estimate the regression´s δ, we get an estimated coefficient that is consistent and converges to the true value at rate *T*, i.e. the OLS estimator is super consistent. This result gives us the legitimation to use Weitzman´s mean standardized method to estimate gamma.

Now, consider that the two random variables y_t and x_t follow a random walk *with drift*:

$$y_t = \alpha_y + y_{t-1} + e_{1t}, \; e_{1t} \sim i.i.d.\,(0, \sigma^2)$$

and (3.8)

$x_t = \alpha_x + x_{t-1} + e_{2t}$, $e_{2t} \sim i.i.d.(0, \sigma^2)$

For start values $y_0, x_0 \neq 0$ we can write them as:

$y_t = \alpha_y t + y_0 + \sum_1^T e_{1t}$ and $x_t = \alpha_x t + x_0 + \sum_1^T e_{2t}$ (3.9)

Suppose that the variables are co-integrated with the co-integrating vector $\phi = (1 - \delta)'$ then based on equation (3.6) we get:

$$y_t = \delta x_t + u_t = \delta\left(\alpha_x t + x_0 + \sum_1^T e_{2t}\right) + u_t$$

By using the common stochastically trend, the co-integrated relationship becomes:

$$\boldsymbol{\beta}'\mathbf{Y_t} = \delta\left(\alpha_x t + x_0 + \sum_1^T e_{xt}\right) + u_t - \delta\left(\alpha_x t + x_0 + \sum_1^T e_{xt}\right) = u_t \quad (3.10)$$

It is not difficult to see that:

$E(y_t) = \delta E(x_t)$, and $E(x_t) = \alpha_x t + x_0$ (3.11)

Then based on (3.4) the Weitzman´s "mean-standardized" variables become:

$$\frac{y_t}{E(y_t)} = \frac{y_t}{\delta E(x_t)}$$

Moreover, from equation (3.6) by dividing both sides with $E(y_t)$ we have:

$$\frac{y_t}{E(y_t)} = \delta \frac{x_t}{E(y_t)} + \frac{u_t}{E(y_t)} = \frac{\delta x_t}{\delta E(x_t)} + \frac{u_t}{\delta E(x_t)} = 1\frac{x_t}{E(x_t)} + \frac{u_t}{\delta E(x_t)} \quad (3.12)$$

The last result tells that, whatever the co-integrating vector is, for the Weitzman´s "mean standardized" variables for these series we expect the regression coefficient to be equal to one. Note that this result is valid when we do not have a constant in the co-integration regression.

If instead the co-integration regression has a *constant*, that is,

$$y_t = c + \delta x_t + u_t, \text{ where } u_t \sim I(0) \quad (3.13)$$

$$x_t = a_x + x_{t-1} + \upsilon_t, \text{ where } \upsilon_t \sim I(0) \quad (3.14)$$

we have the original model (3.1), i.e., without standardization with means.

3.3 Empirical Estimates.

Consider two random variables y_t and x_t that follow a random walk with drift and general co-integration vector: $(y_t - \delta x_t)$

For a start value $y_0, x_0 \neq 0$ we can write as before:

$$y_t = y_0 + \sum_1^T e_{1t} + \alpha_y t \text{ and } x_t = x_0 + \sum_1^T e_{2t} + \alpha_x t \quad (3.15)$$

Now if $\alpha_y, \alpha_x \neq 0$, the averages:

$$\overline{y} = y_0 + \overline{S}_y + \alpha_y \overline{t} \text{ and } \overline{x} = x_0 + \overline{S}_x + \alpha_x \overline{t} \quad ,$$

where $\bar{S}_{(.)} = \frac{\sum\left(\sum_{1}^{T} e_{(.)t}\right)}{T}$, that is, the average of cumulative sum of error terms (3.16)

diverge and do estimate the $E(y_t), E(x_t)$, see Johansen (2007).

3.5 A simple modification

Consider again equation (3.5) with a modification of y_t, x_t so that we have the following equation:

$$y_t^* = (1-\gamma) + \gamma x_t^* + u_t \quad (3.17)$$

It is well known that the OLS estimation of the constant term is:

$$(1-\hat{\gamma}) = \bar{y}^* - \hat{\gamma}\bar{x}^* \quad (3.18)$$

From this, is not difficult to see that the modified variables have to have "means" equal to one to meet the restriction (3.18), i.e.,

$$\bar{y}^* = \bar{x}^* = 1 \quad (3.19)$$

Now, let us look on another simple relationship in the regression between two variables:

$$\hat{\gamma} = r_{x^*,y^*} \frac{s_{y^*}}{s_{x^*}} \quad (3.20)$$

Based on that relationship, for the gamma estimation to meet the restriction $0 < \gamma \le 1$ it must be that $\left(s_{x^*}\right) = \left(s_{y^*}\right)$. In that case the correlation coefficient is equal to gamma.

The simplest way to meet this restriction and equation (3.19), while maintaining the same relationship between the two variables, is to use the following simple transformation:

$$y_t^* = \left[\left(y_t - \overline{y} \right) / s_{y_t} \right] + 1, \quad x_t^* = \left[\left(x_t - \overline{x} \right) / s_{x_t} \right] + 1, \tag{3.21}$$

In this way both variables have the same means and the same standard deviations equal to one. And in that case the correlation of these two variables should be a good estimate of gamma. But we do not need to make all these data transformations, as gamma is just the correlation coefficient in a static regression system.

However, because below we have time series in regression (3.19) the results become more complicated. For that reason we study estimation methods with a Monte-Carlo experiment.

4. Monte-Carlo simulation

We performed a Monte-Carlo experiment by generating data according to the model defined by

$$y_t = c + \delta x_t + e_{2t}, \tag{4.1}$$

$$x_t = a_x + x_{t-1} + e_{1t}, \tag{4.2}$$

Simulations were made for three main versions of this model:

Model 1:

The error components $\left(e_{1t}, e_{2t} \right)'$ in model 1 were generated as

e_{it} ~i.i.d., $\mathrm{E}(e_{it}) = 0$, $\mathrm{E}(e_{it}^2) = 0.25$ and $\mathrm{Cov}\left(e_{1t}\ \mathrm{e}_{2t} \right) = 0$.

Model 2:

The error components $(e_{1t}, e_{2t})'$ in model 2 were generated as

e_{1t}~i.i.d., $\mathrm{E}(e_{1t}) = 0$, $\mathrm{E}(e_{1t}^2) = 0.25$

$e_{2t} = 0.5e_{2t-1} + u_t$ u_t~i.i.d., $\mathrm{E}(u_{1t}) = 0$, $\mathrm{E}(u_t^2) = 0.25$

and $\mathrm{Cov}(e_{1t}\ e_{2t}) = 0$.

That is, there is autocorrelation in the co-integrated regression´s error terms. The drift term is $a_x = 0.23$ in both models.

The variables y_t and x_t are integrated of order one, I(1), and are co-integrated. Without loss of generality, the co-integrating vector for (y_t, x_t)´ is (1, -8.43)´ . This setting is because this is the co-integration vector in the applied example that will be studied later.

For each time series, 50 pre-sample values were generated with zero initial conditions, taking net sample sizes of T = 25, 50, 75 and 200.

Three sub models were also estimated for each model:

1) with c =0, that is, no constant in the co-integrated regression
2) with c =17.25, that is, positive constant in the co-integrated regression
3) with c =-17.25, that is, negative constant in the co-integrated regression

Model 3:

Finally to study the variance effect, the following model was simulated:

The error components $(e_{1t}, e_{2t})'$ in this model 3 are generated;

e_{1t}~i.i.d., $\mathrm{E}(e_{1t}) = 0$, $\mathrm{E}(e_{1t}^2) = 0.0215$

$e_{2t} = 0.5e_{2t-1} + u_t$ u_t~i.i.d., $\mathrm{E}(u_{1t}) = 0$, $\mathrm{E}(u_t^2) = 1.04$

and $\mathrm{Cov}\left(e_{1t}\ \mathrm{e}_{2t}\right) = 0$.

4) With drift term is $a_x = 0.023$ and net sample 62 observations. The cointegrating vector for $(y_t, x_t)'$ is also here (1, -8.43)´ and with c =-17.25, that is, negative constant in the co-integrated regression.

Finally the number of Monte Carlo replications per model is 1,000. The calculations were performed using GAUSS 12.

5. Results of the Monte Carlo simulations.

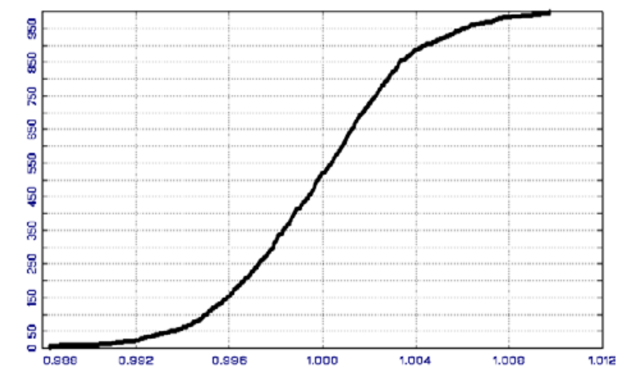
In the simulation for model 1 as was shown in the previous section we expected that estimated coefficient should be near to one, both for *Weitzman*´s "mean standardized" method and the suggested alternative, the *standardized transformed* method.

Figure 1 shows the Weitzman´s method with white noise and with autocorrelation in the errors, model 1 and 2, case 1, that is, without a constant in the co-integrating regression.

The estimates behave as we expected, close to one coefficient even for 25 observations between 0.988 and 1.01, (2 decimal accuracy). Also, the autocorrelation effect is very small between 0.980 and 1.017.

Figure 1: Model 1 and 2 case 1 ***Weitzman´s method for estimating the gamma***

Figure 1a 25 Observations White noise	Figure 1c 25 Observations AR(1) errors



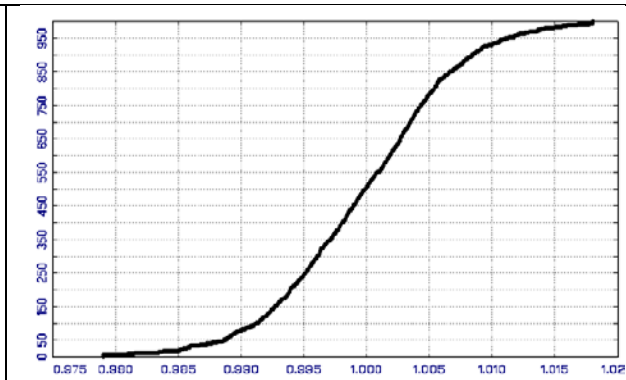
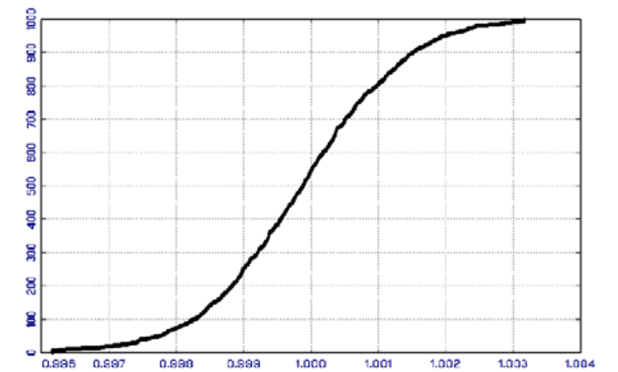


Figure 1b 50 Observations White noise

Figure 1d 50 Observations AR(1) errors



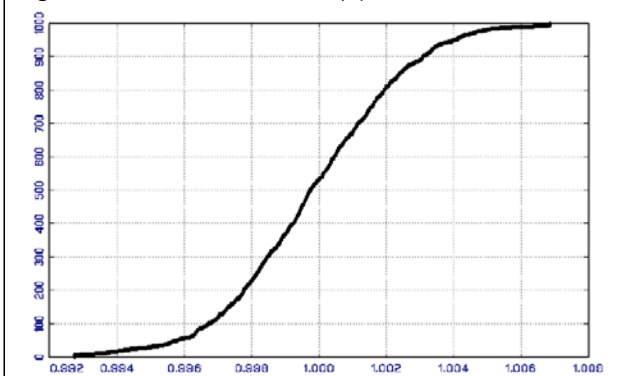
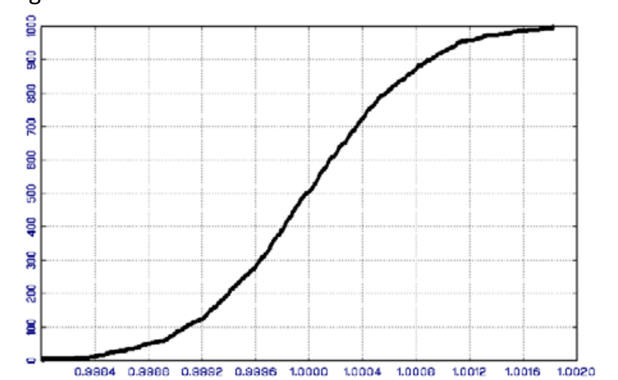


Figure 1c 75 Observations White noise

Figure 1e 75 Observations AR(1) errors



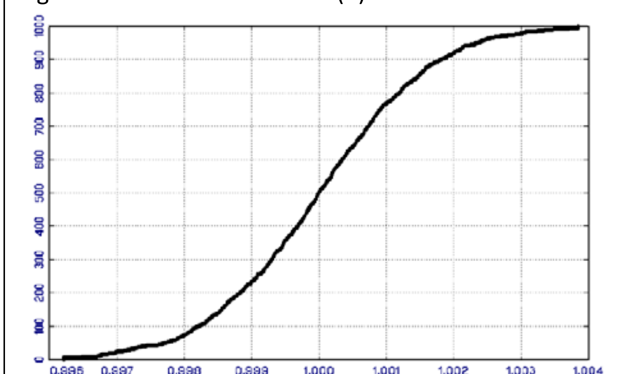
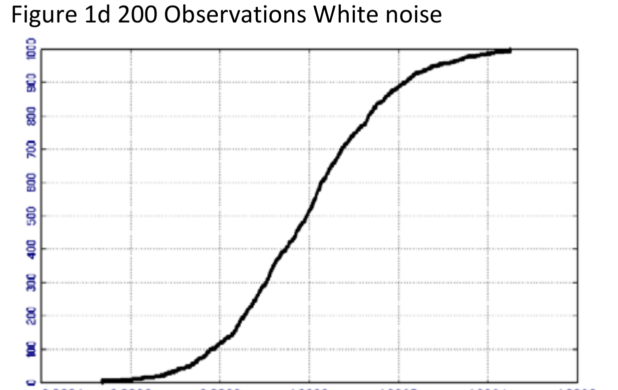


Figure 1d 200 Observations White noise

Figure 1f 200 Observations AR(1) errors



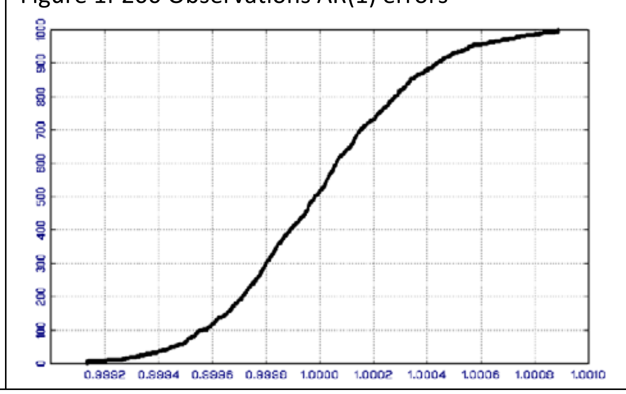


Figure 2: Model 2 case 1 ***Standardized transformed series for estimating gamma***

Figure 2a 25 Observations

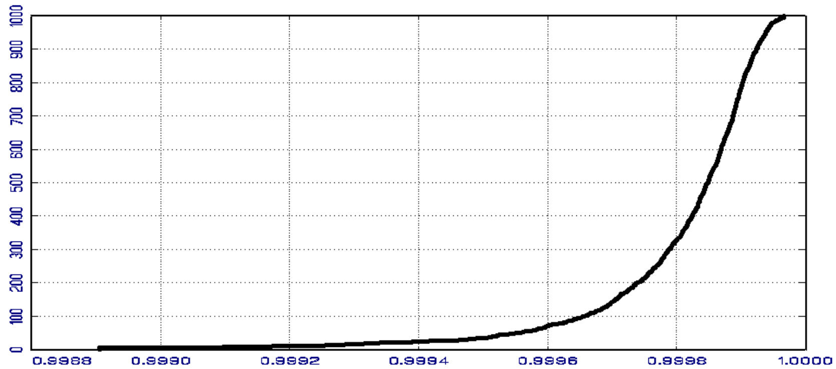


Figure 2b 50 Observations

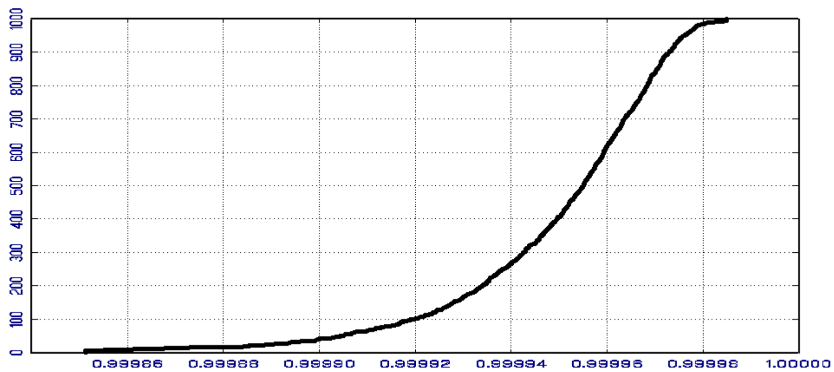


Figure 2c 75 Observations

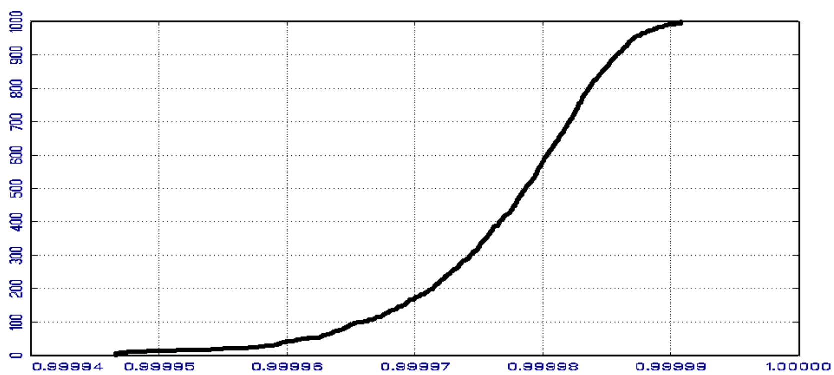
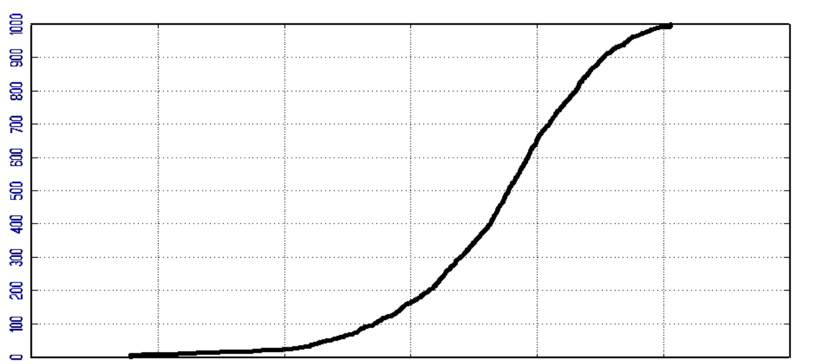


Figure 2d 200 Observations



In the co-integration case, we have super-consistent estimations of the regressions coefficient, which here leads to accuracy at the third decimal, even for a sample of 75 observations, see figures 1c and 1e.

The difference in estimation of gamma between model 1 with white noise and model 2 with autocorrelation in errors is very small, so we show only the “worst” case results from estimation of model 2.

Figure 2 shows the results from estimating gamma on the mean standardized transformed series. It can be observed that as expected (after all, is equal to correlations coefficient) no estimates exceed 1. Moreover, even for 25 observations we have accuracy at the third decimal, between 0.9989 and 0.99989. For 50 observations, we have accuracy of at the fourth decimal and for 200 observations at the sixth decimal.

Summarizing the results for models 1 and 2 in the case 1 both methods perform satisfactorily. However we prefer our method of standardized transformed series for estimating gamma because it´s always less than one and the same time very near to one, that is, more efficient than Weitzman´s method.

Figure 3 shows the results for Model 2 and case 2 for 75 observations. Here the results agree with the results presented in chapter 3.2 case (a). Weitzman´s method shows estimates less than the expected one, between 0.847 and 0.92, while the method of standardized transformed series performs better again with results between 0.999945 and 0.999991, that is, our method is more robust than the Weitzman´s.

Figure 3: Model 2 case 2 ***Weitzman´s and Standardized transformed methods for estimating gamma***

Figure 3a 75 Observations ***Weitzman´s***

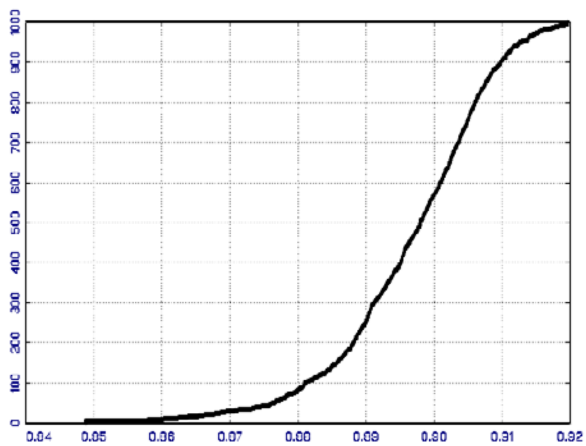


Figure 3b 75 Observations ***Standardized transformed***

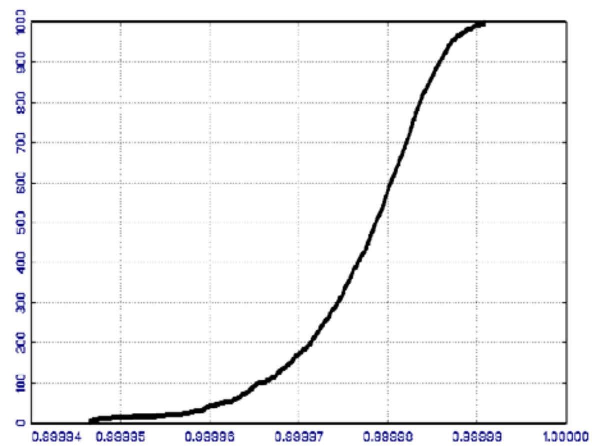


Figure 4 shows the results for Model 2 and case 3 for 75 observations. Weitzman´s method gives estimates that exceed the expected one, between 1.08 and 1.145. Once more our method of standardized transformed series performs better again and it seems not to be affected by the sign of the constant term.

Figure 4 Model 2 case 3 ***Weitzman´s and standardized transformed methods for estimating gamma***

Figure 4a 75 Observations ***Weitzman´s***

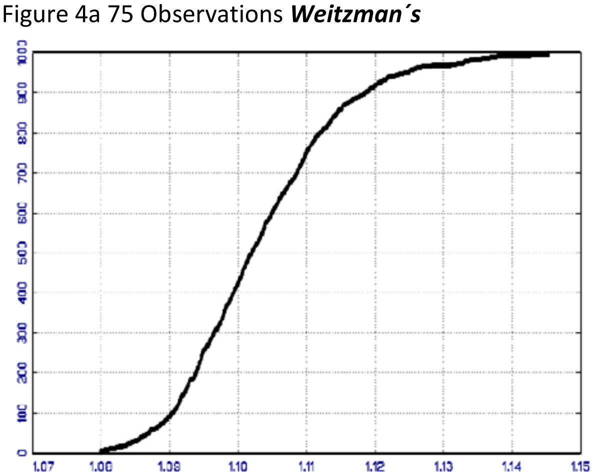
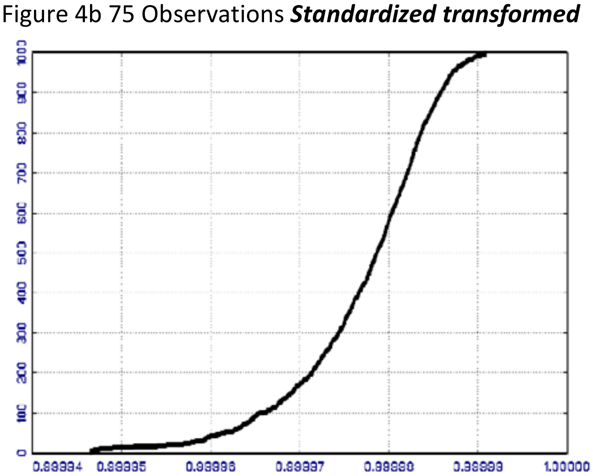


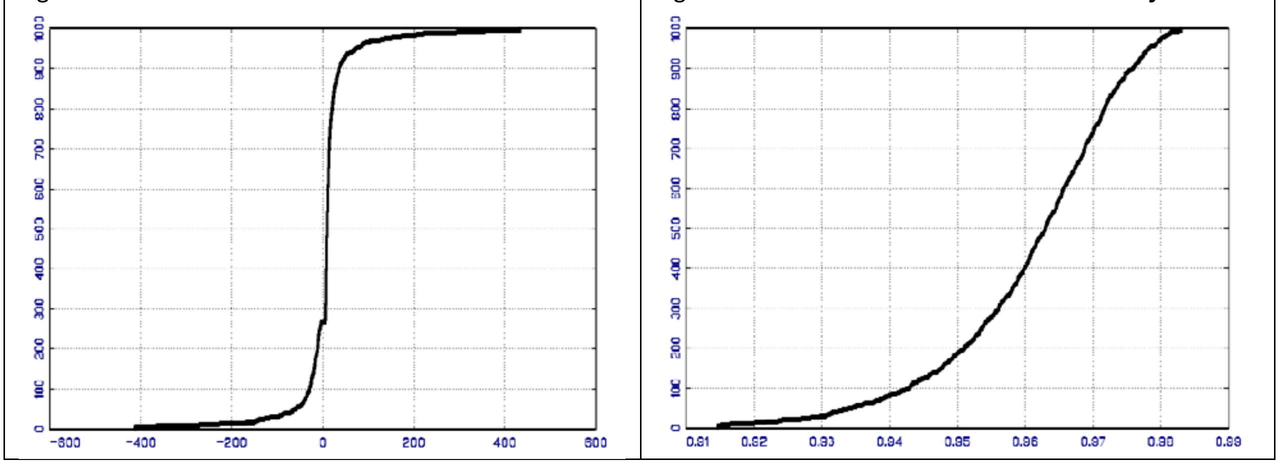
Figure 4b 75 Observations ***Standardized transformed***



Finally figure 5 shows the results of the variance effect for model 3. Here the variance of y_t is quite larger than the variance of x_t . We can observe that Weitzman´s method gives estimates that not only exceed the expected one, but also estimate values less than one. As figure 5a shows that the greatly biased estimation, with a values as big as 400 and negative as -400, of the gamma with Weitzman´s method, while the standardized transformed method for estimating the gamma is still good and near to one with only 1 decimal accurate.

Figure 5 Model 3 ***Weitzman´s and standardized transformed methods for estimating gamma***

Figure 5a 61 Observations ***Weitzman´s*** | Figure 5b 61 Observations ***standardized transformed***



Note that we use 61 observations in figure 5 because this case is similarly to the follow applied example.

Summarizing the results of our simulations is that we prefer the method of standardized transformed series for estimating gamma because it´s more efficient than Weitzman´s method and robust in all the studied cases with or without constant in co-integrating regression. Moreover, it is much more robust in the cases when the two variables have different variances.

6. An empirical example.

The analysis here uses the same data that Hultkrantz et al. (2014) have studied. We use the 61 observations of the logarithm of GDP in Sweden and the series “RailT” annual ton-kilometer data for railroads for the period 1950–2011. We follow the same notation as in Hultkrantz et al. (2014), so the two variables will be called LnGDP and RailT.

Figure 6a shows the original series, while 6b shows the Weitzman´s transformation of the data, finally, 6 c shows the standardized transformed series.

Figure 6a shows the two raw series that we use in our example it is difficult to say anything more that show a kind of trend, stochastic or deterministic. A unit root test showed that the series follow a random walk with drift, (see Hultkrantz et al. 2014).

Table 1 shows the summary statistics and we observe the interesting thing that the standard deviation of the RailT is 10 times larger than the standard deviation of LnGDP. Note also that we have adapted these statistics in our simulation model 3.

Figure 6b shows Weitzman´s transformation while Figure 6c shows the standardized transformed series. The last figure reveals also the possible co-integration of these series.

Figure 6. ***Original, Weitzman´s and standardized transformed, GDP and RailT series.***

Figure 6a

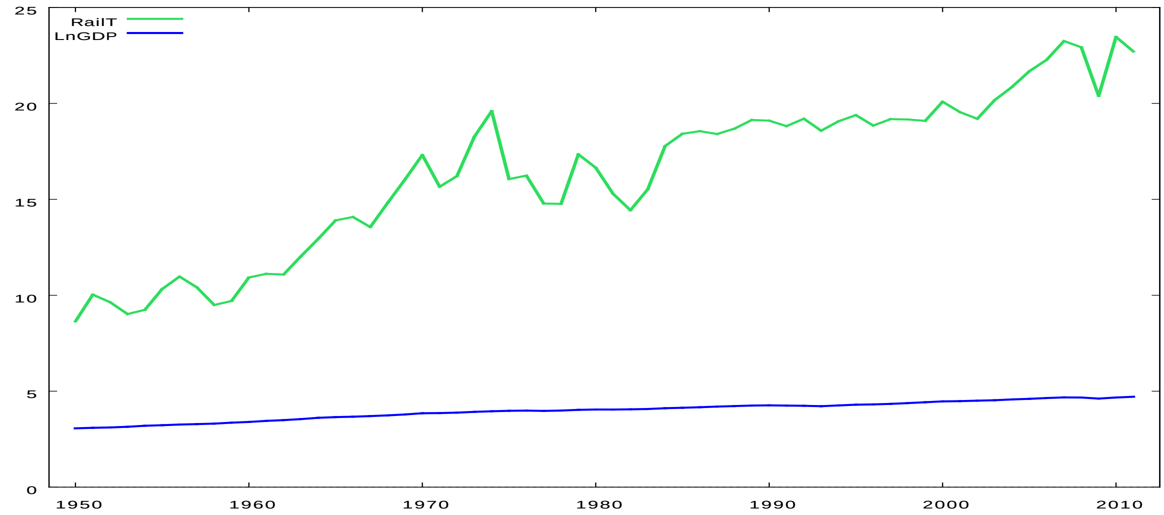


Figure 6b

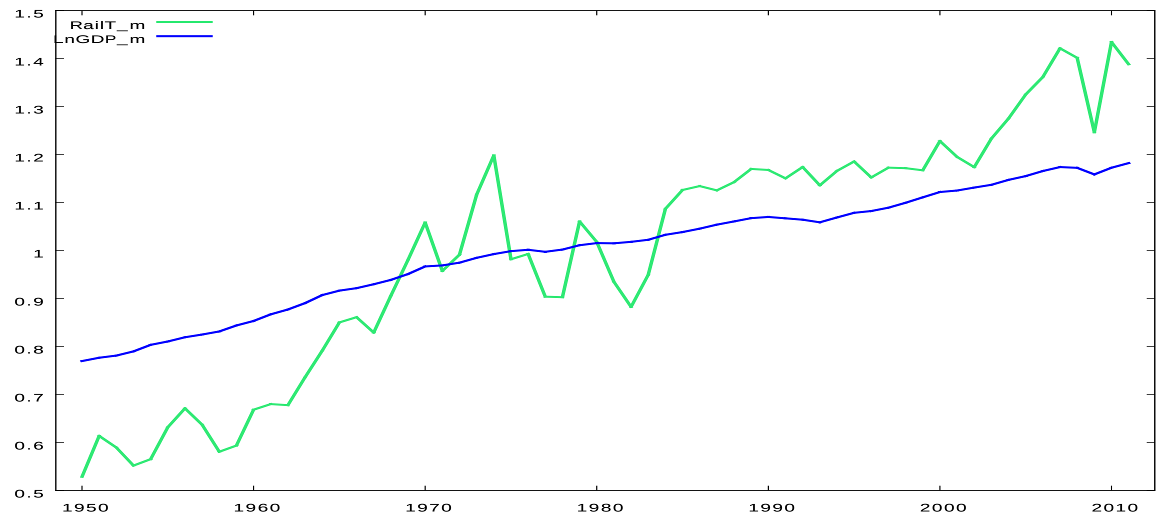


Figure 6c

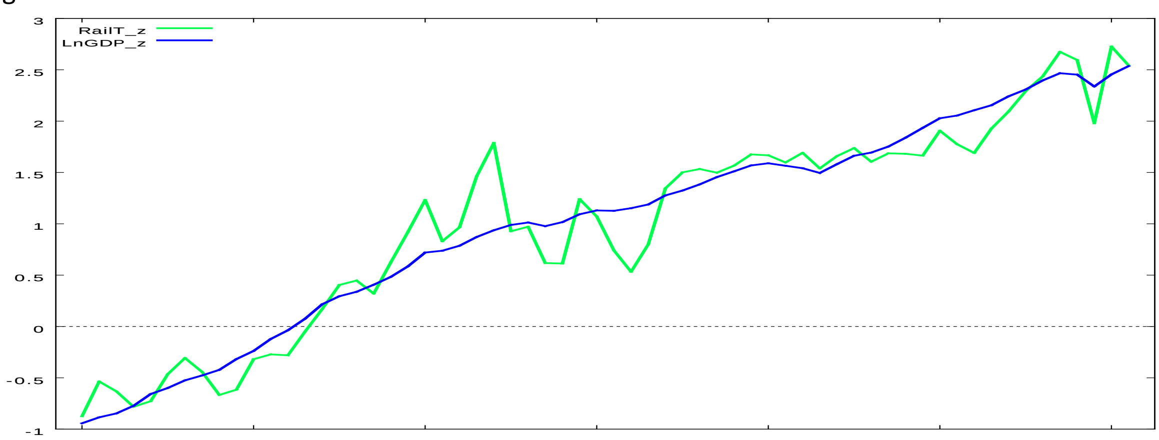


Table 1: Summary Statistics for the variables RailT and LnGDP

RailT (1950 – 2011)				**LnGDP** (1950 – 2011)			
Mean	Median	Minimum	Maximum	Mean	Median	Minimum	Maximum
16.3560	17.3290	8.64000	23.4640	3.98679	4.04743	3.06805	4.71313
Std.Dev.	C.V.	Skewness	Ex. Kurt.	Std. Dev.	C.V.	Skewness	Ex. Kurt.
4.11692	0.251707	-0.332079	-0.933177	0.472864	0.118608	-0.365175	-0.893852

Table 2 shows that the unit-root hypothesis is rejected (t-statistic = **-4.39952**) for the residuals (u-hat) from the co-integrating regression. That is, there is evidence for a co-integrating relationship. Now based on that result we expect that the "gamma" should be near or equal to one. So we estimate "gamma" with Weitzman´s standarized data, Table 3 and with the standardized transformed series, Table 4***.***

Table 2 Engle-Granger 2 steps co-integration test

Step 1: Co-integrating regression

Co-integrating regression - OLS, using observations 1950-2011 (T = 62)   Dependent variable: RailT

	coefficient	std. error	t-ratio	p-value
const	**-17.2505**	1.12884	-15.28	2.37e-022 ***
LnGDP	**8.42946**	0.281207	29.98	8.33e-038 ***

Step 2: testing for a unit root in uhat

Augmented Dickey-Fuller test for uhat including one lag of (1-L)uhat sample size 60
unit-root null hypothesis: a = 1

model: (1-L)y = (a-1)*y(-1) + ... + e
1st-order autocorrelation coeff. for e: 0.015000
estimated value of (a - 1):   -0.558119
test statistic: tau_c(2) =   **-4.399520**
asymptotic p-value   0.001732

As we expected based on our Monte Carlo results, table 3 shows that the estimated gamma with Weitzman´s method is long away from the expected "one" coefficient is more than 2,

while table 4 shows an estimated gamma very near to "one" **(0.968197)** with the standardized transformed series.

Note also by using Maximum likelihood and take in account the autocorrelation of the error term, with standardized transformed series we get gamma equal to **0.978543.**

Table 3 ***Weitzman´s method for estimating gamma***

OLS, using observations 1950-2011 (T = 62)
Dependent variable: RailT_m

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	**-1.05469**	0.069017	-15.2815	<0.00001	***
LnGDP_m	**2.05469**	0.0685443	29.9760	<0.00001	***

Mean dependent var	1.000000	S.D. dependent var	0.251707
Sum squared resid	0.241908	S.E. of regression	0.063496
R-squared	0.937406	Adjusted R-squared	0.936363
F(1. 60)	898.5620	P-value(F)	8.33e-38

Table 4 ***Standardized transformed method for estimating gamma***

OLS, using observations 1950-2011 (T = 62)
Dependent variable: RailT_z

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	**0.0318026**	0.0454932	0.6991	0.48721	
LnGDP_z	**0.968197**	0.0322991	29.9760	<0.00001	***

Mean dependent var	1.000000	S.D. dependent var	1.000000
Sum squared resid	3.818219	S.E. of regression	0.252264
R-squared	0.937406	Adjusted R-squared	0.936363
F(1. 60)	898.5620	P-value(F)	8.33e-38

7. Conclusion and Summary.

In this paper we suggest approaches for how to estimate the Weitzman (2012) gamma in the polar cases when the project return and the macroeconomic variable are co-integrated. We use Weitzman (2012) approach, and our simple data transformation that keeps the “real project gamma” within the zero to one interval.

With a Monte Carlo study we show that our method of Standardized transformed series for estimating gamma is better, because it´s always less than one and the same time very near to one, that is, more efficient than Weitzman´s method.

Finally, we demonstrate the same findings in a Monte Carlo experiment and show the superiority of our method in an application based on Swedish time-series data from 1950-2011 for annual time-series data for rail freight rents and GDP.

Now we are comfortable that by using historical data and the method of standardized transformed series in the polar case when the project returns and the macroeconomic variable are co-integrated, we have a method for estimating gamma that is robust. Moreover, this inspires us to investigate in a new paper the case when the project returns and the macroeconomic variable are not co-integrated.

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