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Choosing Opponents in Skiing Sprint Elimination Tournaments

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Abstract

In this study we analyse data from world cup cross-country skiing sprint elimination tournaments for men and women in 2015-2020. Instead of being assigned a quarterfinal according to a seeding scheme, prequalified athletes choose themselves in sequential order in which of five quarterfinals to compete. Due to a time constraint on the day the competition is held, the recovery time between the knockout heats varies. This implies a clear advantage for the athlete to race in an early rather than in a late quarterfinal to maximize her probability of reaching the podium. The purpose of the paper is to analyse the athletes' choices facing the trade-off between recovery time and expected degree of competition when choosing in which quarterfinal to compete. We find empirical support for the prediction that higher ranked athletes from the qualification round prefer to compete in early quarterfinals, despite facing expected harder competition. Nevertheless, our results also suggest that athletes underestimate the value of choosing an early quarterfinal. In addition, we propose a seeding scheme capturing the fundamental disparity across quarterfinals using the estimates from a logistic regression model.

JEL Classifications: C51, C72, Z20

Keywords: Choosing opponent, sequential choice, seeeding, skiing sprint,

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1. Introduction

Many sports contests take the form of an elimination tournament, in which the matching of contestants in pairs or in groups often is based on their performance in a preceding qualification tournament.[1] The designer's goal when determining the order of the knockout games is often to avoid that higher ranked contestants meet each other in early rounds in the tournament, but instead face lower ranked opponents until it is time for the last thrilling knockout rounds in the tournament. Very seldom, the higher ranked contestants from the qualification stage can choose their opponents during the subsequent (first) phase of the knockout stage. However, this unusual design has been used by the Swedish national hockey league when entering its playoff during the seasons 2006-2014 and the method is still applied in the country's lower hockey leagues. There has also been proposals to apply a similar scheme in the Major League Baseball (*MLB*) in the US, where the top team in each league would pick its playoff opponent.[2] Guyon (2019) presents how a "choose your opponent" format would work for the UEFA Champions League. The argument is that this method would induce stronger incentives for the teams to improve their performance in the preceding qualification tournament (the group stage), thereby making these games more exciting. Also, the occasion at which the teams publicly announce their choice of opponent would attract a lot of media attention and bring a new dimension to the knockout stage.

In this paper we analyse the outcome in cross country skiing sprint contests, a knockout tournament organised by the International Ski Federation (FIS). The organisation previously applied a conventional method to seed qualified athletes. However, the last five years it has adopted an approach based on athletes partly are choosing their opponents in the knockout stage after the preceding qualification round. These cross-country skiing contests, each stretching out only for one day, are held about twelve times per season at various places in Europe, Russia, and Canada. Each contest starts with a qualification round before the elimination stage which includes two rounds besides the final; the quarterfinals and the semifinals. Within the old design, an athlete was assigned one of the five quarterfinals based on his ranking in the qualification round. At the end of the season 2014/2015, the FIS decided to change the design and the qualified athletes were free to choose one of the five quarterfinals themselves, according to a certain order based on their ranking in the qualification round. The main motive behind the change of design was to internalize the positive effect athletes potentially obtained from having a longer recovery time before a future final if they competed in one of two early quarterfinals rather than in one of the two late quarterfinals. As the new design was introduced, the chairman of the FIS cross country committee, Vegard Ulvang, 1992 Olympic champion and one of the creators behind this novelty, articulated the conjecture that the higher ranked athletes, due to the expected longer recovery duration prior to a final, would foremost choose to compete in one of the first two quarterfinals rather than in any

[1] See Groth et al. 2012 for a listing of examples of sports events organised in such a way

[2] https://www.nbcsports.com/philadelphia/phillies/mlb-playoff-format-pick-opponent-14-teams-phillies

of the late quarterfinals although this likely would result in tougher competition already in the quarterfinal.

Our study sets out to test this behavioural presumption, labelled *Ulvang's conjecture.* Are the two first quarterfinals overrepresented by higher ranked athletes from the qualification round? If this conjecture turns out to be correct, the inevitable follow-up question would be if the proportions of higher ranked athletes in different quarterfinals are balanced in such a way that the probability of reaching the podium (top three in the final) is equalized across quarterfinals. If the proportions are not balanced, is it possible, by using data on the athletes' choices of quarterfinals and their performances, to come up with a seeding scheme capturing the asymmetry in recovery duration that follows from the assignment of quarterfinals?

In order to gain an understanding as to the optimal choice of quarterfinal in the current design, we consider a simple knockout tournament model with two rounds – semifinals and final. The aim of the model is to capture an athlete's (henceforth denoted player) trade-off between recovery time and expected degree of competition when choosing in which of two semifinals to compete. In our model four players of two types in sequential order choose semifinal. The winning probabilities are in our model exogenous, that is, players' exerted effort is not strategically allocated across the two rounds. Our model predicts that higher ranked players are more likely to compete against each other in the first semifinal rather than in the second semifinal.

Also, we develop a test statistic suitable for the purpose of testing Ulvang's conjecture, that is, we test whether high ranked athletes from the qualification round to a larger extent choose early quarterfinals rather than late quarterfinals.[3] We also develop a method in order to test whether athletes make choices consistent with the objective of maximizing the probability of reaching the podium, facing the trade-off between expected competition in different quarterfinals and variation in recovery time. Finally, we suggest a seeding scheme capturing the athlete's advantage of being assigned one of the two early quarterfinals. To our best knowledge, no study has yet been conducted on modelling this type of tournament design, or on using contestants' observed choices to propose a seeding scheme.

2. The Skiing Sprint Competition

An individual skiing sprint competition in the cross-country World Cup begins with a prologue, a qualification round, where the 60 to 80 athletes ski a course which has a length of about 1.5 km, each athlete starting at about 15 second intervals. The 30 fastest athletes qualify for the five quarterfinals, with six athletes in each quarterfinal. The athletes who come first and second place in the first two quarterfinals, are placed in the first semifinal, and the athletes coming first and second place in the last

[3] It should be pointed out, that the ranking assigned to an athlete in the qualification round - an order number that we make use of in the empirical analysis - marks out her place in the qualification round. That is, the lower the athlete's ranking number from the qualification round, the higher the athlete's ranking.

two quarterfinals, are placed in the second semifinal. The winner of the third quarterfinal, is placed in semifinal one while the athlete at second place goes to semifinal two. In addition to these top-ten athletes from the five quarterfinals, the two athletes with the best times of the athletes ending up at place 3-6 in the quarterfinals (the lucky losers) are also qualified for the semifinals. The faster of these two is placed in semifinal one while the other athlete is placed in semifinal two. Finally, the top two athletes in each semifinal together with the two lucky losers from the semifinals are qualified for the final. All finals are run on the same course as the qualification round and mass start is applied in all knockout races. The format of the competition is identical for men and women who run the races alternately. The 30 athletes that qualify for the quarterfinals are awarded world cup ranking points, conditioned on their performance in subsequent knockout rounds. The timing of the competition during a day is illustrated in Figure 1.

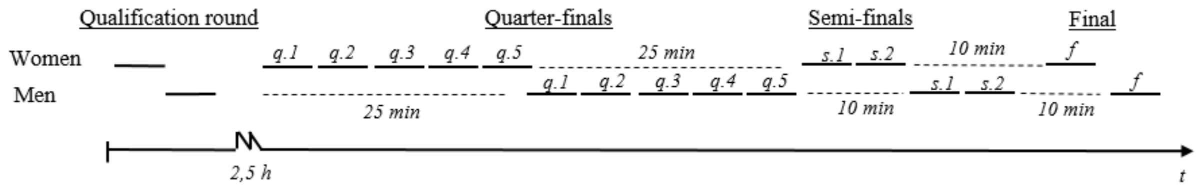

Figure 1. The timing of skiing sprint competition

The sequential ordering of the quarterfinals and semifinals means that the recovery time for those athletes advancing from one stage to the next stage will differ. However, the recovery time between the different races is considered to be long enough not to affect the performance in subsequent heats. An exception is the relative short duration between the semifinal two and the final, where the athletes advancing from semifinal two are disadvantaged relative those athletes coming from semifinal one. Consequently, the assignment of quarterfinal may be crucial for the chances to succeed in a final.

2.1 Seeding in the Old Design

The seeding of the athletes in the quarterfinals, from the start of the skiing sprint competition until the late season 2014/2015, was determined by their ranking from the qualification round, in which the fastest skier was assigned the rank number one, the second fastest skier was assigned rank number two, and so on.

Table 1. Seeding scheme in the quarterfinals, old design

	Quarterfinal (Q)				
	Q_1	Q_2	Q_3	Q_4	Q_5
Ranking of athlete	1	4	5	2	3
	10	7	6	9	8
	11	14	15	12	13
	20	17	16	19	18
	21	24	25	22	23
	30	27	26	29	28
Rank sum	93	93	93	93	93

The seeding method applied can best be described as a standard seeding. The sum of ranks from the six qualified athletes in each of the five quarterfinals was 93, where the five best ranked athletes as well as the five worst ranked athletes among the top 30 athletes from the qualification round were placed in separate quarterfinals. The seeding scheme is shown in Table 1.

In 2014 the FIS decided to change the seeding method as it turned out that athletes assigned the first two quarterfinals were overrepresented among the medallists (top three in the final) in the competitions. The effect of a shorter recovery time before the final, when advancing from semifinal two rather than from semifinal one, became evident in the result lists.

Tables 2a and 2b show how medallists were distributed across the five quarterfinals for men and women for the seasons 2009/2010 – 2014/2015.

Table 2a. Share of medallists –men - from the quarterfinals within the old design

	Season						
Quarterfinal	09/10	10/11	11/12	12/13	13/14	14/15	Total
q.1	0.33	0.21	0.31	0.27	0.21	0.25	0.27
q.2	0.28	0.27	0.38	0.20	0.33	0.25	0.29
q.3	0.11	0.15	0.15	0.27	0.33	0.25	0.21
q.4	0.19	0.27	0.0	0.13	0.03	0.08	0.12
q.5	0.08	0.09	0.15	0.13	0.09	0.17	0.12

Source: FIS

Table 2b. Share of medallists –women - from the quarterfinals within the old design

	Season						
Quarterfinal	09/10	10/11	11/12	12/13	13/14	14/15	Total
q.1	0.31	0.33	0.36	0.23	0.30	0.33	0.31
q.2	0.08	0.18	0.10	0.27	0.12	0.21	0.15
q.3	0.19	0.12	0.13	0.10	0.18	0.17	0.15
q.4	0.17	0.12	0.28	0.17	0.21	0.12	0.18
q.5	0.25	0.24	0.13	0.23	0.18	0.17	0.20

Source: FIS

From Table 2a, it is clear that the athletes assigned to one of the two early quarterfinals had an advantage over those athletes placed in a late quarterfinal. The percentages of medallists coming from the two first quarterfinals are more than twice as large as the corresponding share coming from quarterfinals four and five. As can be seen from Table 2b, the pattern for women is not as clear as for men. Yet, the share of medallists coming from the first quarterfinal dominates.

2.2 The Current Design

To eliminate the advantage of being assigned one of the two first quarterfinals more or less by pure luck, a new approach to distribute the qualified athletes across the five quarterfinals was adopted at the end of the season 2014/2015. Instead of letting a predetermined seeding scheme decide which ranked athlete that is assigned a certain quarterfinal, the athletes now choose the quarterfinal themselves, according to a predetermined order. The ordering of choices is based on the ranking from the qualification round, where the eleven highest ranked athletes first make their choices in descending ranking order, starting with the 11th ranked athlete, $\{11,10...2,1\}$. The remaining 19 athletes then make their choices in ascending ranking order, starting with the 12th ranked athlete, $\{12,13...29,30\}$. The choices are revealed immediately, that is, an athlete who is about to choose her quarterfinal knows about the previous choices.

In an interview for the Norwegian television channel NRK (2014), the chairman of the FIS cross country committee, Vegard Ulvang, addressed the new tactical dimension now entering the skiing sprint competitions:

> "*Then you are rewarded for a good performance in the prologue. The fastest athletes have the possibility of choosing whether they wish to meet the toughest competitors in an early heat or if they prefer shorter recovery time and weaker competitors in a later heat. This might be a way of making things a bit fairer. It will be a tactic assessment the performer has to make, thereby increasing the pressure on the athlete*".

Ulvang conjectured that a larger fraction of the high ranked athletes from the qualification round would choose to compete in the first quarterfinals rather than in the late quarterfinals. The disadvantage of expected tougher competition in an early quarterfinal, relative a late quarterfinal, would be outweighed by the advantage of facing longer recovery time before the final.

3. Literature

This study is related to the body of literature in physiology on the effects of recovery duration in cross country skiing sprint. To bring down the accumulation of fatigue, that is, to reduce the concentration of blood lactate, the recovery time between the heats is essential. A comprehensive survey on factors influencing the performance of skiing sprint is provided by Hébert-Losier et al. (2017). The outcome from experimental tests, reflecting the format of skiing sprint competitions - e.g., type of participants, duration of exercises, equipment, number of heats- indicates that the recovery time should be about 20 minutes to fully ensure that a break does not impact the athlete's performance in subsequent heat. Zory et al. (2006) carried out an experiment with seven male skiers from the Italian national sprint team. Each individual skied in three heats a distance of 1380 m on a double-poling ergometer (~2:50 min/heat). Between the heats the skiers had a recovery period of 12 minutes. Their results show a significant decrease in upper-body power output as well as in force, suggesting a presence of fatigue. Vesterinen et al. (2009) conducted a similar experiment in which sixteen male cross-country skiers, on roller skis, skied four 850-m heats (~2:20 min/heat) separated by 20 minutes of recovery. The authors find no differences in performance between the heats, that is, no evidence of accumulation of fatigue. Moxnes and Moxnes (2014) develop a mathematical model showing how anaerobic portion of total energy depends on time. Using the relationship between blood lactate levels and oxygen consumption, they derive the time it takes for a given level of blood lactate concentration to come down to a concentration level that equals the level at the start of the race. Given a racing time around 3 minutes and 20 seconds, they find that if the recovery time is below 20 minutes, then performance in a subsequent heat will deteriorate.

A common approach when modelling the effects of fatigue and recovery upon performance in tournaments is to assume dependence across matches or heats. Players or teams are constrained how much effort they can exert during a tournament. Hence, the predicted outcome of a tournament is considered to be the result of how players, often assumed to have asymmetric abilities, strategically choose to allocate their effort across matches (see Ryvkin (2011) for a survey). Theoretical and empirical support can be found both for the "burnout hypothesis" - players do not withhold effort but instead perform at their best at any stage of the tournament (e.g Amegashie et al (2007) – and for the hypothesis that players strategically exert different effort at various stages of the tournament (e.g. Groh et al. (2012)). Harbaugh and Klumpp (2005) find that the introduction of a rest day between matches in the NCAA men's basketball tournament increased the probability of winning for the superior team relative the inferior team. The reason is that the need to allocate effort across the matches disappeared, implying that both teams could exert their full strength in each match.

In our model we assume effort independence across matches or heats, that is, a player does not choose her effort level, but instead performs her maximum in each round. The duration between each round is assumed long enough for complete recovery, with one important exception. In our model, a player's choice in which of two semifinals she wants to compete, will be dependent on that particular exception. In other words, in maximizing her probability of winning the tournament, a player is assumed to act strategically when choosing between matches.

The last part of our study deals with seeding in elimination tournaments. As noted above, the seeding method applied by FIS until the change of design in the late season 2014/2015, resembles a standard seeding, albeit the presence of more than two participants in each quarterfinal. The standard seeding, where top ranked athletes are matched head-to-head against lower ranked athletes in the first round, often serves as a reference point when analysing the properties of various design of elimination tournaments (e.g Hwang 1982, Marchand 2011, Groth et al. 2012, Karpov 2016). The standard seeding is widely used because the draw seeks to delay the confrontation between the tournament's best athletes until the very last rounds, increasing the quality of the matches as the tournament progresses. If spectators' preferences for the overall strength of the athletes are stronger than their preferences for competitive intensity (balance in strength) in a match, then Dagaev and Suzdaltsev (2018) show that standard seeding is optimal. In other words, the probability of winning the tournament should be increasing in athletes' ranking. However, a number of studies provides evidence that this property of monotonicity does not hold in general in the standard seeding method. Theoretical contributions by Hwang (1982) and Horen and Riezman (1985) show that if the number of participants in the elimination tournament is more than four, then there exists no unique draw that satisfies monotonicity. The optimal draw depends on the probability strength matrix , that is the matrix capturing the probability that athlete *i* wins against athlete *j*. Empirical analyses of elimination tournaments indicate that the probability of winning is positively related to the difference in ranking, but the probability does not increase monotonically (e.g Boulier and Stekler (1999), Khatibi et al. (2015)). An alternative to the standard seeding, the equal gap seeding, is proposed by Karpov (2016). In contrast to the standard seeding, where the sum of players' ranking in any of the matches in the first round is the same, the equal gap seeding sets out that the difference in players' ranking in any of the matches in the first round is the same. Given the assumption of the domain of winning probabilities, Karpov shows that the equal gap seeding satisfies a number of tournament ideals.

Besides the criteria of finding a seeding that delays the confrontation between the two highest ranked athletes, Groh et al. (2012) consider other goals that the designer may want to meet with the seeding. In their model, four players are seeded according to their ranks and simultaneously play pairwise in two semifinals, where the winners compete in the final. The winning probabilities are endogenous, that is, the outcome of the tournament depends on the players' strategic choice of effort exerted in each match. Their theoretical point of departure, when analysing which of the three possible seedings {*A*:1-4, 2-3; *B*:1-3, 2-4; *C*:1-2, 3-4} that optimizes a specific goal, is the behaviour in an all-pay auction. They show that seeding *A* and *B* maximize different goals and both seedings are superior to seeding *C* in all goals considered.

Our contribution to the existing literature on seeding is a result of our empirical analysis of observed behaviour in an elimination tournament, where players to some extent may choose their opponents in the first knockout round. Making use of our estimated regression parameters, given our sample of athletes, we propose a seeding scheme where the goal is to internalize the inherent disadvantage of being assigned one of the two last quarterfinals in skiing sprint competitions. However, our seeding does not indicate in which of the quarterfinals a ranked athlete is assigned. Instead, it stipulates what the athletes' ranking from the qualification round should sum up to in each of the five quarterfinals.

4. Modelling the Choice

In this section we set up a simple knockout tournament model where (some) players can choose their opponents. To make the model computational tractable, we consider a tournament with two rounds – semifinals and final - and head-to-head competition in each round, that is, the number of players is limited to four players. The purpose with the model is to analyse how a player's choice between two alternatives is affected when varying two parameters capturing the degree of players' competitiveness and recovery duration.

The four players, $i = A, B, C, D$ compete head-to-head in two semifinals, the first semifinal (s.1) and the second semifinal (s.2). The winner of each semifinal advances to the final. The objective of each player is to win the final. The players are of two types: high ranked players (*H*) and low ranked players (*L*). Players A and B are assumed to be of type *H* whereas players C and D are of type *L*. The players choose, in sequential order, which one of the two semifinals to compete in. The ordering of the choices is: B, A, C, D.[4] A player's choice of semifinal becomes public prior to the next player's choice. Hence, there are six possible settings $(j = 1...6)$ of semifinals, including the mirroring of identical plays, albeit in different order. Figure 2 illustrates the decision tree of our model. We denote the player's probability of winning the whole tournament as $p_{i,j}$. For example, $p_{C,4}$ denotes the probability that player C will win the tournament given that she faces player A in semifinal 1. A player of type *H* will beat a player of type *L* with probability $p > 0.5$, and if two players of the same type compete, the probability is 0.5 to win against the other.

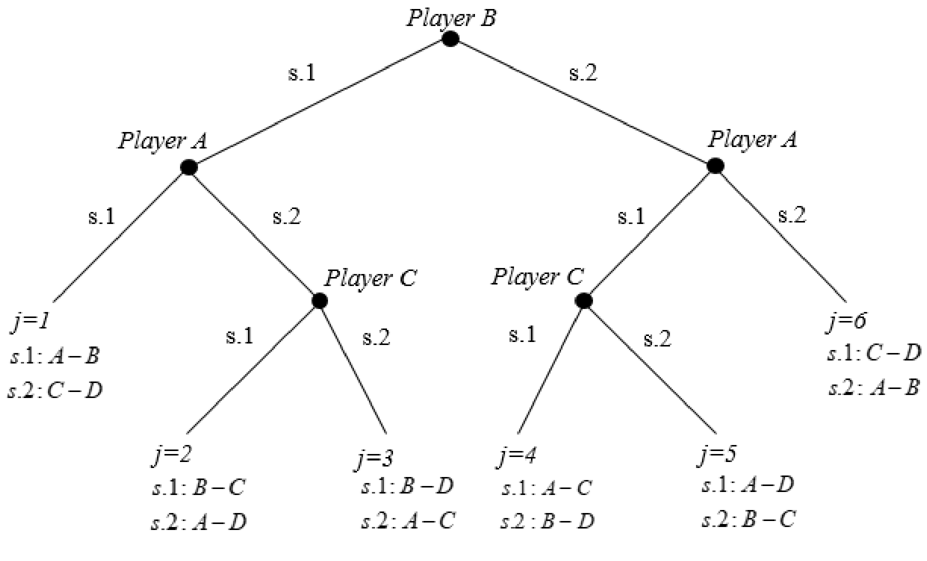

Figure 2. The sequential order of choices and possible settings of semifinals

To capture the effect of having a shorter recovery time when the player advances into the final from the second semifinal, we multiply that player's probability of winning the final with a constant *c*, where $0 < c < 1$. The lower value of *c*, the larger is the negative effect - henceforth the recovery effect - of having advanced into the final from the second semifinal rather than from the first semifinal. Thus, even

[4] Player D never makes a choice and player C can only choose her opponent when player B chooses the opposite semifinal as player A does.

though semifinal settings *j=1* and *j=6* imply identical plays, we have $p_{A,1} = p_{B,1} > p_{A,6} = p_{B,6}$ and $p_{C,1} = p_{D,1} < p_{C,6} = p_{D,6}$ due to the recovery effect c. The players are assumed to have full information on the values of the probabilities defined above, as well as the value of c.

Proposition 1 *Player B will always choose the first semifinal and play against player A if and only if*

$$c < f(p) = \frac{-0.5}{p^3 - 1.5p^2 + 0.5p - 0.5}.$$

Otherwise, player B will play against player C in the first semifinal.

Proof See appendix.

In Figure 3 we illustrate the result graphically. Points below the convex graph indicate combinations of levels of the recovery effect and the probability p for which player A chooses to compete in the first semifinal. For values of c up to about 0.91, the degree of competition from low ranked players has no effect upon player A's choice. Player A might choose to avoid player B in the first semifinal for higher values of c. For example, for $c = 0.95$, shown as a horizontal line in the figure, A will choose semifinal 2 for values of p in between 0.61 and 0.94, rounded to two decimal places.

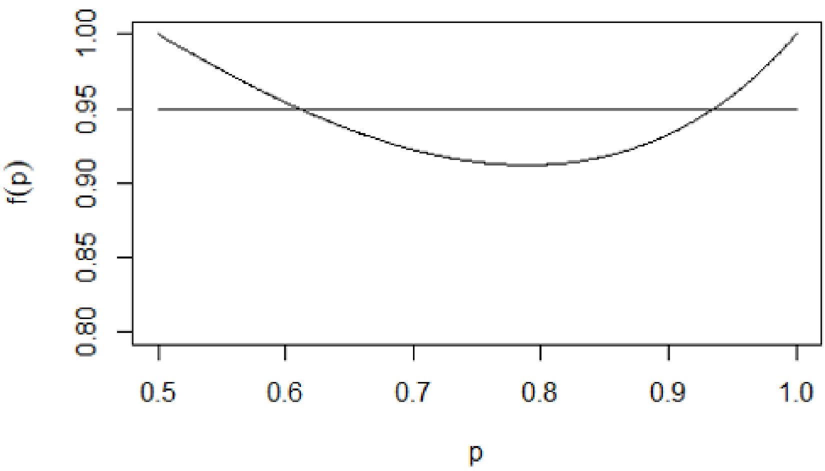

Figure 3. Player A's choice of semifinal for various combinations of p and c

To understand the mechanism behind A's choice for high values of c, once B has chosen semifinal 1, we initially assume that $c = 1$ and $p = 0.5$. Thus, no recovery effect is assumed, and competition is equalized across all players. This combination of c and p obviously makes player A indifferent between the two semifinals. Now, letting p increase, still assuming no recovery effect, A will choose semifinal 2 in order to have a positive probability of avoiding player B in the tournament, now the single most competitive opponent. Thus, as p increases, for A to be indifferent, a compensation in terms of a decrease in c is necessary, and we are moving downwards along the graph of the convex function in the figure. As p further increases, holding c fixed, there is a positive effect on the incentive for A to choose semifinal two in that the importance of avoiding B in the tournament increases due to a decrease of the relative competitiveness of type L players. However, there is also a negative effect, since the probability increases for A of ending up weakened in the final with type H player B. For p about 0.79 these opposite effects cancel each other out. This negative effect outweighs the positive effect for larger values of p, meaning that we are moving upwards along the graph for indifference. For a further increase in p, A needs to be compensated by an increase in c, i.e., a lower recovery effect, to still be indifferent and not choosing semifinal 1. For values of p close to 1, player A expects to face player B in the final with almost certainty, both advancing from different semifinals. Player A is then better off playing against player B already in the first semifinal, unless the recovery effect is negligible, i.e., c is close to 1, making A indifferent.

5. Empirical Methods

In this chapter we outline the methodology to answer the questions raised in the introductory section. We propose a test statistic for testing Ulvang's conjecture that high ranked athletes to a larger extent choose early quarterfinals. Furthermore, a logistic regression model is specified, where certain parameter restrictions correspond to rational choices of quarterfinals are to be tested with a Wald test. Finally, using the information from the observed choices of quarterfinal and the outcome from the competitions, we also identify a method how to create a quarterfinal seeding scheme that internalizes the athlete's advantage of being assigned one of the two early quarterfinals vis-a-vis .

5.1 A Test for Ulvang's Conjecture

In this section we propose a test statistic for testing the null hypothesis that the athletes choose quarterfinals in a pure random way against the alternative that higher ranked athletes from the qualification round to a larger extent choose early quarterfinals rather than late quarterfinals. Define U_k and V_k as the rank sum from the qualification round for the early quarterfinals, one and two, and late quarterfinals, four and five, respectively, for competition k, $k = 1, 2, \ldots, 56$.

The null hypothesis and the alternative hypothesis can be formulated as

$$H_0: E(U_k) - E(V_k) = 0 \quad \text{against} \quad H_A: E(U_k) - E(V_k) < 0\,.$$

An appropriate test statistic is given by

$$Z = \frac{\overline{R}_E - \overline{R}_L}{\sqrt{465/14}},$$

where $\bar{R}_E = \frac{\sum_{k=1}^{56} U_k}{56}$ and $\bar{R}_L = \frac{\sum_{k=1}^{56} V_k}{56}$. The test statistic follows approximately a standard normal distribution under the null hypothesis. Moreover, since we reject H_0 in favour of H_A for large negative values of *z*, the rejection region is $RR = \{z < -z_\alpha\}$ where z_α is such that $P(Z > z_\alpha) = \alpha$.

Proposition 2

The test statistic $Z = \frac{\bar{R}_E - \bar{R}_L}{\sqrt{465/14}} \overset{appr}{\sim} N(0,1)$ *under* $H_0 : E(U_k) - E(V_k) = 0$.

Proof See appendix.

Alternative test statistics could be used. The sign test and Wilcoxon's matched-pairs signed rank test are two non-parametric alternatives, while a paired t-test is another option where the population standard deviation of difference is estimated. However, since the assumptions under the null hypothesis are fulfilled to apply a parametric test, where the population variance can be derived, the test statistic given in Proposition 2 is preferred in terms of power.

5.2 A Test for Rational Choice

In the previous section a test was suggested to investigate whether high ranked athletes choose early quarterfinals to a larger extent than late quarterfinals. If so, a natural follow-up question would be whether the proportions in different quarterfinals of high ranked athletes from the qualification round, are balanced in a way, such that the chance of reaching the podium is irrespective of choice of quarterfinal, controlling for the capacity of the athlete.

For this question to be answered, we adopt a logistic regression approach, where a rational behaviour among the athletes as a group, in terms of balanced proportions of high ranked athletes in different quarterfinals, corresponds to certain parameter restrictions within a model to be presented below.

The athlete's probability of reaching the podium is modelled as a function of her choice of quarterfinal, her individual capacity relative other athletes (short-term, middle-term and long-term capacity) and individual specific effects.

The dependent variable to be used is *Podium*, a binary variable taking the value one if podium is reached, zero otherwise. The first type of explanatory variables considers quarterfinals, where a dummy variable is used to indicate the choice of one of four quarterfinals (Q_1, Q_2, Q_4, Q_5). Quarterfinal three (Q_3) serves as reference.

The second type of explanatory variables captures the athlete's capacity relative other athletes for different time perspectives. These variables are:

- *Rankqual*: The athlete's ranking number from the qualification round (short-term capacity).
- *Rankqualsq*: *Rankqual* squared.
- *Podium_1*: A dummy variable indicating if the athlete reached the podium in the latest competition (middle-term capacity).
- *Rankwcp*: The athlete's ranking number among the 30 quarterfinalists based on the current world cup sprint points achieved in previous world cup sprint competitions the current season (long-term capacity).
- *Rankwcpsq*: *Rankwcp* squared.

In addition, we also include individual specific dummy variables ($I_1, I_2,..,I_m$) for m out of those $m+1$ athletes having variation in the dependent variable ($m = 29$ for men and $m = 30$ for women).

Thus, the linear predictor can be written as

$$\eta = \beta_0 + \beta_{Q_1} Q_1 + \beta_{Q_2} Q_2 + \beta_{Q_4} Q_4 + \beta_{Q_5} Q_5$$

$$+\beta_{Rankqual} Rankqual + \beta_{Rankqualsq} Rankqualsq + \beta_{Rankwcp} Rankwcp$$

$$+\beta_{Rankwcpsq} Rankwcpsq + \beta_{Podium\_1} Podium\_1 + \beta_{I_1} I_1 + ... + \beta_{Im} I_m .$$

The parameters are to be estimated using the method of maximum likelihood based on a total of *1680* observations for both men and women (*56* competitions with *30* qualified athletes in each competition).

We expect the ranking number from the qualification round, as well as ranking number based on current world cup sprint points, to have a negative declining effect on the probability of reaching the podium.[5] Thus, $\beta_{Rankqual}$ and $\beta_{Rankwcp}$ are both expected to be negative, while $\beta_{Rankqualsq}$ and $\beta_{Rankwcpsq}$ are expected to be positive. Furthermore, a good performance in the previous competition is expected to have a positive effect, implying β_{Podium_1} should have a positive sign.

Now, assume that higher ranked athletes to a larger extent choose early quarterfinals instead of late quarterfinals. Then, there is a possibility that those relatively few high ranked athletes, choosing a late quarterfinal, will be fully compensated for the shorter recovery time prior to a final thanks to weaker competition in the late quarterfinal. Such a behaviour - the athletes' choice of quarterfinal as a group, making the probability of reaching the podium irrespective of quarterfinal conditioning on the athlete's

[5] Again, note that the athlete's historical performance in terms of her world cup points is inversely related to her assigned ranking number.

capacity - corresponds to four parameter restrictions in the specified model, formulated in the null hypothesis below

$$H_0: \beta_{Q_1} = \beta_{Q_2} = \beta_{Q_4} = \beta_{Q_5} = 0$$

H_A: At least one of β_{Q_1}, β_{Q_2}, β_{Q_4} and β_{Q_5} is not equal to zero.

The alternative hypothesis corresponds to a behaviour of choice where the probability of reaching the podium, conditioned on the capacity of the athlete, differs for at least one of the five quarterfinals. The null hypothesis is to be tested with a Wald test, which approximately follows a chi-squared distribution with 4 degrees of freedom under H_0.

5.3 Seeding instead of Choosing Quarterfinal

The old design as well as the current design have both been subject to critique. The old design did not take the unfair difference of recovery time into account, while the current design, by some athletes, is considered requiring a tactical skill not belonging to the sport (Östersunds-Posten 2014).

Therefore, we look into the possibility of coming up with a new design that might solve the problem of the former design, as well as the problem of the current design. The idea of the new design is to apply the old design of assigning quarterfinals in a somewhat modified way. Instead of assigning quarterfinals to make the quarterfinals equal in terms of competition, i.e. equal rank sums, we propose a seeding scheme where the sum of ranks differs across quarterfinals, with low rank sums attached to early quarterfinals and high rank sums attached to late quarterfinals. Hence, the sum of ranks will reveal the degree of competition in each quarterfinal. The implication is that higher ranked athletes from the qualification, who are likely to be placed in the first two quarterfinals, will face relatively strong competition already in the first knockout round but then, in case of advancement, be compensated by a longer recovery time before the final. Likewise, lower ranked athletes, placed in the late quarterfinals, pay the weaker competition in their first round with a shorter recovery time prior to a final.

To implement the seeding scheme discussed above, we include the variable *Rankqualsum* in the model set out in the previous section. This variable is defined as the sum of the individual ranks in specific athlete's quarterfinal. The aim of including this variable is to capture and control for the competition in the quarterfinal, following the idea used in the old design to define competition for seeding purposes.

The linear predictor in the logistic regression model is now extended to:

$$\eta = \beta_0 + \beta_{Q_1} Q_1 + \beta_{Q_2} Q_2 + \beta_{Q_4} Q_4 + \beta_{Q_5} Q_5$$

$$+\beta_{Rankqual} Rankqual + \beta_{Rankqualsq} Rankqualsq + \beta_{Rankwcp} Rankwcp + \beta_{Rankwcpsq} Rankwcpsq$$

$$+\beta_{Podium\_1} Podium\_1 + \beta_{I_1} I_1 + ... + \beta_{Im} I_m + \beta_{Rankqualsum} Rankqualsum.$$

The sign of *Rankqualsum* is expected to be positive. A larger rank sum indicates weaker competition in the quarterfinal which should be associated with a high probability of reaching the podium when controlling for the type of quarterfinal and the athlete's capacity. Given the control for competition, we expect β_{Q_1} and β_{Q_2} to be positive, and β_{Q_4} and β_{Q_5} to be negative, due to the differences in recovery time.

Before we get into details of how to assign the rank sums across different quarterfinals, an assumption underlying the new proposal will be discussed.

The way athletes progress during the final rounds in combination with the presumed recovery effect forms a physiological basis for the claim that athletes should prefer early quarterfinals to late quarterfinals when we control for the competition. Likewise, an early quarterfinal is expected to be better than quarterfinal three, while quarterfinal three is to be preferred to late quarterfinals. However, when it comes to a comparison between quarterfinal one and quarterfinal two, no quarterfinal should be preferred to the other. The reason for that is that the assignment of semifinals is the same for both quarterfinals. The top two athletes in each of these quarterfinals qualify for the first semifinal, thereby getting long recovery between the semifinal and the final, provided the final is reached. Certainly, the recovery time between the quarterfinal and the semifinal is longer for those athletes going in the first quarterfinal compared with the second one, which might be an argument for the first quarterfinal to be more favourable. However, the time is long enough for athletes in both quarterfinals to fully recover. A similar argument holds for not separating quarterfinals four and five.

Thus, differences in the probability of reaching the podium from different quarterfinals when we control for the capacity of the athlete and the competition in the quarterfinal are not expected, neither between the two early quarterfinals nor between the two late quarterfinals. A difference is expected between early quarterfinals, quarterfinal 3 and late quarterfinals, only. Henceforth we denote these three types of quarterfinals by *1*, *2* and *3*, respectively.

The claim, that an athlete should be indifferent between the two early quarterfinals as well as being indifferent between the two late quarterfinals, corresponds to the parameter restrictions:

$$H_0 : \beta_{Q_1} = \beta_{Q_2}, \beta_{Q_4} = \beta_{Q_5}$$

H_A : At least one of the restrictions under H_0 does not hold.

The null hypothesis is to be tested with a Wald test, which approximately follows a chi-square distribution with 2 degrees of freedom under H_0.

Provided the null hypothesis above is true, we may look upon the five quarterfinals as three types of quarterfinals: early quarterfinals (Q_E), quarterfinal $3(Q_3)$, and late quarterfinals (Q_L), where Q_E and Q_L are defined as dummy variables, Q_3 being the reference.

The linear predictor from the previous section can now be written as

$$\eta = \beta_0 + \beta_{Q_E} Q_E + \beta_{Q_L} Q_L + \beta_{Rankqual} Rankqual + \beta_{Rankqualsq} Rankqualsq + \beta_{Rankwcp} Rankwcp$$
$$+ \beta_{Rankwcpsq} Rankwcpsq + \beta_{Podium\_1} Podium\_1 + \beta_{I_1} I_1 + ... + \beta_{1m} I_m + \beta_{Rankqualsum} Rankqualsum.$$

Now, let x_j define the rank sum for the chosen quarterfinal of type *j*, $j = 1,2,3.$ Conditioning on the three types of quarterfinals, the linear predictor can be written as

$$\begin{aligned}
(\eta | \textit{quarterfinal of type 1}) &= \beta_0 + \beta_{Q_E} + \beta_{Rankqualsum} x_1 + ... \\
(\eta | \textit{quarterfinal of type 2}) &= \beta_0 + \beta_{Rankqualsum} x_2 + ... \\
(\eta | \textit{quarterfinal of type 3}) &= \beta_0 + \beta_{Q_L} + \beta_{Rankqualsum} x_3 + ... \quad .
\end{aligned}$$

The probability of reaching the podium will be the same, irrespective of type of quarterfinal and conditioning on a certain capacity, if

$$(\eta | \textit{quarterfinal of type 1}) = (\eta | \textit{quarterfinal of type 2})$$

and

$$(\eta | \textit{quarterfinal of type 3}) = (\eta | \textit{quarterfinal of type 2}),$$

which can be written as

$$\beta_{Q_E} + \beta_{Rankqualsum} x_1 = \beta_{Rankqualsum} x_2 \text{ and } \beta_{Q_L} + \beta_{Rankqualsum} x_3 = \beta_{Rankqualsum} x_2.$$

Solving for x_1 and x_3, we obtain

$$x_1 = x_2 - \frac{\beta_{Q_E}}{\beta_{Rankqualsum}} \text{ and } x_3 = x_2 - \frac{\beta_{Q_L}}{\beta_{Rankqualsum}}.$$

The first relation gives us combinations of x_1 and x_2, i.e., combinations of competition in terms of rank sums in a quarterfinal of type *1* and type *2*, such that an athlete should be indifferent between the two types of quarterfinals provided the goal is to maximize the probability of reaching the podium. The second relation is interpreted analogously.

By adding the restriction

$$2x_1 + x_2 + 2x_3 = 1 + 2 + ... + 30 = 465$$

we obtain an equation system with three equations and three unknowns, x_1, x_2 and x_3, which can be solved for in terms of the parameters β_{Q_E}, β_{Q_L} and $\beta_{Ranksumqual}$.

Solving the system yields

$$x_1 = 93 + \frac{2\beta_{Q_L} - 3\beta_{Q_E}}{5\beta_{Rankqualsum}}, \; x_2 = 93 + \frac{2(\beta_{Q_E} + \beta_{Q_L})}{5\beta_{Rankqualsum}}, \; x_3 = 93 + \frac{2\beta_{Q_E} - 3\beta_{Q_L}}{5\beta_{Rankqualsum}}.$$

Expecting $\beta_{Q_L} < 0, \beta_{Q_E} > 0$ and $\beta_{Rankqualsum} > 0$ it follows that we should expect $x_1 < x_2 < x_3$. Thus, in order to make the athletes indifferent between the three types of quarterfinals, the rank sums should increase with type of quarterfinal. Moreover, from the expected signs it follows that the rank sum for early quarterfinals should be smaller than *93*, while the rank sum for late quarterfinals should be larger than *93*, which is the value assigned to all quarterfinals in the former system.

Once maximum likelihood estimates of x_1, x_2 and x_3 are obtained, rounded to integer values, the new design is implemented as follows: For a certain value of $x_{j,}$ $j = 1, 2, 3$, one out of all possible samples of six unique qualification ranks summing up to x_j , is randomly drawn. These six numbers are assigned to the specific type of quarterfinal.

6. Data

Official results for all world cup skiing sprint competitions as well as data on individual athletes are collected from the International Ski Federation's website (FIS Ski, 2020).

The data includes the results from 121 competitions – for men as well as for women - during eleven seasons, 2009/2010 - 2019/2020, separated into two periods. The first period, 09/10-14/15, contains the results under the old design, whereas the second period, 15/16 – 19/20, consists of the results under the current design.[6] The data from the first period – 65 competitions - is mainly used for presenting the skewed distribution of medallists across the five quarterfinals in Table 2, whereas the data from the second period – 56 competitions - is the source for our empirical analysis of the athletes' choices.

Table 3. Average rank sums for different quarterfinals by season and sex for the second period

				Season			
Quarterfinal	14/15	15/16	16/17	17/18	18/19	19/20	Total
Women							
1	61	86	91	85	93	89	88
2	83	87	89	88	87	90	88
3	102	95	95	92	96	92	94
4	102	95	92	96	91	97	94
5	117	102	98	104	98	97	100
Men							
1	86	89	87	89	83	96	89
2	88	90	87	84	86	91	87
3	92	86	92	101	94	93	93
4	92	97	93	93	95	91	94
5	108	103	106	98	107	94	102

For each of these competitions, we have data on the 30 athletes competing in the quarterfinals. We observe their choice of quarterfinals, their achieved results, and their official ranking. The entire data

[6] The current design was implemented under the last two competitions during the season 2014/2015.

set contains 7260 observations, of which 3360 observations, divided into men and women equally, are from the second period.

Table 3 shows the average rank sums, rounded to integer values, for different quarterfinals divided by season. The rank sums for early quarterfinals are low compared to late quarterfinals, indicating stronger competitions in the first two quarterfinals. The pattern is quite stable over time, except for the latest season where the rank sum for the first quarterfinal is high for men.

In Table 4 we show the share of medallists coming from each of the five quarterfinals during the seasons in which the current design has been applied.

Table 4. Share of medallists from the quarterfinals by season and sex for the second period

				Season			
Quarterfinal	14/15	15/16	16/17	17/18	18/19	19/20	Total
Women							
1	0.17	0.31	0.37	0.40	0.28	0.30	0.32
2	0.33	0.31	0.27	0.33	0.25	0.33	0.30
3	0.33	0.06	0.13	0.20	0.25	0.07	0.15
4	0.17	0.22	0.10	0.03	0.14	0.10	0.12
5	0.00	0.11	0.13	0.03	0.08	0.20	0.11
Men							
1	0.17	0.33	0.30	0.53	0.58	0.50	0.44
2	0.50	0.36	0.13	0.23	0.19	0.17	0.27
3	0.17	0.17	0.20	0.13	0.06	0.13	0.14
4	0.17	0.03	0.13	0.03	0.14	0.17	0.10
5	0.00	0.11	0.03	0.07	0.03	0.03	0.05

The figures exhibit that athletes from early quarterfinals are overrepresented among medallists. The share of medallists coming from the two first quarterfinals is even higher under the current design than we observe under the old design.[7]

7. Results

In this section we apply the methods described in section 5 on our data. The results from testing Ulvang's conjecture and testing for rational choice are presented. We also present estimated rank sums to be assigned to the various quarterfinals, used in the revision of the old seeding scheme, intending to capture the recovery effect.

[7] The figures from the season 2014/2015 are based on two competitions only.

7.1 Test of Ulvang's Conjecture

The two rows in Table 5, also shown as the last column in Table 3, show the average qualification rank sum for each of the five quarterfinals based on 56 competitions. As expected, the rank sum increases in the ordering of quarterfinals.

Table 5. Average qualification rank sum for different quarterfinals under the current design, n=56

	Quarterfinal				
	1	2	3	4	5
Women	88.04	87.93	94.30	94.32	100.41
Men	88.68	87.48	92.93	93.84	102.07

We apply the test statistic outlined in section 5.1 to test Ulvang's conjecture, that is, to test the null hypothesis that the choices of quarterfinal are made randomly against the alternative hypothesis that the expected rank sum from early quarterfinals is smaller than for late quarterfinals.

Women: $\overline{R}_E$ = 88.04+87.93 = 175.96 and $\overline{R}_L$ = 94.32+100.41=194.73

$$z = \frac{\overline{R}_E - \overline{R}_L}{\sqrt{465/14}} = \frac{175.96 - 194.73}{\sqrt{465/14}} = -3.26 \quad (p < 0.01)$$

Men: $\overline{R}_E$ = 88.68+87.48 = 176.16 and $\overline{R}_L$ = 93.84+102.07=195.91

$$z = \frac{\overline{R}_E - \overline{R}_L}{\sqrt{465/14}} = \frac{176.16 - 195.91}{\sqrt{465/14}} = -3.43 \quad (p < 0.01)$$

Our tests reject the null hypothesis, indicating that data supports Ulvang's conjecture for both women and men. Facing the trade-off between expected weaker competition in late quarterfinals or longer recovery time of choosing early quarterfinals, our test suggests that higher ranked athletes to a larger extent prefer the later alternative to the former.

7.2 Testing for Rational Choice

We make use of the regression model specified in Section 5.2 to find the partial effect from the choice of quarterfinal upon the probability of reaching the podium. In Table 6 the results from estimation of three logistic regression models for both women and men are presented with the variable *Podium* as dependent variable. In the first model only the quarterfinal dummy variables are included. Model 2 also includes those variables aiming to capture the capacity of the athlete, except for individual specific dummy variables. In the third model individual specific dummy variables are included as well.[8] Looking at the first model, the parameter estimates corresponding to the early quarterfinals attain large significantly positive values for both men and women. The parameter estimates corresponding to late

[8] Estimates on individual dummy coefficients are available on request.

quarterfinals are negative, although not significantly negative. These results imply that the probability of reaching the podium is larger for early quarterfinalists than for those athletes going in the third and late quarterfinals.

Table 6. Results for logistic regression models, podium as response variable

Sex		*Men*			*Women*	
Model	*(1)*	*(2)*	*(3)*	*(1)*	*(2)*	*(3)*
Variable						
Intercept	*-2.61*** (0.216)*	*0.359 (0.397)*	*-4.44*** (1.08)*	*-2.52*** (0.208)*	*1.18*** (0.412)*	*-2.85*** (1.14)*
Q_1	*1.35*** (0.253)*	*0.976*** (0.287)*	*0.800*** (0.344)*	*0.846*** (0.256)*	*0.583** (0.305)*	*0.657** (0.351)*
Q_2	*0.744*** (0.269)*	*0.429* (0.303)*	*0.322 (0.338)*	*0.846*** (0.256)*	*0.601** (0.304)*	*0.661** (0.348)*
Q_4	*-0.321 (0.330)*	*0.0123 (0.370)*	*-0.111 (0.403)*	*-0.239 (0.310)*	*0.00630 (0.357)*	*-0.104 (0.406)*
Q_5	*-0.982** (0.401)*	*-0.285 (0.465)*	*-0.0770 (0.497)*	*-0.411 (0.323)*	*0.107 (0.366)*	*-0.116 (0.442)*
Rankqual		*-0.295*** (0.0510)*	*-0.251*** (.0631)*		*-0.198*** (0.0467)*	*-0.244*** (0.0541)*
Rankqualsq		*0.00502*** (0.00192)*	*0.00383* (0.00244)*		*0.00290*** (0.00180)*	*0.00430** (0.00200)*
Rankwcp		*-0.0638* (0.0472)*	*-0.0136 (0.0556)*		*-0.303*** (0.0517)*	*-0.162*** (0.0624)*
Rankwcpsq		*0.000197 (0.00175)*	*0.000643 (0.00204)*		*0.00692*** (0.00191)*	*0.00331* (0.00233)*
Podium_1		*0.475** (0.274)*	*-0.00190 (0.289)*		*0.521** (0.275)*	*-0.0508 (0.301)*
No quarter final effect (p-value)		0.0005	0.0521		0.100	0.0706
No individual effect (p-value)			0.0013			< 0.0005

Standard error in parenthesis. *Significant at 10%; significant at 5%; *** significant at 1%

The most likely explanation for these results is the fact that high ranked athletes in the qualification round are overrepresented in early quarterfinals, amplified by the positive effect on performance of longer recovery time between semifinal and final for those athletes going in the early quarterfinals.

Consider the results from estimation of the second model, where variables to control for the individual's capacity are included. The signs of the parameter estimates for these variables are as expected, although the estimates corresponding to variables concerning world cup points are not significant for men. As can be seen from Table 6, an overall test of no quarterfinal effect, described in Section 5.2, is rejected at the 10 % level of significance. The pattern of large positive parameter estimates for early quarterfinals is still observed, albeit not as clearly as before. To summarize, the probability of reaching the podium, when controlling for the capacity of the athlete, seems to be higher for early quarterfinals.

Turning to the results from estimation of the third model, to which individual specific dummy variables are added as well, there is no dramatic change of the parameter estimates. For both men and women, a Wald test supports the inclusion of the individual dummy variables as a group. The support for rejecting the null hypothesis of no quarterfinal effect is decreasing a bit for men. However, the result is still significant at the 10 % level for both men and women. Thus, the results indicate that the way athletes choose quarterfinals, an individual athlete has a better chance of reaching the podium going in an early quarterfinal.

7.3 A Revised Seeding Scheme

In this section we present estimation results of rank sums for the three types of quarterfinals $\left(Q_E, Q_3, Q_L\right)$ that would make an athlete indifferent between different types of quarterfinals, using the regression models specified in Section 5.3.

Consider Table 7. In both models the variable *Rankqualsum* is included to measure the effect of competition on the performance, controlling for types of quarterfinals. As expected, the sign associated with this variable is positive. This means that a large rank sum, i.e., a weak competition, is associated with a high probability of reaching the podium, conditioning on a certain athlete going in a specific quarterfinal. For the women this effect is not as strong as for the men, although the effect is significantly positive at the 10 percent level.

Controlling for the competition, in terms of the qualification rank sum, the parameter estimates corresponding to the quarterfinal dummies are not interpreted in the same way as for the model without the rank sum variable. Now, irrespective of the athletes' choice, if there truly is an effect of recovery time on performance, this effect should be reflected in these estimates. Therefore, contrary to the model where the variable *Rankqualsum* is not included, the parameter estimates corresponding to early quarterfinals are expected to be positive. The parameter estimates for late quarterfinals are expected to be negative. This is also consistent with our findings, although the estimates for late quarterfinals are not significantly negative.

Table 7. Results for logistic regression models including degree of competition, podium as response variable

Sex	*Men*		*Women*	
Model	*(1)*	*(2)*	*(1)*	*(2)*
Variable				
Intercept	*-6.98**** *(1.44)*	*-6.27**** *(1.40)*	*-4.21**** *(1.49)*	*-4.19**** *(1.47)*
Q_1	*0.966**** *(0.354)*		*0.750*** *(0.358)*	
Q_2	*0.449** *(0.340)*		*0.750*** *(0.354)*	
Q_4	*-0.171* *(0.416)*		*-0.126* *(0.407)*	
Q_5	*-0.343* *(0.507)*		*-0.244* *(0.452)*	
Q_E		*0.680*** *(0.306)*		*0.751**** *(0.320)*
Q_L		*-0.230* *(0.366)*		*-0.173* *(0.366)*
Rankqual	*-0.282**** *(0.0635)*	*-0.282**** *(0.0627)*	*-0.258**** *(0.0550)*	*-0.259**** *(0.0550)*
Rankqualsq	*0.00450*** *(0.00240)*	*0.00456*** *(0.00237)*	*0.00466**** *(0.00200)*	*0.00467**** *(0.00200)*
Rankwcp	*-0.0144* *(0.0564)*	*-0.0216* *(0.0549)*	*-0.163**** *(0.0625)*	*-0.160**** *(.0590)*
Rankwcpsq	*0.000943* *(0.00206)*	*0.00111* *(0.00204)*	*0.00341** *(0.00234)*	*0.00334** *(0.00227)*
Podium_1	*-0.0158* *(0.301)*		*-0.0597* *(0.302)*	
Ranksumqual	*0.0291**** *(0.00915)*	*0.0282**** *(0.00923)*	*0.0156** *(0.0112)*	*0.0150** *(0.0110)*
Equal recovery effect within types of quarterfinal (p-value)	*0.231*		*0.965*	
No quarter final effect (p-value)	*0.0013****	*0.0047****	*0.0330****	*0.0051****
No individual effect (p-value)	*0.0076****	*0.0009****	*<0.0005****	*<0.0005****

Standard error in parenthesis; *Significant at 10%; significant at 5%; *** significant at 1%

The result from the chi square test set out in section 5.3 reveals no support for the recovery effect to differ between the first two quarterfinals or between the last two quarterfinals. This is true for men as well as for women. Thus, only three types of quarterfinals will be considered; early quarterfinals, quarterfinal three, and late quarterfinals.

Following the approach described in section 5.3 and using estimates from the second model specification in Table 7, it is possible to calculate estimates of those rank sums making a certain athlete indifferent between quarterfinals.[9] These estimates are provided in Table 8.

*Table 8. Estimated rank sums for indifference between quarterfinals for men and women**

	Quarterfinal				
	1	2	3	4	5
Men	75	75	99	108	108
Woman	58	58	109	120	120

**Rounded to nearest integer*

Comparing with the actual average rank sum for different quarterfinals based on *56* competitions in Table 3 the estimates in Table 8 suggest high ranked athletes should pick early quarterfinals to an even greater extent than they do. On the margins, the positive recovery effect from such a behaviour outweighs the negative effect of tougher competition.

There is a difference in the estimated rank sums between men and women. For women the competition in terms of a low rank sum should be as low as *58* for early quarterfinals compared to *75* for men. The reason for this difference is the difference in the estimated effect of the variable *Rankqualsum*. For men, a change in the value of this variable by a certain amount has a larger effect on the performance than a corresponding change for women. A wider spread between rank sums for early and late quarterfinals is needed for the women to compensate for differences in recovery time.

8. Conclusions and Discussion

The old seeding design in skiing sprint elimination tournaments was regarded unfair because the chances of reaching the podium were higher if the athlete more or less by pure luck was seeded in an early rather than in a late quarterfinal. Athletes competing in an early quarterfinal can later in the tournament, prior to a final, benefit from a longer recovery time than those competing in a late quarterfinal. This advantage was revealed in the result lists, where the athletes assigned the first two quarterfinals were overrepresented as medallists in the competitions. The motive to adopt the design used today, where

[9] In this specification the variable *Podium_1* has been dropped due to an unexpected sign.

each prequalified athlete chooses her quarterfinal instead of being assigned a quarterfinal through a seeding scheme, was to internalize this recovery effect.

Our empirical analysis of the current design, shows that higher ranked athletes tend to choose an early quarterfínal rather than a late one, and that the probability of reaching the podium still is higher when choosing an early quarterfinal, conditioning on capacity, despite the impact of an increased competition in the early quarterfinals. Hence, our findings indicate that athletes would benefit further from choosing an early instead of a late quarterfinal, suggesting they are making irrational choices. The optimal choice of quarterfinal, given the set of information, is a complex problem. Two athletes with the same set of information and identical ranking, may make different choices due to their differences in tackling the computational burden and handling the tactical dimension inherent in the design.

Holding on to seeding, we present a revision of the old seeding scheme. Instead of letting the sum of athletes' ranking from the qualification round be equal, i.e., the sum 93, across quarterfinals, this sum should differ across quarterfinals to adjust for the variation in recovery time prior to a final. The earlier the quarterfinal, the lower is the sum of raking. Compared to the current design, this revised seeding would thus lift a tricky strategic element out of the competition but still capture the fundamental disparity across quarterfinals. Our proposal does not point out in which of the quarterfinals a competitor with a certain ranking from the qualification round should compete. For each quarterfinal, our seeding only specifies a total sum of ranking, arising from a number of possible combinations of six numbers, adding up to the specified sum.

Clearly, an assertion that irrationality could be behind the result is not without objection. First and foremost, it is reasonable to assume that for some of the athletes, the objective function behind the choice of quarterfinal may not be to maximize the probability of ending up on the podium, but instead to maximize the expected award of world cup ranking points. The expected number of ranking points awarded may be maximized by choosing a late quarterfinal with expected lower competition, even if it leads to a shorter recovery time in case of advancement to the final, thus lowering the chances of ending up on the podium. Secondly, the athlete's choice of quarterfinal surely to a large extent depends on her private perception of her own capacity and on her subjective perception of the competitors' capacity on the current competition day. In other words, the basis for the athlete's decision, may deviate from our data on the athletes' official ranking when assessing their capacity. Thirdly, an athlete's relative performance in the qualification round, which takes the form of an individual race against the clock, may not always reflect the individual's tactical ability to perform in a race with a mass start where the goal is to be the first or the second to cross the finish line. Fourth, we do not consider whether a skiing sprint competition has been run as a classic style competition or as a skate style competition, a factor that likely has an impact on some athletes' competitiveness on the ski course. This may give rise to misleading information in the variables supposed to capture the athlete's capacity.

One natural step in this research would be to adopt another objective function behind the athletes' choices of quarterfinals. As suggested above, a plausible point of departure for the analysis would be to assume that the athletes seek to maximize the expected world cup points rather than maximizing the probability of ending up on the podium.

Another step would be to use comparable data from the same skiing sprint competitions held the years before the season 2014/2015 to investigate whether an athlete's probability of reaching the podium has been affected by the FIS's decision to switch design. Given that an athlete's performance is observed under both regimes, it may be possible to assess - conditioning on the athlete's various rankings - to what extent the athlete's choice of quarterfinal has improved her outcome in the competitions under the current design vs. the old design.

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Appendix

Proof of Proposition 1

Given that player B initially chooses the first semifinal (s.1), we write the probability of player A winning the tournament when choosing s.1 as

$$p_{A,1} = 0.5 \times 0.5\left(1-\left(1-p\right)c\right) + 0.5 \times 0.5\left(1-\left(1-p\right)c\right)$$

The first part is the probability that player A beats player B, player C beats player D in s.2, and in the final player A beats player C. Note that player C's probability of beating player A in the final is reduced by the factor *c*, that is $\left(1-p\right)c$. The second part identifies the same probabilities as the first part, but now it is player D who advances to the final.

If player A instead chooses s.2, then player C is better off choosing s.1 than s.2. To see this, we have

$$p_{C,2} = \left(1-p\right)p\left(1-pc\right) + \left(1-p\right)\left(1-p\right)\left(1-0.5c\right)$$

and

$$p_{C,3} = \left(1-p\right)p\left(1-p\right)c + \left(1-p\right)\left(1-p\right)0.5c,$$

where

$$p_{C,2} - p_{C,3} = \left(1-p\right)\left(1-c\right) > 0.$$

Thus, the probability that player A will win the tournament given that player B chooses s.1 and player A chooses s.2 can be written as

$$p_{A,2} = p \times p0.5c + p\left(1-p\right)pc.$$

Player A will choose to compete against player B in s.1 if $p_{A,1} - p_{A,2} > 0$. Using the expressions for $p_{A,1}$ and $p_{A,2}$, we obtain the condition for player A choosing to compete against player B in s.1 as

$$c < \frac{-0{,}5}{p^3 - 1.5p^2 + 0.5p - 0.5}.$$

Now, given this condition, the probability of player B winning the tournament if he chooses s.1 is

$$p_{B,1} = 0.5\left[0.5\left(1-\left(1-p\right)c\right) + 0.5\left(1-\left(1-p\right)c\right)\right].$$

Otherwise, player B will meet player C in s.1, generating the corresponding probability

$$p_{B,2} = p \times p\left(1-0{,}5c\right) + p\left(1-p\right)\left(1-\left(1-p\right)c\right).$$

Turning to the case where player B initially chooses the second semi-final (s.2), and player A chooses s.1, it can easily be verified that player C prefers to compete against player A in s.1 rather than facing player B in s.2. The price player C has to pay, in case of winning against player B in s.2, is a reduced probability $\left((1-p)c \text{ or } 0.5c\right)$ of winning the final either against player A or against player D. We have

$$p_{C,4} - p_{C,5} = (1-p)(1-c) > 0.$$

Player A's probability of winning the tournament when choosing s.1 is then

$$p_{A,4} = pp(1-0.5c) + p(1-p)\left(1-(1-p)c\right)$$

Making use of the inequalities $(1-0.5c) > 0.5$ and $1-(1-p)c > 0.5$, we get s

$$p_{A,4} = p \times p(1-0.5c) + p(1-p)\left(1-(1-p)c\right) > p \times p0.5 + p(1-p)0.5 = 0.5p.$$

Since player A's probability of winning the tournament when choosing s.2 is $p_{A,6} = 0.5pc$, we have established that $p_{A,4} - p_{A.6} > 0$. To summarize, if player B initially chooses s.2 then he will face player D in this semifinal. Player B's probability of winning the tournament when choosing s.2 is then

$$p_{B,4} = p \times p0.5c + p(1-p)pc.$$

However, player B will never choose s.2. For the case $c < \frac{-0.5}{p^3 - 1.5p^2 + 0.5p - 0.5}$, it is relevant for player B to compare $p_{B,1}$ with $p_{B,4}$. For this case, we have earlier found that $p_{A,1} > p_{A,2}$. By definition of the plays we also have $p_{A,1} = p_{B,1}$ and $p_{A,2} = p_{B,4}$. Hence, $p_{B,1} > p_{B,4}$.

For the case $c > \frac{-0.5}{p^3 - 1.5p^2 + 0.5p - 0.5}$ we compare $p_{B,2}$ with $p_{B,4}$. It is easily verified that

$$p_{B,2} - p_{B,4} = p(1-c) > 0.$$

Proof of Proposition 2

To show that the test statistic follows a standard normal distribution under the null hypothesis we need to prove that

$$\bar{R}_E - \bar{R}_L \overset{appr}{\sim} N\left(0, \frac{465}{14}\right) \text{ under } H_0 .$$

First, from the way the null hypothesis is formulated we have

$$E\left(\bar{R}_E - \bar{R}_L\right) = E\left(\bar{R}_E\right) - E\left(\bar{R}_L\right) = 0 .$$

Second, to prove that $V\left(\bar{R}_E - \bar{R}_L\right) = 465/14$, we define X_{ik} and Y_{ik}, $i = 1, 2, \ldots 12$, for the k^{th} competition, as the rank for the i^{th} athlete in an early quarterfinal and in a late quarterfinal, respectively. Thus,

$$U_k = \sum_{i=1}^{12} X_{ik} \text{ and } V_k = \sum_{i=1}^{12} Y_{ik} .$$

From the way a competition is designed, under the assumption of the null hypothesis that the athletes choose quarterfinals at random, it follows that X_{ik} and Y_{ik} are discrete uniformly distributed from 1 to 30, implying that $E\left(X_{ik}\right) = E\left(X_{ik}\right) = 15.5$ and $V\left(X_{ik}\right) = V\left(Y_{ik}\right) = 74\frac{11}{12}$ using well-known results for the discrete uniform distribution (see for example Casella and Berger (2002)).

Referring to the same assumption of random choice, it also follows that X_{ik} and X_{jm} are independent for all combinations of (i, j) and (k, m), except for combinations where $k = m$. This result holds true for Y_{ik} and Y_{jm}, as well as for X_{ik} and Y_{jm}. For combinations where $k = m$ and $i \neq j$ we get

$$\begin{aligned} Cov\left(X_{ik}, X_{jk}\right) &= E\left[\left(X_{ik} - E\left(X_{ik}\right)\right)\left(X_{jk} - E\left(X_{jk}\right)\right)\right] = E\left(X_{ik}X_{jk}\right) - E\left(X_{ik}\right)E\left(X_{jk}\right) \\ &= \sum_{x_{ik}=1}^{30}\sum_{x_{jk}=1}^{30} x_{ik}x_{jk} P\left(X_{ik} = x_{ik}, X_{jk} = x_{jk}\right) - (15.5)^2 \\ &= \frac{1}{870}\sum\sum_{x_{ik} \neq x_{jk}} x_{ik}x_{jk} - (15.5)^2 = -2\frac{7}{12}, \end{aligned}$$

where the second to last step follows from the fact that X_{ik} and X_{jk} cannot take on the same value and there are 870 possible outcomes $\left(x_{ik}, x_{jk}\right)$, all equally likely to occur. The same result holds for $Cov\left(Y_{ik}, Y_{jk}\right)$. The result is also valid for $Cov\left(X_{ik}, Y_{jk}\right)$, here for the case $i = j$ as well.

Now, making use of the results for $V\left(X_{ik}\right)$ and $Cov\left(X_{ik}, X_{jk}\right)$, we get

$$V\left(U_k\right)=V\left(\sum_{i=1}^{12}X_{ik}\right)=\sum_{i=1}^{12}V\left(X_{ik}\right)+2\sum\sum_{1\le i\le j\le 12}Cov\left(X_{ik},X_{jk}\right)$$
$$=12V\left(X_{ik}\right)+132Cov\left(X_{ik},X_{jk}\right)=558.$$

Likewise, we obtain $V\left(V_k\right)=558$.

We also get

$$Cov\left(U_k,V_k\right)=Cov\left(\sum_{i=1}^{12}X_{ik},\sum_{j=1}^{12}Y_{jk}\right)=\sum_{i=1}^{12}\sum_{j=1}^{12}Cov\left(X_{ik},Y_{jk}\right)$$
$$=144Cov\left(X_{ik},Y_{jk}\right)=-372.$$

In order to find $V\left(\bar{R}_E-\bar{R}_L\right)$, we also need $V\left(\bar{R}_E\right),V\left(\bar{R}_L\right)$ and $Cov\left(\bar{R}_E,\bar{R}_L\right)$.

Since $U_1,U_2,...,U_{56}$ are *i.i.d*, we have

$$V\left(\bar{R}_E\right)=V\left(\frac{\sum_{k=1}^{56}U_k}{56}\right)=\frac{1}{56^2}\sum_{k=1}^{56}U_k=\frac{1}{56^2}\times 56V\left(U_k\right)=\frac{279}{28}\text{ using the result }V\left(U_k\right)=558.$$

Likewise, we obtain $V\left(\bar{R}_L\right)=\dfrac{279}{28}$.

Using that $Cov\left(U_k,V_k\right)=372$ and $Cov\left(U_k,V_m\right)=0$ for $k\ne m$, we have

$$Cov\left(\bar{R}_E,\bar{R}_L\right)=Cov\left(\frac{\sum_{k=1}^{56}U_k}{56},\frac{\sum_{m=1}^{56}V_m}{56}\right)=\frac{1}{56^2}\sum_{k=1}^{56}\sum_{m=1}^{56}Cov\left(U_k,V_m\right)$$
$$=\frac{1}{56^2}\sum_{k=1}^{56}Cov\left(U_k,V_k\right)=\frac{1}{56^2}\times 56Cov\left(U_k,V_k\right)=-\frac{93}{14}.$$

Thus, combining the results above, we obtain the variance $V\left(\bar{R}_E-\bar{R}_L\right)$ as

$$V\left(\bar{R}_E-\bar{R}_L\right)=V\left(\bar{R}_E\right)+V\left(\bar{R}_L\right)-2Cov\left(\bar{R}_E,\bar{R}_L\right)=\frac{465}{14}.$$

Third, to prove that $\bar{R}_E-\bar{R}_L$ follows an approximate normal distribution, we note that $U_1,U_2,...,U_{56}$ as well as $V_1,V_2,...,V_{56}$ are *i.i.d* , meaning that $\bar{R}_E$ and $\bar{R}_L$ are approximately normal distributed by the Central Limit Theorem. Therefore, the difference between $\bar{R}_E$ and $\bar{R}_L$ - a linear combination - is also normal distributed.