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Dynamic relationship between Stock and Bond returns: A GAS MIDAS copula approach

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Abstract

Stock and bond are the two most crucial assets for portfolio allocation and risk management. This study proposes generalized autoregressive score mixed frequency data sampling (GAS MIDAS) copula models to analyze the dynamic dependence between stock returns and bond returns. A GAS MIDAS copula decomposes their relationship into a short-term dependence and a long-term dependence. While the long-term dependence is driven by related macro-finance factors using a MIDAS regression, the short-term effect follows a GAS process. Asymmetric dependence at different quantiles is also taken into account. We find that the proposed GAS MIDAS copula models are more effective in optimal portfolio allocation and improve the accuracy in risk management compared to other alternatives.

JEL-codes: C32, C52, C58, G11, G12

Keywords: GAS copulas, MIDAS, asymmetry.

1 Introduction

Stock and bond are the two most crucial assets for portfolio allocation and risk management and their dependence is one of the integral parts of asset allocation and investment strategies. For that reason, their joint relationship has been investigated intensively by Guidolin and Timmermann (2007), Christiansen and Ranaldo (2007), Yang et al. (2009), Baele et al. (2010), David and Veronesi (2013), among others. While a stock is considered as a risky asset, a treasury bond is often used as a hedge during the recession and crisis periods (Campbell et al., 2017). Previous research highlights the predictive power of various macro-finance variables for stock–bond comovement. For example, Campbell and Ammer (1993) consider that the stock-bond correlation has been driven by the changes in the economic conditions and monetary policy. More recently, Bekaert et al. (2020) show that macroeconomic uncertainty and risk aversion are important determinants of stock and bond prices. Such comovements are also found to be asymmetric when exposing to market information with equities responding stronger than bonds to joint bad news (see, for example, Cappiello et al. (2006)). Therefore, understanding this dependence not only helps investors to derive their optimal portfolio but also serves as an early warning signal for the changes in the macroeconomic conditions.

This study proposes a generalized autoregressive score mixed frequency data sampling (GAS MIDAS) copula approach to analyze the dynamic relationship between stock returns and bond returns. Following Conrad and Kleen (2020), we first use a multiplicative GARCH MIDAS model for the return distributions. Then, the standardized innovations of stock and bond returns are assumed to relate to each other through a copula model. Our proposed GAS MIDAS copula decomposes the stock-bond relation into a short-term dependence and a long-term dependence. While the long-term effect is updated at a lower frequency using a MIDAS regression, the short-term effect follows a GAS process. This approach allows us to combine daily stock and bond returns with macro-finance variables recorded at different frequencies. The macroeconomic explanatory variables are divided into four main groups, such as inflation and interest rates (II), state of the economy (SE), market uncertainty (UC), and illiquidity (IL). Asymmetric dependence at different quantiles is also taken into account using measures of asymmetric association. These factors, and in particular their forecasts, are found to be good at forecasting both in and out of sample stock–bond dependence.

Based on the standard rational pricing model, stock and bond are valued based on the future cash flows and the discount rate. It follows that there are fundamental economic factors that can affect the stock-bond comovements. Ilmanen (2003) considers that discount rates would be more stable when the inflation is low which pushes down the stock-bond correlation. And at a high inflation rate, the discount rate changes rule out the volatility and result in a positive correlation. On the other hand, economic growth and uncertainty affect the correlation reversely. Connolly et al. (2005) found that the uncertainty in the stock market decreases the correlation due to the flight-to-quality phenomenon. Furthermore, Baele et al. (2010) consider not only state variables but also uncertainty measures from survey data. The changes in the dependence can also be explained by the state of fundamental factors based on the theoretical macroeconomic models. For example, Song (2017) uses a consumption-based equilibrium model to identify the driving forces of the monetary policy to the changes in the stock-bond relation. Alternatively, Campbell et al. (2020) utilize a consumption-based habit formation model to explain the switch sign correlation in the 2000s while Li et al. (2020) consider both monetary policy and fiscal policy for the explanation.

However, one difficulty of using the macroeconomic conditions to explain the stock-bond relation is that the macroeconomic variables are often released at lower frequencies (say, monthly or quarterly). Fortunately, the mixed-data sampling (MIDAS) introduced by Ghysels et al. (2004) and Ghysels et al. (2006) can solve the frequency mismatch issue. MIDAS offers a framework to incorporate macroeconomic variables sampled at different frequencies along with the financial series, where macroeconomic variables enter directly into the specification of the long-term component. To model dependence between stock and bond returns via involving MIDAS structure, the Dynamic Conditional Correlation approach coupled with MIDAS structure has been popular in the existing literature. This approach allows the decomposition of dynamic correlation into long and short term components in addition to incorporating macroeconomic factors into the model. Asgharian et al. (2016) apply a DCC MIDAS model and suggest four main driven factors for the stock-bond correlation, which includes, the inflation and interest rate, the state of the economy, the illiquidity, and the uncertainty. In another study and through international evidence, Conrad and Stürmer (2017) shows that the stock-bond correlation is mainly driven by inflation and interest rate expectations as well as a flight-to-safety during times of stress in financial markets.

As the returns of financial series are characterized by fat tails and asymmetry (Christoffersen et al., 2012). Restricted multivariate Gaussian model, such as the original DCC-MIDAS approach, can ignore or misinterpret the effect of extreme observations to the stock-bond dependence. Alternatively, copulas have been successfully utilized in the literature as multivariate probability distributions for non-linear dependence structures, see Joe (2014) and Czado (2019), Nguyen et al. (2020), among others. Copulas help to separate the marginal distributions and model the dependence by switching from the domain of the data to the unit hypercube (Smith, 2011). Noticing that the copulas allow more parameters to capture the tail dependence, they are preferred over the classical multivariate distributions. Besides, there are a large number of bivariate copula functions (Joe, 1997), which are suitable for various dependence behaviors of data.

To the extent, several dynamic copula models have been proposed for financial returns. Since the introduction of MIDAS framework and noticing it's importance in finance research, several attempts have been made to couple copula structure with it. Most recently, Jiang et al. (2020) propose a time varying Copula MIDAS GARCH with exogenous explanatory variables to model the joint distribution of returns, where the short-term component is updated based on the restricted ARMA(1,10) process. Close to our proposal, Gong et al. (2020) suggest a Copula-MIDAS-X model that incorporates low-frequency explanatory variables into a high-frequency dynamic copula model to investigate the impacts of economic factors on the relationship between oil and stocks. Deviating from existing literature, we propose a Copula model where the GAS process is used to capture short-term dependence within the MIDAS framework, and further extend it to account for the asymmetric effect as well. Different from the DCC model where the updated term is based on the first or second moment of the most recent observations, the GAS process utilizes the complete density of the copula function. The defining feature of GAS models is that the time-varying parameters are driven by the score of the predictive log-likelihood function, which can be viewed as a Newton–Raphson update that delivers a better fit for the next period conditional on past and current information (Gorgi et al. (2019)). By relying on the density structure to update the time-varying parameters, GAS models take into account all information in the data distribution. Through empirical evidence, Koopman et al. (2016) supports that the GAS updated process outperforms other observation driven processes in terms of predictive accuracy.

Our proposed GAS MIDAS copula model are compared with the Exponential Weight Moving Average (EWMA), the Dynamic Conditional Correlation (DCC) model (Engle, 2002), and the GAS copula model (Creal et al., 2013) in different stress test scenarios. We find that the GAS MIDAS model can effectively integrate the information of the long-term dependence and give a better in-sample correlation fit. We illustrate our proposed asymmetric GAS MIDAS copula model for the dynamic stock-bond dependence using several macroeconomic explanatory variables. We find that the inflation and interest rate, the state of the economy, and the illiquidity contribute to the long-term dependence. We also find evidence of different regimes where the effects in the contraction and expansion are asymmetric. Other sources of soft information from the Survey of Professional Forecasters also help to incorporate a forward-looking of the business cycle and growth outlook to explain the long-term dependence. In general, the principal component of inflation and interest rate combined with the principal component of illiquidity can serve as a good predictor for the long-term change in the stock-bond dependence. The investor who utilizes the GAS MIDAS copula models can improve the accuracy in risk management and optimize the portfolio allocation.

The rest of the paper is organized as follows. Section 2 introduces the econometric framework of the GAS MIDAS copula model. We present the performance of GAS MIDAS copula models with simulated stress tests in Section 3. In Section 4, we analyze the fundamental factors that affect the dependence of stock returns and bond returns. Section 5 compares the optimal portfolio allocation and risk management based on the GAS MIDAS copula models. Finally, conclusions are drawn in Section 6.

2 GAS MIDAS Copula models

In this section, we present the GAS MIDAS copula models for the dynamic dependence of stock returns and bond returns. Following Conrad and Kleen (2020), we first employ the multiplicative GARCH model to estimate the marginal distribution of individual returns then explain the stock-bond dependence through a bivariate copula function. We allow for explanatory variables to justify the long-term changes in the dependence through a MIDAS regression whereas the short-term dependence follows a GAS process.

2.1 Model Specification

To model the returns series for stock (or bond), we use a class of component GARCH model based on the MIDAS regression (Engle et al. (2013)), for which the importance has been highlighted in several research studies. For example, Asgharian et al. (2013) show that the addition of a business cycle proxy in the GARCH-MIDAS specification improves the model's forecasting ability compared to the conventional GARCH modifications. Engle et al. (2013) report that the inclusion of a business cycle latent variable affects both the volatility components, i.e. long-term and the short-term variance components. Taken together, the inclusion of macroeconomic variables can depict the underlying dependence dynamics more accurately.

Let r_{it} be daily time series returns of stock (or bond) at time t, for $i = 1, 2$. Following Conrad and Kleen (2020) we utilize the multiplicative GARCH model to explain the long-term changes ($\kappa_{i\tau}$) in the volatility using explanatory variables. The short-term component (g_{it}) is intended to explain (say, daily) clustering of volatility and is assumed to follow a mean-reverting unit-variance GJR-GARCH(1,1) process.

$$
\begin{aligned}
r_{it} &= \mu_i + \sqrt{\kappa_{i\tau} g_{it}}\epsilon_{it}, \\
g_{it} &= (1 - \alpha_i - 0.5\gamma_i - \beta_i) + (\alpha_i + \gamma_i \mathbf{I}_{\{\epsilon_{i,t-1}<0\}}) g_{i,t-1}\epsilon_{i,t-1}^2 + \beta_i g_{i,t-1}, \\
\kappa_{i\tau} &= \exp\left(m_i + \sum_{j=1}^{N_i} \delta_{i,j} \left[\sum_{k=1}^{K_j} \phi_k(\omega_{i,j,1}, \omega_{i,j,2}) X_{i,j,\tau-k} \right] \right),
\end{aligned}
\tag{1}
$$

where τ is a time indicator of the explanatory variables (usually in low-frequency); $\kappa_{i\tau}$ is the long-term component of volatility and is constant across all days within the period τ. The low-frequency τ is related to the daily time points through $\tau = \lfloor t/L \rfloor$ with L is the number of days during a release period of the explanatory variable. Let $(\alpha_i, \beta_i, \gamma_i)$ be the set of parameters that governs the short-term volatility component g_{it} such that $0 < \alpha_i + 0.5\gamma_i + \beta_i < 1$ and $\alpha_i, \beta_i > 0$ for a stationary condition. In each period t, short-term volatility is updated through the past innovation $\epsilon_{i,t-1}$ that follows a distribution F_i with zero mean and unit variance. The leverage effect of "bad news" is controlled by the parameter $\gamma_i > 0$ in which a negative shock has a higher impact to the volatility than a positive shock. Secondly, a set of explanatory variables $X_{i,j}$, for $j = 1, \ldots, N_i$, help to refine the long-term volatility component $\kappa_{i\tau}$ in the spirit of MIDAS regression and filtering, see Engle et al. (2013). The weighting function $\phi_k(\cdot)$ creates a weighting scheme and regularizes for the effect of the last K_j observations of the explanatory variable $X_{i,j}$. In the Appendix A, we provide several commonly used weighting functions that allow for different pattern effects, for example the beta-polynomial function is written as,

$$\phi_k(\omega_{i,j,1},\omega_{i,j,2}) = \frac{\big(k/(K_j+1)\big)^{\omega_{i,j,1}-1}\big(1-k/(K_j+1)\big)^{\omega_{i,j,2}-1}}{\sum_{l=1}^{K_j}\big(l/(K_j+1)\big)^{\omega_{i,j,1}-1}\big(1-l/(K_j+1)\big)^{\omega_{i,j,2}-1}}. \tag{2}$$

where $\omega_{i,j,1}$ and $\omega_{i,j,2}$ are the weights to be estimated for each j variable. Using the flexible/unrestricted beta smoothing function, the long-term volatility of daily returns in the above equation is expressed as a weighted average of lower-frequency financial and/or macroeconomic variables. This beta-polynomial is independently estimated for each MIDAS regression and for each explanatory variable therein. Following Engle et al. (2013) and Asgharian et al. (2013), we use the restricted version of the beta weighting scheme by fixing $\omega_{i,j,1} = 1$. The restricted beta weighting scheme ensures a decaying pattern whereas the size of $\omega_{i,j,2}$ determines the speed of decay: large (small) values of $\omega_{i,j,2}$ generate an accelerating (decelerating) decaying pattern for the lagged values of explanatory variable(s) in the MIDAS filter.

2.2 A GAS Copula model

Assuming that we have specified an appropriate marginal model for stock returns and bond returns, the next step is to model the joint dependence between them through a copula function. Let $u_{1t} = F_1(\epsilon_{1t})$, and $u_{2t} = F_2(\epsilon_{2t})$ be sequences of independent random variables that follow a uniform marginal distribution. Using Sklar (1959) theorem, there exists an unknown copula density function $c_t(u_{1t}, u_{2t})$ that satisfies $f(\epsilon_{1t}, \epsilon_{2t}) = f_1(\epsilon_{1t})f_2(\epsilon_{2t})c_t(u_{1t}, u_{2t})$. As the copula density function can be dynamic, the copula dependence parameters are allowed to be time-varying while the copula function remains unchanged (Patton (2006), Hafner and Manner (2012), Nguyen et al. (2019)). An important feature of any dynamic model is to specify how the parameters evolve through time. Cox (1981) classifies such models into two classes: observation-driven and parameter-driven specifications. The parameter-driven specifications, such as stochastic copula models (see Hafner and Manner (2012)) allow the varying parameters to evolve as a latent time series process (with idiosyncratic innovations). The observation-driven specifications, such as ARCH-type models for volatility (see Engle 1982) and related models for copulas (see, Patton (2006), Creal et al. (2013) and references therein) model the varying parameters as some function of lagged dependent variables as well as contemporaneous and lagged exogenous variables. In this approach, the parameters evolve randomly over time, but they are perfectly predictable one step ahead given past information. The likelihood function for such models is also available in closed form. Another advantage of the latter approach over the former is that it avoids the need to "integrate out" the innovation terms driving the latent time series processes.

However, within the class of observation-driven specifications, the choice of an appropriate *function* of lagged dependent variables is to be made. For models of the conditional variance, the lagged squared residual (the ARCH-family of models) comes as an obvious choice, but for models with parameters that lack an obvious interpretation, the choice is less clear. To overcome this problem, we follow Creal et al. (2013) and Harvey (2013), and allow the time-varying parameter θ_t to follow the generalized autoregressive score (GAS) process. The process adopts the score vector of the predictive model density to update the time-varying parameters. This choice is motivated by the fact that the GAS model belongs to a class of observation-driven models with a similar degree of generality as obtained for non-linear, non-Gaussian state-space models. By relying on the density structure to update the time-varying parameters, GAS models take into account full information in the data distribution. Koopman et al. (2016) provide empirical evidence that the GAS updated process outperforms other observation-driven processes in terms of predictive accuracy. Following Creal et al. (2013), the process can be written as,

$$\begin{aligned}(u_{1t}, u_{2t}) &\sim c_t(u_{1t}, u_{2t}|\theta_t),\ \ \theta_t = \Lambda(\lambda_t),\\ \lambda_{t+1} &= \lambda_0(1-\beta) + \alpha\frac{\partial \log c_t(u_{1t}, u_{2t}|\lambda_t)}{\partial \lambda_t} + \beta\lambda_t,\end{aligned} \tag{3}$$

where λ_t is an observation-driven process which is mapped from the real unrestricted domain to the restricted domain of the copula parameter through a transformation function Λ, e.g., if $\theta_t > 0$, $\Lambda(\lambda_t) = \exp(\lambda_t)$. And $(\lambda_0, \alpha, \beta)$ are the set of fixed parameters that control for the dynamic behavior of the process such that $|\beta| < 1$ for stationarity. The process λ_t will vary around the mean λ_0 and rely on its past value with a persistent parameter β and be updated with an adjustment term calculated as a score function. This updating procedure is similar to the Newton-Raphson algorithm that maximizes the predicted likelihood given the current and past information (Gorgi et al., 2019). Blasques et al. (2015) show that the use of the score functions leads to the minimization of the Kullback–Leibler divergence between the true conditional density and the model-implied conditional density, hence becomes more robust to the model misspecification.

In the bivariate context, there are several copula functions that allow for a flexible dependence (see, Joe (2014)). Among those, elliptical copula and Archimedean copula families are most commonly used in finance due to parsimonious specification and their ability to capture tail dependence. In this study, we employ the bivariate Gaussian copula, the Student copula, the Clayton copula, the Gumbel copula, the Frank copula, the Joe copula to model the dynamic dependence of stock returns and bond returns. In order for the Archimedean copulas that can capture both positive and negative dependence, we first create a symmetric Archimedean copula density function as an equally weighted of the Archimedean copula and its 180-degrees rotated Archimedean copula, see Appendix B. For $\lambda_t > 0$, this symmetric Archimedean copula is used to model the capture positive dependence and for $\lambda_t < 0$ the 90-degrees rotated symmetric Archimedean copula is used for the negative dependence. Also, the degrees of freedom ν is kept fixed in the Student copula model and we let the correlation parameter vary instead. The appropriate bivariate copula function for the stock-bond dependence is chosen by the Akaike information criterion (AIC) and the Bayesian information criterion (BIC).

2.3 A GAS MIDAS Copula model

The GAS copula model assumes a fixed level of long-term dependence which might be very restricted empirically. One straight extension to the GAS copulas, which is our second contribution to the literature, is to allow for the long-term dependence parameter to be driven by other explanatory variables. This makes sense in reality since new information in the market can create a permanent shift in the level of expected cash flow hence affect the dependence. This proposal is also motivated by the contributions made in Colacito et al. (2011) to model dynamic correlations with a short- and long-term component specification, and Asgharian et al. (2016) where, in the latter, the authors investigate stock–bond correlation using a DCC-MIDAS model and documented the importance of using macro-finance forecasts to predict future long-term stock–bond correlation. Together with proposing the time-varying parameter θ_t to follow the GAS structure, we incorporate low-frequency explanatory variables into the copula model to explain the evolution of the dynamic dependence structure. Most particularly, we incorporate the changes in the long-term dependence through a MIDAS regression equation, which would allow us to investigate the direct effect of macroeconomic information on dependence obtained through copula as follows,

$$\begin{aligned} (u_{1t}, u_{2t}) &\sim c_t(u_{1t}, u_{2t}|\theta_t),\ \ \theta_t = \Lambda(\lambda_t), \\ \lambda_{t+1} &= \lambda_\tau(1-\beta) + \alpha \frac{\partial \log c_t(u_{1t}, u_{2t}|\lambda_t)}{\partial \lambda_t} + \beta\lambda_t, \\ \lambda_\tau &= \lambda_0 + \sum_{j=1}^{N} \delta_j \left[\sum_{k=1}^{K_j} \phi_k(\omega_{j,1}, \omega_{j,2}) X_{j,\tau-k} \right], \end{aligned} \tag{4}$$

where the last part of Eq. 4 couples the MIDAS framework within copula. λ_t represents the fundamental or secular causes of time variation in the dependence, and can evolve by a range of low-frequency explanatory variables $X = (X_1, \cdots, X_N)$, such as macroeconomic variables, $\phi_k(\cdot)$ is the weighting scheme of the variable j on its k lag, for $k = 1, \ldots, K$, as defined in Eq. 2. Similar to the marginal model, this GAS MIDAS extension allows for the flexible changes in the long-term dependence through a set of explanatory variables X_j. The significance of the contribution of variable X_j can be seen through the significance of the regression coefficient δ_j and the weighting parameters $\omega_{j,1}$ and $\omega_{j,2}$ regulates for how long the effect lasts for explanatory variables X_j. Here, the GAS MIDAS copula model reduces to the GAS copula model when $\delta_j = 0, \forall j$.

2.4 An asymmetric GAS MIDAS Copula model

It is well known to financial practitioners that the vast majority of financial data show various systematic asymmetries (see, for example Perez-Quiros and Timmermann (2001) and Babsiri and Zakoian (2001)). Among them two have been the subject of more thorough studies, namely asymmetry in the distribution of returns and asymmetry in the way volatility responds to positive and negative (relatively to the mean) returns. The second form of asymmetry stems from the fact that the market is prone to react differently to positive as opposed to negative returns. The impact of negative news is different on the volatility and correlation than the positive news. The concept of volatility asymmetry was initially pointed out by Black (1976) and Christie (1982). This phenomenon is famously discussed in financial literature under the name of "leverage effect", which states that a drop in the price value of the stock leads to negative returns, which increases the financial leverage (debt-to-equity ratio) and consequently makes the stock riskier and increases its volatility.

Cappiello et al. (2006) extends this phenomenon to assess asymmetry in correlations and show that the sign of innovation induces the stock-bond dependence differently. For example, when a systematic negative shock comes to the equity returns, the investors will allocate a higher weight to safety assets, hence decreases the stock-bond dependence. Following the directions and motivated by the findings reported in Cappiello et al. (2006), we extend further our proposed model to account for asymmetric effect in the dependence structure at flexible quantiles $0 < q_1, q_2 \leqslant 1$. We present an asymmetric GAS MIDAS Copula model as follows,

$$\begin{aligned} (u_{1t}, u_{2t}) &\sim c_t(u_{1t}, u_{2t}|\theta_t), \\ \theta_t &= \Lambda(\lambda_t), \\ \lambda_{t+1} &= \lambda_\tau(1-\beta) + \alpha \frac{\partial \log c_t(u_{1t}, u_{2t}|\lambda_t)}{\partial \lambda_t} + \beta\lambda_t + \gamma\left(v_t - \bar{v}\right), \\ \lambda_\tau &= \lambda_0 + \sum_{j=1}^{N} \delta_j \left[\sum_{k=1}^{K_j} \phi_k(\omega_{j,1}, \omega_{j,2}) X_{j,\tau-k} \right], \end{aligned} \tag{5}$$

where γ is the parameter that controls for the asymmetry, v_t is a measure of association related to "bad news" at time t and $\bar{v} = \mathbf{E}(v_t)$. Following Joe (2014), we propose several measures of asymmetric association between u_{1t} and u_{2t} such as,

(a) Normal score:

$$v_t = \left[\Phi^{-1}(u_{1t})\mathbf{I}_{\{u_{1t}<q_1\}}\right]\left[\Phi^{-1}(u_{2t})\mathbf{I}_{\{u_{2t}<q_2\}}\right].$$

The expectation of the normal score is related to the Pearson correlation between two transformed normal random variables at a lower quadrant. As the dependence can be positive or negative, we also check with a different quadrant where $v_t = \left[\Phi^{-1}(u_{1t})\mathbf{I}_{\{u_{1t}<q_1\}}\right]\left[\Phi^{-1}(u_{2t})\mathbf{I}_{\{u_{2t}>q_2\}}\right]$.

(b) Spearman's rank:

$$v_t = \left[(u_{1t} - 0.5)\mathbf{I}_{\{u_{1t}<q_1\}}\right]\left[(u_{2t} - 0.5)\mathbf{I}_{\{u_{2t}<q_2\}}\right].$$

In this case, $\bar{v}$ is actually the Spearman's rank correlation between stock and bond innovations at a lower quadrant.

(c) Spearman's footrule:

$$v_t = |u_{1t} - u_{2t}|\, \mathbf{I}_{\{u_{1t}<q_1\}}\mathbf{I}_{\{u_{2t}<q_2\}}.$$

This measure of association is inversely related to the Spearman's footrule as the concordance ordering of copulas, see Patton (2006). However, the Spearman's footrule is calculated based on the Manhattan distance between two sets of ranks and suffers from asymmetry in the sense that the Spearman's footrule is equal 0, 1/3, 0.5 under perfect positive dependence, independence, perfect negative dependence respectively.

(d) Gini's gamma:

$$v_t = (|1 - u_{1t} - u_{2t}| - |u_{1t} - u_{2t}|)\mathbf{I}_{\{u_{1t}<q_1\}}\mathbf{I}_{\{u_{2t}<q_2\}}.$$

Salama and Quade (2001) and Genest et al. (2010) consider the Gini's gamma is an extension of the Spearman's footrule whereas it is a symmetric measure. Hence, the parameter estimation can be less sensitive to the quantile specification.

In any cases, note that the expectation of the asymmetric association $\bar{v} = \mathbf{E}(v_t)$ is feasible and can be calculated with the sample analogue $\bar{v} = \frac{1}{T}\sum_{t=1}^{T} v_t$, see Cappiello et al. (2006).

2.5 Estimation

The total log-likelihood function can be decomposed as a sum of the log likelihood of marginal returns and the copula log-likelihood function,

$$\begin{aligned}\mathcal{L}(\Theta) = &\sum_{t=1}^{T} \log f_t(r_{1t}, r_{2t}; \Theta)\\ &\sum_{t=1}^{T} \log f_{1t}(r_{1t}; \Theta_1) + \sum_{t=1}^{T} \log f_{2t}(r_{2t}; \Theta_2) + \sum_{t=1}^{T} \log c_t(F_{1t}(r_{1t}), F_{2t}(r_{1t}); \Theta_c)\end{aligned}$$

where $\Theta = (\Theta_1', \Theta_2', \Theta_c')'$ is a vector of the parameters of the marginal distributions of stock returns Θ_1 and bond returns Θ_2 and parameters of the copula Θ_c. The estimation of a GAS MIDAS copula model is implemented based on the two-stage estimation procedure, see Joe (2005). In the first stage, we fit the multiplicative GARCH MIDAS for marginal return series and in the second stage, we obtain the copula data using the empirical CDF function. Note that, the marginal distribution can be substituted with any GARCH or stochastic volatility specifications, however the two-stage estimation procedure is statistically efficient (Chen and Fan, 2006).

3 Simulation study

To provide strong empirical motivation to our proposed model, a simulation study is conducted where we compare the proposed GAS MIDAS copula models with the EWMA (Appendix C), the DCC (Engle, 2002), the GAS (Creal et al., 2013) when the true correlation structure is known, in different stress scenarios based on the proposal of Engle (2002). We simulate $T = 2000$ observations from a bivariate Gaussian copula with time-varying correlation parameter ρ_t. Following five scenarios are considered for the behavior of ρ_t such that,

1. Constant: $\rho_t = 0.8$.
2. Sine: $\rho_t = 0.5cos(2\pi t/250)$.
3. Fast Sine: $\rho_t = 0.5cos(2\pi t/25)$.
4. Step: $\rho_t = 0.5 - I(t > 1000)$.
5. Ramp: $\rho_t = ((t \mod 200) - 100)/101$.

Figure 1 illustrates the ρ_t processes for different stress tests. Engle (2002) considers that these stress tests mimic different realistic contexts that the correlation can be constant, gradual changes, rapid changes, and abrupt changes. We generate 200 datasets for each stress test and obtain the estimate of the correlation process $\hat{\rho}_t$. The 22-day realized correlation (RCor) is calculated as a low-frequency explanatory variable for the long-term change in the correlation. The accuracy of each model as assessed based on the mean absolute error (MAE) and the mean-squared error (MSE),

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |\hat{\rho}_t - \rho_t|,$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T} (\hat{\rho}_t - \rho_t)^2.$$

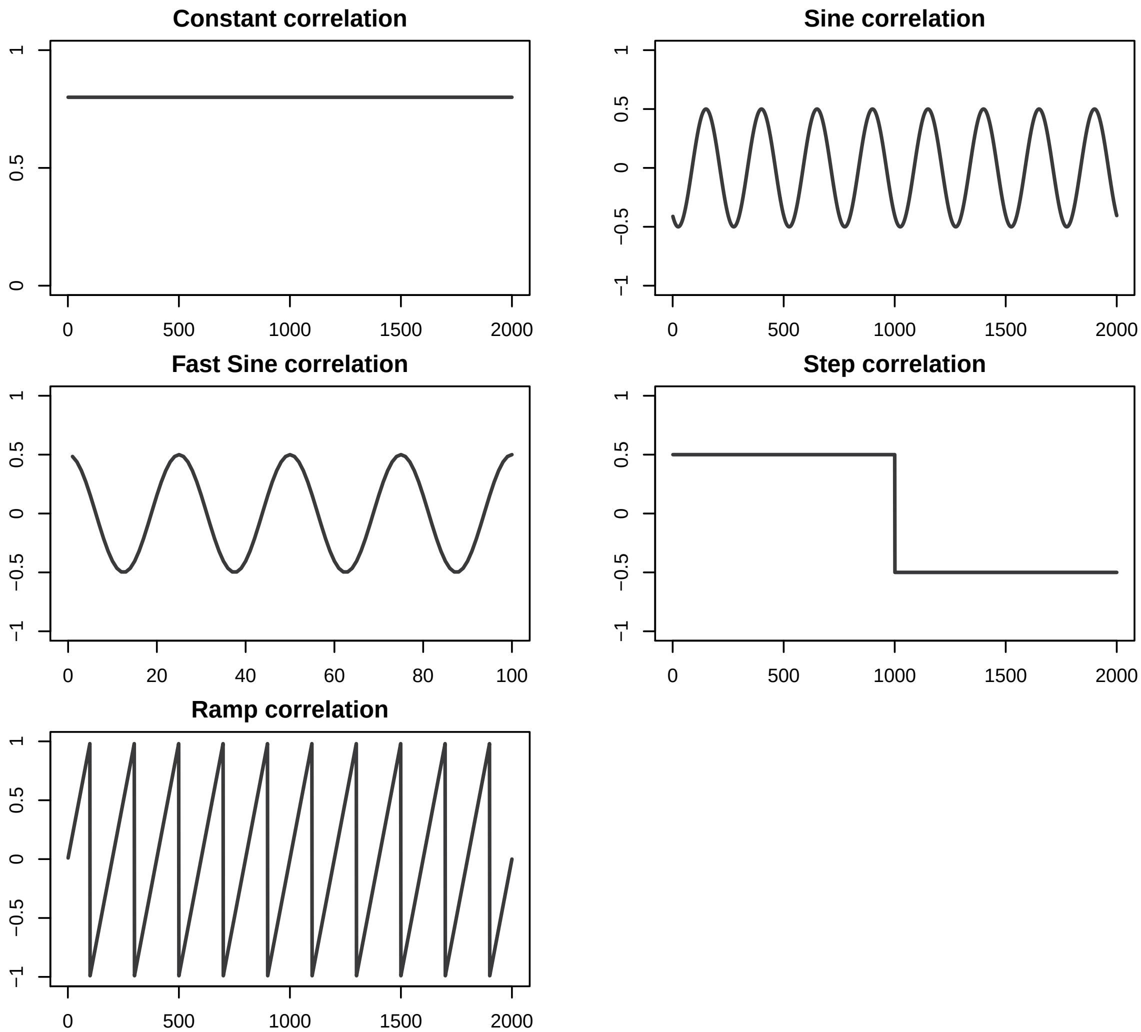

Figure 1: The ρ_t processes for different test scenarios

Table 1 compares the relative MAE and MSE of the estimation of the correlation using the EWMA, the GAS, the GAS MIDAS over the benchmark DCC model. In most of the considered cases, the GAS MIDAS model has the smallest MAE and MSE. For the constant correlation case though, the GAS MIDAS model does not seem very helpful as there are redundant parameters in the estimation. In general, the long-term changes in the correlation can be explained by the RCor in most of the occasions, which shows that the GAS MIDAS (highlighted in bold) can effectively incorporate this source of information to give a better correlation fit.

Table 1: MAE and MSE results: a simulation study

	Constant	Sine	Fast sine	Step	Ramp
			(a) MAE		
EWMA	5.067	1.184	1.180	1.003	1.357
DCC	1.000	1.000	1.000	1.000	1.000
GAS	0.798	1.002	0.988	0.875	0.940
GAS MIDAS	0.943	**0.609**	**0.986**	**0.866**	**0.875**
			(b) MSE		
EWMA	25.007	1.341	1.399	0.868	1.902
DCC	1.000	1.000	1.000	1.000	1.000
GAS	0.666	0.999	0.978	0.863	0.919
GAS MIDAS	0.980	**0.387**	0.978	**0.850**	**0.828**

The table shows the relative MAE and MSE of the estimation of the correlation using the EWMA, the GAS, the GAS MIDAS over the benchmark DCC model. We use the restricted beta weighting function and $K = 9$ lags of monthly RCor as a low-frequency explanatory variable for the long-term change in the correlation. We generate 200 pseudo datasets for each stress test and calculate the average of MAE and MSE. The entries less than 1 indicate that the given model is better.

We also implemented other robust simulation studies on the choice of copula functions, the lag number of the explanatory variables, weighting functions as well as the incorporation of the asymmetry effect[1].

4 Empirical illustration

4.1 Data description

In this section, we illustrate our proposal to model the dependence of stock returns and bond returns during the period from 01/01/1990 to 31/03/2021. The dataset is obtained through Macrobond database. The stock returns are calculated as the first difference log of the S&P 500 index multiplied by 100. The bond returns are calculated based on the yield-to-maturity of the 10 year Treasury bonds, see Swinkels (2019). As can be seen, the chosen period has several recessions and crises period such as the early 1990s recession, the Dot-com bubble, the subprime mortgage

[1]For brevity, the results are not reported here but are available upon request.

crisis (2007–2009) and the recent Covid-19 pandemic (2020 - 2021). Figure 2 shows the evolution of daily stock returns and bond returns. We can see that the most extreme return observations are coincident with the periods of recessions and crises. Also, the realized correlation was positive during the early 1990s recession but became mostly negative during the recent crises periods. The Covid-19 pandemic also shares a similar pattern with the recent recessions, the realized correlation was negative and then became positive in the first quarter of 2021.

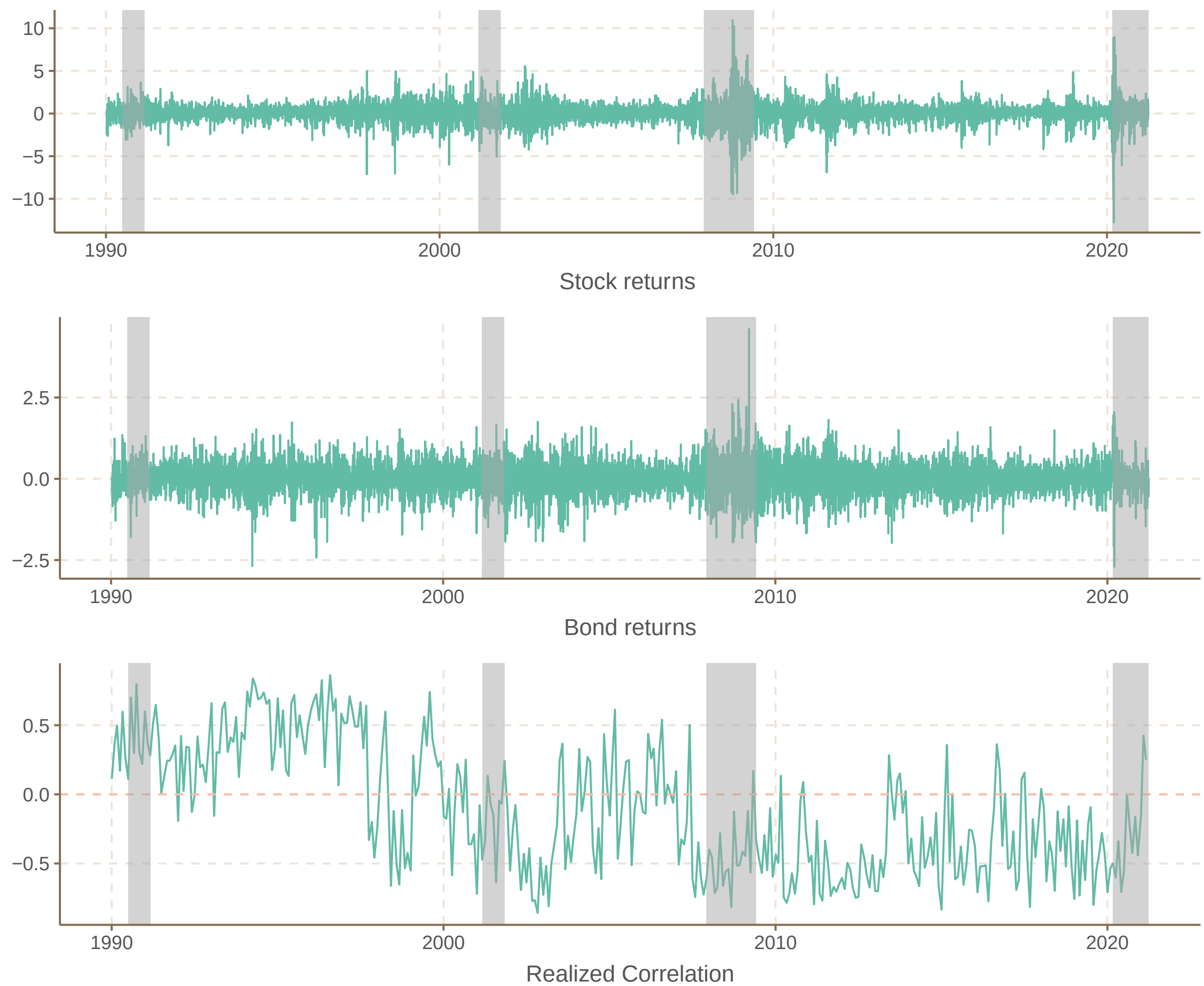

Figure 2: S&P 500 stock returns and 10Y Government bond returns.

The figures show the daily S&P 500 stock returns and the daily 10 year Government bond returns during the period from 01/01/1990 to 31/03/2021. The stock returns are calculated as the log difference of the S&P 500 index multiplied by 100. The bond returns are calculated based on the yield-to-maturity of the 10 year Treasury bonds, see Swinkels (2019). The shaded areas highlight the recession periods based on the NBER indicators.

Table 2(a) describes the summary statistics of the S&P 500 returns and the 10Y Government bond returns. Daily stock returns are higher than bond returns on average and more volatile with a higher degree of fat tails and asymmetry. The other measures confirm the presence of serial correlation and the ARCH effect in both the series. Table 2(b) reports the Pearson correlation of stock returns and bond returns during the subperiods and the total period of chosen data. The correlation was positive before 2000 and became negative after that. The 5% exceedance correlations at four different quadrants are denoted by $\rho^{I}_{0.05}, \rho^{II}_{0.05}, \rho^{III}_{0.05}, \rho^{IV}_{0.05}$. Before the 2000s, the exceedance correlations in the first and the third quadrants were significant due to the positive correlation while they both decreased after that. Also after the 2000s, the second and the fourth quadrants observed a strong negative correlation which represents the flight to safety effect between stock and bond. The test statistics for the symmetric exceedance correlation between (III-I) quadrants and between (II-IV) quadrants are calculated following Hong et al. (2007). There is slight evidence for the asymmetric exceedance correlation between stock returns and bond returns.

Table 2: Summary statistics

(a) Summary statistics of S&P 500 stock returns and 10Y Government bond returns (01/01/1990 - 31/03/2021)

	Mean	Standard deviation	Skewness	Kurtosis	Minimum	Maximum	J-B test	Ljung-Box(12)	ARCH(12)
Stock returns	0.031	1.149	-0.410	11.311	-12.765	10.957	42.2***	109.0***	2248.1***
Bond returns	0.019	0.462	-0.027	3.146	-2.707	4.608	3.2***	28.0***	496.5***

(b) Tests for exceedance correlations symmetry

Period	$\bar{\rho}$	$\rho^{III}_{0.05}$	$\rho^{I}_{0.05}$	Test (III-I)	p.val (III-I)	$\rho^{II}_{0.05}$	$\rho^{IV}_{0.05}$	Test (II-IV)	p.val (II-IV)
1990 - 1999	0.267	0.428	0.468	0.013	0.909	0.244	-0.356	2.026	0.155
2000 - 2009	-0.328	-0.69	-0.005	2.31	0.129	-0.542	-0.073	3.672*	0.055
2010 - 2021	-0.421	-	-	-	-	-0.552	-0.41	0.693	0.405
1990 - 2021	-0.224	0.117	0.005	0.277	0.599	-0.539	-0.184	4.396**	0.036

Panel (a) reports the summary statistics of S&P 500 stock returns and 10Y Government bond returns (01/01/1990 - 31/03/2021). The stock returns are calculated as the log difference of the S&P 500 index multiplied by 100. The bond returns are calculated based on the yield-to-maturity of the 10 year Treasury bonds, see Swinkels (2019). We also report the test statistics of the Jarque-Bera test for normality, the Ljung-Box test for the serial correlation and ARCH test for the GARCH effect. Panel (b) reports the Pearson correlation of stock returns and bond returns in subperiods. The 5% exceedance correlations at different quadrants are denoted by $\rho^{I}_{0.05}, \rho^{II}_{0.05}, \rho^{III}_{0.05}, \rho^{IV}_{0.05}$. The test statistics for the symmetric exceedence correlation between (III-I) quadrants and between (II-IV) quadrants are calculated following Hong et al. (2007). ***,**,* denote significant at 1%, 5%, 10% level.

4.2 Marginal distribution

In this section, we model the marginal returns using the multiplicative GARCH model by Conrad and Kleen (2020). The long-term changes in the volatility are driven by several explanatory variables such as the daily 22-day rolling Realized volatility (RV), the daily S&P 100 Volatility Index (VXO), the daily S&P 500 Volatility Index (VIX), the weekly National Financial Conditions Index (NFCI), the monthly National Activity Index (NAI), the monthly Industrial Production (IP). On the other hand, the short-term change is explained by a GJR GARCH model (Glosten et al., 1993). Following the specifications for the lag length of the explanatory variables and the weighting scheme by Conrad and Kleen (2020), we first estimate the marginal model using one measure of volatility, then using one measure of volatility combined with another macroeconomic variable. Table 3 reports the estimation results of the multiplicative component GARCH-MIDAS model for the stock returns and bond returns. The parameters are quite stable across specifications for both stocks and bonds. The estimate of $\omega_{i,j,2}$ $i = 1, 2; j = 1, 2$, indicates that the degree of smoothing varies across the macro-finance factors, where a smaller value leads to a larger degree of smoothing over the lagged observations. The asymmetric effect is found to be highly significant and higher for stock returns than for the bond returns. The signs of the estimated regression parameters δs for realized volatility, the VXO, the VIX, and the macroeconomic variables are in line with findings in the previous literature, i.e., higher levels of financial volatility tend to increase long-term volatility. We found that the VXO has a better explanation for long-term change in the volatility than the VIX and the RV in terms of log-likelihood (LLH) or Bayesian information criterion (BIC).

Since the NFCI and the macroeconomic variables, in particular, capture the lower frequency movements, it would be interesting to estimate GARCH-MIDAS models with the VXO jointly with any of these components. This would allow us to formally check whether the NFCI, the NAI and the IP contain information that is complementary to the VXO. When controlling for the VXO, only the slope parameter associated with NFCI is significant for stock volatility. Even that the two-factor GARCH MIDAS model results in better log-likelihood but based on the BIC, we choose the GARCH MIDAS using VXO as the appropriate marginal model for stock returns and bond returns. In the next stage, we obtain the copula data using the empirical CDF function of the standardized innovations. Then, we compare the dynamic dependence of stock and bond using our proposal GAS MIDAS models with different specifications.

Table 3: GARCH-MIDAS models for the marginal distributions

(a) The multiplicative component GARCH-MIDAS for the marginal distribution of Stock returns (01/01/1990 - 31/03/2021)

	μ_i	α_i	β_i	γ_i	m_i	$\delta_{i,1}$	$\omega_{i,1,2}$	$\delta_{i,2}$	$\omega_{i,2,2}$	K	LLH	BIC
RV Stock	0.029***	0.000	0.828***	0.215***	-1.243***	1.171***	2.840***			264	-1.301	2.610
	(0.009)	(0.009)	(0.016)	(0.023)	(0.119)	(0.091)	(0.665)					
VXO	0.026***	0.000	0.854***	0.086***	-2.057***	1.435***	3.789***			3	-1.286	2.580
	(0.009)	(0.014)	(0.022)	(0.018)	(0.075)	(0.052)	(0.743)					
VIX	0.025***	0.000	0.849***	0.098***	-2.158***	1.547***	3.417***			3	-1.289	2.586
	(0.009)	(0.012)	(0.023)	(0.019)	(0.085)	(0.061)	(0.627)					
VXO+NFCI	0.025***	0.000	0.851***	0.091***	-1.898***	1.355***	3.768***	0.151**	2.176	52	-1.286	2.582
	(0.009)	(0.014)	(0.023)	(0.018)	(0.102)	(0.063)	(0.723)	(0.064)	(1.659)			
VXO+NAI	0.024***	0.000	0.858***	0.091***	-1.936***	1.323***	3.766***	-0.197	7.039	36	-1.286	2.582
	(0.009)	(0.013)	(0.022)	(0.017)	(0.088)	(0.065)	(0.71)	(0.098)	(5.07)			
VXO+IP	0.026***	0.000	0.847***	0.087***	-2.085***	1.463***	3.787***	-0.008	33.645	36	-1.286	2.582
	(0.009)	(0.014)	(0.024)	(0.018)	(0.089)	(0.057)	(0.753)	(0.009)	(45.377)			

(b) The multiplicative component GARCH-MIDAS for the marginal distribution of bond returns (01/01/1990 - 31/03/2021)

	μ_i	α_i	β_i	γ_i	m_i	$\delta_{i,1}$	$\omega_{i,1,2}$	$\delta_{i,2}$	$\omega_{i,2,2}$	K	LLH	BIC
RV Bond	0.021***	0.050***	0.941***	-0.013*	-1.884***	0.351*	1.000			264	-0.587	1.181
	(0.005)	(0.003)	(0.001)	(0.007)	(0.167)	(0.185)	(0.633)					
VXO	0.021***	0.028***	0.974***	-0.006	-1.142***	0.570***	4.947			3	-0.581	1.171
	(0.005)	(0.002)	(0.000)	(0.004)	(0.190)	(0.084)	(24.412)					
VIX	0.020***	0.029***	0.961***	-0.001	-2.345***	0.619***	2.790			3	-0.581	1.171
	(0.005)	(0.011)	(0.002)	(0.008)	(0.294)	(0.086)	(3.506)					
VXO+NFCI	0.019***	0.028***	0.961***	0.000	-2.196***	0.557***	3.226	0.245	1.834	52	-0.581	1.171
	(0.005)	(0.005)	(0.001)	(0.006)	(0.172)	(0.066)	(6.687)	(0.204)	(1.535)			
VXO+NAI	0.019***	0.027***	0.958***	0.002	-2.375***	0.580***	5.375	-0.429	1.000	36	-0.580	1.171
	(0.005)	(0.006)	(0.000)	(0.007)	(0.160)	(0.127)	(50.156)	(0.463)	(1.644)			
VXO+IP	0.019***	0.028***	0.961***	-0.000	-2.296***	0.579***	3.331	-0.017	3.851	36	-0.580	1.171
	(0.005)	(0.005)	(0.001)	(0.006)	(0.150)	(0.077)	(11.096)	(0.025)	(10.599)			

The tables report the estimation results of the multiplicative component GARCH-MIDAS model for the stock returns and bond returns proposed by Conrad and Kleen (2020). There are 7 variables are chosen to explain the long-term component of the volatility such as the daily RV, the daily S&P 100 Volatility Index (VXO), the daily S&P 500 Volatility Index (VIX), the weekly National Financial Conditions Index (NFCI), the monthly National Activity Index (NAI), the monthly Industrial Production (IP). The lag length K of the explanatory variables are set based on Conrad and Kleen (2020) and the weighting scheme is the restricted beta function. The lag length of VXO in the two-factor GARCH MIDAS is equal to 3. The values of the maximum likelihood (LLH) and the Bayesian information criteria (BIC) are normalized for the number of observations which shows that the GARCH-MIDAS with VXO index is preferred for the marginal distribution. ***,**,* denote significant at 1%, 5%, 10% level.

4.3 Model comparisons

In this section, we aim to select an appropriate GAS MIDAS copula model for stock returns and bond returns. Several specifications need to be determined, for example, the appropriate copula function, the set of explanatory variables, the choice of lags and weighting functions for the MIDAS regression, and a measure of asymmetric association. We first find out the suitable copula function using a GAS MIDAS copula model with the monthly RCor as an explanatory variable. Then we investigate the MIDAS effect by a sufficient lag observation with a weighting function. Furthermore, we verify if the model can be improved when accounting for an asymmetric effect. And finally, we include more other explanatory variables to see if they could explain the change in the long-term dependence.

Table 4 reports the estimation results of the GAS MIDAS copula models for the dependence of stock returns and bond returns in comparison to the DCC and DCC MIDAS Gaussian copula models. We choose the RCor with the restricted beta weighting scheme function to explain the long-term component of the stock-bond dependence. The lag length K is selected such that the maximum likelihood becomes insensitive to the choice of the lag explanatory variable. The δs measures the impact that the lagged X has on the long-term stock and bond dependence and $\omega_{1,2}$ measures how long the impact would. In general, the copula functions that can capture tail dependence are favored. Based on the model selection criteria, LLH and BIC, the GAS MIDAS Student-t copula model is selected for the dynamic dependence of stock returns and bond returns. In Appendix E, we compare the effect of different weighting scheme functions to the GAS MIDAS copula models. We find that the restricted Beta weighting function is good enough to smooth the effect of the explanatory variables. We keep the restricted Beta function as a weighting scheme of the GAS MIDAS copula for further analysis.

Table 5 reports the maximum likelihood of the GAS MIDAS copula models for the dependence of stock returns and bond returns using different association measures of asymmetry. We estimate the asymmetric effect at quadrants along with different quantiles, for example Quadrant II ($u_{1t} < 0.5, u_{2t} > 0.5$) or Quadrant III ($u_{1t} < 0.5, u_{2t} < 0.5$) or both. In general, accounting for the asymmetric effect of "bad news" improves the model's goodness of fit over the symmetric model. Among our proposal of association, none of the measures is preferred for the asymmetric effect in all

Table 4: Comparison of DCC MIDAS Gaussian copulas and GAS MIDAS Copulas

	α	β	λ_0	δ_1	$\omega_{1,2}$	ν	K_1	LLH	AIC	BIC
DCC	0.036***	0.960***						674.2	-1344.5	-1330.5
	(0.004)	(0.005)								
DCC MIDAS	0.065***	0.862***	0.013	1.009***	6.686***		24	702.4	-1394.8	-1359.9
	(0.007)	(0.023)	(0.019)	(0.050)	(1.806)					
GAS MIDAS Gaussian	0.213***	0.927***	0.011	1.987***	6.392***		24	701.0	-1392.0	-1357.1
	(0.023)	(0.018)	(0.039)	(0.105)	(1.705)					
GAS MIDAS Student	0.253***	0.934***	0.006	2.016***	6.410***	8.649***	24	759.8	**-1507.6**	**-1465.8**
	(0.032)	(0.020)	(0.045)	(0.122)	(1.903)	(0.937)				
GAS MIDAS sClayton	0.183***	0.961***	-0.032***	1.489***	2.933***		24	717.8	-1425.6	-1390.7
	(0.004)	(0.001)	(0.002)	(0.021)	(0.045)					
GAS MIDAS sGumbel	0.045***	0.958***	0.014***	0.881***	3.122***		24	735.5	-1461.0	-1426.2
	(0.000)	(0.000)	(0.001)	(0.005)	(0.017)					
GAS MIDAS Frank	1.624***	0.986***	-0.257***	4.852***	1.006***		24	653.2	-1296.3	-1261.5
	(0.024)	(0.000)	(0.004)	(0.027)	(0.000)					
GAS MIDAS sJoe	0.155***	0.969***	-0.035***	1.037***	2.094***		24	699.0	-1387.9	-1353.1
	(0.001)	(0.000)	(0.001)	(0.007)	(0.008)					

The table reports the estimation results of the DCC MIDAS and the GAS MIDAS copula model for the dependence of stock returns and bond returns in comparison to the benchmark DCC model. We choose the RCor with the restricted beta weighting scheme function to explain the long-term component of the dependence. The lag length is selected such that the maximum likelihood becomes insensitive to the choice of K. ***,**,* denote significant at 1%, 5%, 10% level.

Table 5: Comparison among different association measures of asymmetry for the GAS MIDAS copula models

		Gaussian	Student	sClayton	sGumbel	Frank	sJoe
Symmetric		701.0	759.8	717.8	735.5	653.2	699.0
Quadrant II & III ($u_{1t} < 0.5, u_{2t} < 1$)	Normal Score	729.8	783.8	752.7	768.8	680.6	736.2
	Spearman's rank	737.7	*788.5*	751.6	762.4	682.9	738.2
	Spearman's footrule	732.2	783.3	749.0	761.8	666.1	733.7
	Gini's gamma	737.0	785.8	752.6	764.3	687.2	734.3
Quadrant III ($u_{1t} < 0.5, u_{2t} < 0.5$)	Normal Score	705.0	762.7	710.9	738.6	611.7	695.6
	Spearman's rank	706.1	764.5	722.5	739.6	649.3	704.7
	Spearman's footrule	704.3	754.4	714.1	737.0	652.1	699.1
	Gini's gamma	708.3	762.4	720.3	739.7	652.6	708.9
Quadrant II ($u_{1t} < 0.5, u_{2t} > 0.5$)	Normal Score	723.5	778.9	748.6	761.9	655.6	734.6
	Spearman's rank	729.7	778.3	749.0	763.5	683.4	730.5
	Spearman's footrule	738.8	787.9	752.1	767.1	674.7	733.1
	Gini's gamma	730.5	782.6	748.5	761.3	669.9	730.4

The table reports the maximum likelihood of the GAS MIDAS copula models for the dependence of stock returns and bond returns using different association measures of asymmetry. We estimate the asymmetric effect at quadrants along with different quantiles. We choose the RCor to explain the long-term component of the dependence. The lag length is selected such that the maximum likelihood becomes insensitive to the choice of $K = 24$. ***,**,* denote significant at 1%, 5%, 10% level.

cases even though we will not be mistreated with any of them. We choose the Spearman's rank as an asymmetric measure. As the dynamic dependence between stock returns and bond returns changes the sign from positive to negative, concentrating on just one quadrant, like Cappiello et al. (2006), may not be sufficient. Table 5 also shows that we should take into account the "bad news" effect in both Quadrant II and III which results in a higher maximum likelihood as well as a higher BIC. Generally, the negative shocks to stock returns carry a higher impact to the stock-bond dependence. We select the asymmetric GAS MIDAS Student-t copula with a restricted Beta function weighting scheme for further analysis.

4.4 Variable selections

As the fundamental factors affect the discount rate and the future cash flow of stock and bond, the stock-bond dependence is driven by the changes in the macroeconomic conditions. Existing literature highlights this fact and here we report a few as supportive evidence. Li (2002) considers that expected inflation is the most important factor that moves stock and bond returns in the same direction while the real interest rate is less important. However, the uncertainty of expected inflation and the real interest rate also increases the correlation. Ilmanen (2003) mentions that stock and bond market is sensible to the business cycle, inflation, volatility, and monetary policy conditions. In theory, inflation shocks are negative for bonds and have no impact on stock if the rising cash flow matches the discount rate. However, in practice, high inflation brings a detrimental effect on real earnings growth. On the other hand, David and Veronesi (2013) reason that inflation news carries information on the real economic growth that affects the stock-bond correlation. Guidolin and Timmermann (2006) model the joint distribution of stock and bond using a regime switch model where the regimes are linked to the economic conditions. Connolly et al. (2005) and Bansal et al. (2010) find that the stock market uncertainty is strongly connected to the correlation because of the flight-to-quality phenomenon. Since the stock and bond can be considered as investment substitutes, Baele et al. (2010) emphasis the importance of the liquidity factor over macroeconomic fundamentals.

Knowing the importance of changes in macroeconomic conditions on stock-bond dependence, in this section, we consider to account for it in our analysis. Following Asgharian et al. (2016), we divide the macroeconomic explanatory variables into four main groups, such as inflation and interest rates (II), state of the economy (SE), market uncertainty (UC), and illiquidity (IL). However in contrast to Asgharian et al. (2016), we use monthly explanatory variables rather than quarterly variables, therefore more information is included in our analysis. Additionally, we also take advantage of the soft information from the Survey of Professional Forecasters (SPF) about the Inflation and Interest rate (SPF II), GDP growth and Industrial production growth (SPF SE), and SPF Uncertainty (SPF UC) in the K lead quarters. As these are quarterly variables, we restrict the survey values to be the same in each month.

We use the first principal component (PC) to summarize the information of the variables in each group. In the II group, we calculate the inflation rate as the yearly change of the consumer price index; the term spread as the difference between 10Y government bond and 3-month treasury bill; and the short-term interest rate as the 3-month treasury bill. In the SE group, we estimate the industrial production growth as the yearly change of the industrial production index, the Aruoba-Diebold-Scotti business conditions index as a proxy for the real business conditions, the yearly change of the coincident economic activity index as a proxy for employment and salaries. In the UC group, we include the VXO, and the realized volatility of stock and bond. In the IL group, we calculate the illiquidity of stock and bond based on the daily close, high, and low prices of the S&P 500 future and the 10Y government bond future, see Abdi and Ranaldo (2017). We measure the PC SPF UC based on the interquartile ranges of SPF inflation and GDP growth forecasts as a proxy for the future uncertainty.

Table 6 reports the correlation matrix of explanatory variables. In general, the PC II and the PC SE are positively correlated with the RCor in which the stock-bond correlation is higher when the economy is in good condition or the inflation and interest rate are high. On the other hand, the PC UC and the PC IL are negatively related with the RCor that explains the flight to quality phenomenon during recession and crisis periods. Among the PC of explanatory variables, the PC II is strongly and positively correlated with the PC SPF II. The PC SE is relatively strong but negatively correlated with PC SPF UC, while for the other variables, the magnitude of the correlation is either mild or low. In the lower panel of the table and within groups, the short-term interest contributes most to the PC II with Inflation joining and Term spread comes after that.

The industrial production and the coincident index contributed heavily to the PC SE indicator. The PC UC is mainly driven by VXO, RV Stock and RV Bond. The stock illiquidity and the bond illiquidity seem to move in the opposite direction. The PC of the SPF variables are more related to the explanatory variables in the same group than to the variables in other groups with few exceptions. The future expectation in the SPF II is positively correlated with the state of economy while the sign for which is negative for the PC SPF UC. As SPF provides soft information from one to several quarters ahead, we put a higher weight to the recent lead quarter in the MIDAS regression scheme following Asgharian et al. (2016).

Table 6: Correlation matrix of explanatory variables

	RCor	PC II	PC SE	PC UC	PC IL	PC SPF II	PC SPF SE	PC SPF UC
PC II	0.429					0.807	0.046	-0.119
PC SE	0.264	0.375				0.296	0.432	-0.636
PC UC	-0.277	-0.175	-0.492			-0.086	-0.312	0.355
PC IL	-0.437	-0.083	-0.368	0.435		-0.087	-0.106	0.231
Inflation	0.323	0.789				0.725	0.084	-0.014
Term spread	0.013	-0.607				-0.277	0.025	0.086
Short-term interest	0.595	0.900				0.793	0.034	-0.173
Industrial Production	0.246		0.935			0.319	0.465	-0.586
ADS Index	0.114		0.363			-0.009	0.131	-0.041
Coincident Index	0.228		0.892			0.270	0.332	-0.657
VXO	-0.253			0.936		0.001	-0.320	0.380
RV Stock	-0.308			0.947		-0.032	-0.292	0.333
RV Bond	-0.176			0.805		-0.222	-0.221	0.232
Stock Illiquidity	-0.267				0.640	-0.041	-0.271	0.309
Bond Illiquidity	0.307				-0.672	0.072	-0.125	-0.001

The table reports the correlation matrix of explanatory variables. We divide 11 variables into 4 main groups such as Inflation and Interest rate (II), the State of Economy (SE), Uncertainty (UC) and Illiquidity (IL). We report the soft information from SPF on the next quarter of II and SE. The PC SPF UC is calculated based on the interquartile ranges of SPF inflation and GDP growth forecasts as a proxy for the future uncertainty. The first part of the panel shows the correlations among the principal components, and the second part of each panel shows the correlations between the principal components and the macro-finance variables used to construct them.

Table 7 reports the estimation results of the asymmetric GAS MIDAS Student-t copula model for the dependence of stock returns and bond returns using one explanatory variable. To summarize, the model selection criteria show that the RCor is the most preferred macro-finance factor for the full sample data. However, other fundamental factors such as PC II, PC SE and PC IL also significantly contributes to the long-term dependence. The signs of the δ coefficients also match with the previous analysis of the correlation matrix. Even though, we do not find a significant contribution of the PC

UC and the PC SPF UC to the long-term dependence, the effect of uncertainty is negative, which means that the long-term stock–bond dependence is low when market uncertainty is high. On the other hand, the PC SE and the PC SPF SE are both positively related to the long-term dependence, which is in accordance with the flight-to-quality phenomenon. However among SPF variables, only PC SPF II can significantly predict the stock-bond long-term dependence. The degrees of freedom are very similar among the GAS copula models with and without the MIDAS effect which shows that the dependence of stock and bond are heavy tails with an asymmetric effect of "bad news" shock. The estimate of $\omega_{1,2}$ indicates that the degree of smoothing varies across the macro-finance factors, where a smaller value leads to a larger degree of smoothing over the lagged observations. For example, the effect of PC II fades out in a slower rate than that of PC IL. The results are robust to the choice of weighting functions, the choice of lag numbers and the asymmetric measure of association.

Table 7: The asymmetric GAS MIDAS Student-t Copula with one explanatory variable

	α	β	λ_0	γ	δ_1	$\omega_{1,2}$	ν	K_1	LLH	AIC	BIC
RCor	0.118***	0.904***	-0.056*	1.576***	1.514***	5.285***	9.2***	24	788.5	-1563.1	-1514.3
	(0.028)	(0.020)	(0.033)	(0.243)	(0.119)	(1.623)	(1.071)				
PC II	0.105***	0.980***	-0.209***	0.735***	0.152***	2.815	8.5***	12	763.0	-1511.9	-1463.1
	(0.019)	(0.004)	(0.051)	(0.136)	(0.044)	(3.250)	(0.917)				
PC SE	0.108***	0.982***	-0.220***	0.697***	0.088*	2.605***	8.5***	12	759.2	-1504.4	-1455.7
	(0.019)	(0.003)	(0.055)	(0.131)	(0.048)	(0.008)	(0.904)				
PC UC	0.109***	0.984***	-0.226***	0.652***	-0.021	7.592***	8.4***	18	757.7	-1501.4	-1452.6
	(0.019)	(0.003)	(0.059)	(0.126)	(0.053)	(0.046)	(0.890)				
PC IL	0.111***	0.976***	-0.214***	0.802***	-0.358***	5.075***	8.9***	18	764.2	-1514.4	-1465.6
	(0.019)	(0.005)	(0.048)	(0.158)	(0.085)	(0.079)	(1.001)				
PC SPF II	0.125***	0.970***	-0.126**	0.867***	0.282***	2.960	8.3***	6	765.7	-1517.5	-1468.7
	(0.022)	(0.007)	(0.050)	(0.167)	(0.075)	(2.723)	(0.876)				
PC SPF SE	0.111***	0.984***	-0.231***	0.645***	0.037	6.541***	8.3***	5	757.6	-1501.2	-1452.4
	(0.018)	(0.003)	(0.061)	(0.132)	(0.067)	(0.023)	(0.871)				
PC SPF UC	0.110***	0.984***	-0.231***	0.649***	-0.102	5.422	8.5***	4	758.1	-1502.2	-1453.5
	(0.018)	(0.003)	(0.059)	(0.125)	(0.121)	(6.115)	(0.902)				

The table reports the estimation results of the asymmetric GAS MIDAS Student-t copula model for the dependence of stock returns and bond returns. We choose one explanatory variable with the restricted beta weighting scheme function to explain the long-term component of the dependence. The lag length is selected such that the maximum likelihood becomes insensitive to the choice of K. The values of the LLH, the AIC, the BIC show that RCor is the most preferred for the dynamic dependence of stock returns and bond returns. ***,**,* denote significant at 1%, 5%, 10% level.

For the easy of comparison, we report the dependence in terms of correlation for the GAS MIDAS Student copula. Figure 3 plots the long-term dependence between stock returns and bond returns using the GAS MIDAS copula model with one explanatory variable as well as the two-year rolling correlation (RC2Y) as a proxy. The RC2Y is depicted in the blue dash line in each plot. It can be seen that the estimated long-term dependence using the GAS MIDAS copula with RCor generally follows the same pattern, while the other models are not as good at capturing the dynamics. The estimated long-term dependence series based on the different macro-finance factors are generally smoother than those by the RCor, except for the PC II and the PC IL, which explain better the long-term change of the dependence in comparison to others and fluctuate closely to realized correlations.

Figure 3: long-term dependence between stock and bond

The figure shows the long-term dependence between stock returns and bond returns using GAS MIDAS copula models with one explanatory variable in comparison to the RC2Y. The PC factor is shown as a bar chart. The shaded areas highlight the recession periods based on the NBER indicators.

To proceed further, we extend the model and add another explanatory variable in order to investigate if this can improve the performance any further. Table 8 reports the estimation results of the asymmetric GAS MIDAS Student-t copula model using two explanatory variables. In Panel (a), we choose the RCor with another explanatory variable to explain the long-term component of the dependence. The parameter δs for the PC factors are not significant except for the PC IL, which not only indicates the importance of these two variables in terms of long-term dependence but also supports by model selection criteria. In Panel (b), we assess the model with PC II and another variable. When combining economic and forecast factors together with PC II, we notice that, only PC IL and PC SPF II are found to be significant, where the parameter δ for PC SPF II is positive, which implies that when the forecasted inflation and interest rate factor raises, then long-term stock–bond dependence also increases. The same goes for PC SPF SE though this factor is not significantly influencing the relationship. The rest of the factors are not found significant and rather influence the long-term stock-bond dependence differently. The combination of PC II and PC IL is found to be significantly improving the relationship as suggested by the model selection criteria. Here again, the estimate of weight $\omega_{j,2},\ j = 1, 2$, indicates that the degree of smoothing varies across different PC-based indicators, where a smaller value leads to a larger degree of smoothing over the lagged observations. The tail parameter ν is found to be significant in all the choices, thus, supporting our choice of Student copula models. Hence, our proposed GAS MIDAS copula models lend a support that the inflation, interest rate and illiquidity are the main factors that drive the long-term change in the stock-bond dependence.

Figure 4 plots the long-term dependence between stock returns and bond returns using the GAS MIDAS copula model with two explanatory variables as well as the RC2Y as a proxy. For brevity, we only report the situations for which the corresponding δ parameters are significant. The RC2Y is depicted in the blue dash line in each plot. In panel (a), the GAS MIDAS copula model with RCor and PC IL is compared with the proxy and it can be seen that the long-term dependence obtained by this model follows the fluctuations very closely. In Panel (b), the PC II model extended with external variables (IL and SPF II) is presented, which, though significant, do not add much to the long-term stock-bond dependence compared to the model with RCor-PC IL.

Figure 5 shows that the total time varying dependence between stock and bond is quite identical

Table 8: The GAS MIDAS Student-t Copula with two explanatory variables

	α	β	λ_0	γ	δ_1	$\omega_{1,2}$	δ_2	$\omega_{2,2}$	ν	K_2	LLH	AIC	BIC
(a) RCor with another explanatory variable													
PC II	0.124***	0.908***	-0.068**	1.519***	1.476***	4.575**	0.022	1.330	9.2***	12	788.7	-1559.4	-1496.6
	(0.027)	(0.021)	(0.034)	(0.242)	(0.138)	(1.869)	(0.035)	(4.174)	(1.070)				
PC SE	0.126***	0.904***	-0.064*	1.516***	1.497***	5.725***	0.025	1.740	8.9***	12	788.9	-1559.7	-1497.0
	(0.028)	(0.020)	(0.034)	(0.238)	(0.127)	(1.783)	(0.033)	(4.084)	(1.001)				
PC UC	0.121***	0.906***	-0.066*	1.594***	1.496***	4.437**	-0.010	1.345	8.9***	18	788.3	-1558.6	-1495.9
	(0.027)	(0.021)	(0.035)	(0.251)	(0.140)	(1.755)	(0.069)	(3.630)	(0.997)				
PC IL	0.112***	0.904***	-0.103***	1.594***	1.270***	6.756***	-0.198**	1.628*	9.4***	18	791.4	-1564.7	-1502.0
	(0.027)	(0.020)	(0.035)	(0.250)	(0.146)	(2.320)	(0.080)	(0.912)	(1.113)				
PC SPF II	0.108***	0.931***	-0.056*	1.342***	1.495***	2.139***	-0.017	3.909***	9.3***	6	785.1	-1552.2	-1489.5
	(0.025)	(0.012)	(0.033)	(0.210)	(0.177)	(0.095)	(0.049)	(0.022)	(1.097)				
PC SPF SE	0.116***	0.928***	-0.093**	1.264***	1.463***	3.302	-0.006	2.921***	8.4***	5	786.5	-1555.0	-1492.3
	(0.030)	(0.023)	(0.043)	(0.221)	(0.172)	(2.136)	(0.043)	(0.267)	(0.884)				
PC SPF UC	0.118***	0.902***	-0.067**	1.590***	1.525***	5.386***	-0.042	2.454***	9.1***	4	788.6	-1559.3	-1496.5
	(0.028)	(0.020)	(0.033)	(0.247)	(0.118)	(1.653)	(0.072)	(0.025)	(1.050)				
(b) PC II with another explanatory variable													
PC SE	0.105***	0.978***	-0.206***	0.772***	0.144***	6.347***	0.045	2.556***	8.4***	12	763.3	-1508.6	-1445.9
	(0.019)	(0.004)	(0.049)	(0.139)	(0.045)	(0.021)	(0.045)	(0.008)	(0.897)				
PC UC	0.105***	0.979***	-0.208***	0.741***	0.145***	6.373***	-0.016	2.549***	8.5***	18	763.0	-1508.0	-1445.3
	(0.019)	(0.004)	(0.051)	(0.137)	(0.044)	(0.039)	(0.051)	(0.009)	(0.921)				
PC IL	0.104***	0.956***	-0.197***	1.181***	0.197***	1.326	-0.405***	7.399	9.1***	18	778.0	-1538.1	-1475.4
	(0.021)	(0.009)	(0.036)	(0.199)	(0.031)	(0.986)	(0.087)	(6.484)	(1.057)				
PC SPF II	0.130***	0.977***	-0.055	0.717***	0.134***	1.813	0.134***	1.813	9.6***	6	760.8	-1507.7	-1458.0
	(0.021)	(0.005)	(0.067)	(0.178)	(0.028)	(1.124)	(0.028)	(1.124)	(1.182)				
PC SPF SE	0.137***	0.983***	0.009	0.549**	0.198***	3.025	0.109	2.601***	8.6***	5	756.1	-1494.2	-1431.5
	(0.025)	(0.007)	(0.145)	(0.242)	(0.059)	(3.481)	(0.104)	(0.051)	(0.950)				
PC SPF UC	0.107***	0.977***	-0.192***	0.874***	0.137***	8.411***	-0.104	2.445***	8.2***	4	762.4	-1506.8	-1444.1
	(0.018)	(0.005)	(0.048)	(0.162)	(0.041)	(0.031)	(0.123)	(0.016)	(0.840)				

The table reports the estimation results of the asymmetric GAS MIDAS Student-t copula model for the dependence of stock returns and bond returns. We choose the RCor with another explanatory variable to explain the long-term component of the dependence. The lag length is selected such that the maximum likelihood becomes insensitive to the choice of K and the restricted beta weighting scheme function is chosen based on previous analysis. We choose $K_1 = 24$ for the RCor variable and $K_1 = 12$ for the PC II variable. ***,**,* denote significant at 1%, 5%, 10% level.

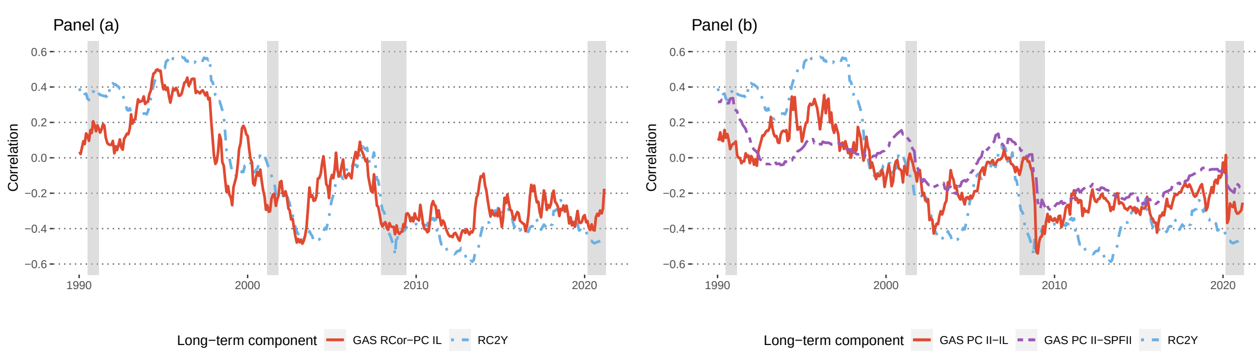

Figure 4: long-term dependence between stock and bond

The figure shows the long-term dependence between stock returns and bond returns using GAS MIDAS copula models with two explanatory variables in comparison to the two-year rolling correlation. The shaded areas highlight the recession periods based on the NBER indicators.

across the GAS MIDAS copula models. Since the total time varying dependence is driven mainly by the short-term dependence due to the decomposition, it indicates that low-frequency macro-finance factors are not important for estimation of the total time varying fluctuations in the stock-bond dependence. However, for long-term strategies, the observation for long-term dependence is different and worth noticing as discussed earlier. In the three last recent recessions and crises from the 2000s, the dependence started at a negative value and remained mostly below zero, which shows bond could be a hedge for stock in such a situation. The dependence then seemed to recover to the zero level nearly to the of considered data, which actually covers the recent Covid 19 pandemic, where the total dependence was below -0.7 during this period. In the next section, we show an application of the GAS MIDAS copula models for the optimal portfolio allocation and risk management.

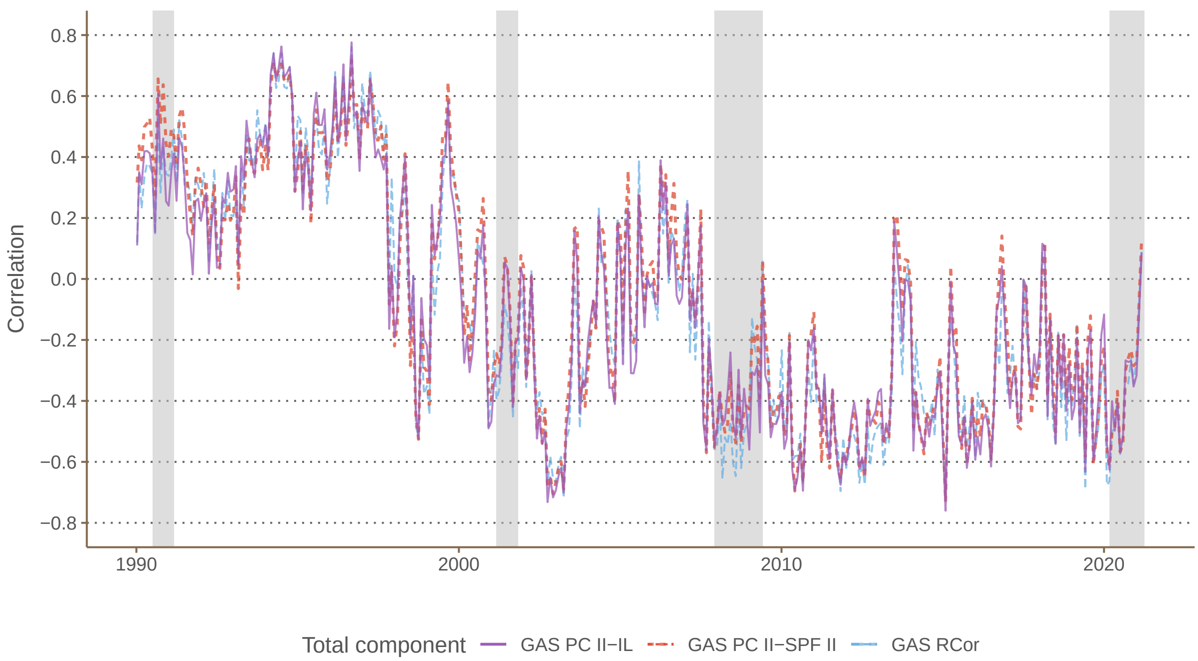

Figure 5: Total dependence between stock and bond

The figure shows the time varying dependence between stock returns and bond returns using the GAS MIDAS copula models with RCor, PC II - PC SPF II, PC II - PC IL. The shaded areas highlight the recession periods based on the NBER indicators.

5 Portfolio allocation and Risk Management

In this section, we illustrate how the GAS MIDAS copula models can be utilized for the return forecasts of the stock-bond portfolio. We split the full samples (01/1990 - 03/2021) into the in-sample period (01/1990 - 04/2017) for the parameter estimation and the last $T = 1000$ observation for the out-of-sample period (04/2017 - 03/2021). The models have been reestimated for the in-sample data. We quantify the associated risk measures of the portfolio such as the quantile loss distribution for a given horizon (VaR), and the conditional expected loss above a quantile (ES). Finally, based on the out-of-sample data, we evaluate the economic value of the GAS MIDAS copula model on the optimal portfolio allocation.

5.1 Return prediction

In each period of the out-of-sample forecast, we fix the estimated parameters and simulate S scenarios based on the assumed model data generating process. Let $\hat{\Theta}_i = (\hat{\mu}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i, \hat{m}_i, \hat{\delta}_i, \hat{\omega}_i)$ be the set of marginal parameters in the GARCH MIDAS models that have been estimated in the previous analysis, we simulate the one step ahead prediction of the returns using Equation 1,

$$
\begin{aligned}
r_{it}^{(s)} &= \hat{\mu}_i + \sqrt{\kappa_{i\tau} g_{it}}\epsilon_{it}^{(s)}, \\
g_{it} &= (1 - \hat{\alpha}_i - 0.5\hat{\gamma}_i - \hat{\beta}_i) + (\hat{\alpha}_i + \hat{\gamma}_i \mathbf{I}_{\{\epsilon_{i,t-1}<0\}}) g_{i,t-1}\epsilon_{i,t-1}^2 + \hat{\beta} g_{i,t-1}, \\
\kappa_{i\tau} &= \exp\left(\hat{m}_i + \sum_{j=1}^{N_i} \hat{\delta}_{ij} \left[\sum_{k=1}^{K_j} \phi_k(\hat{\omega}_{i,j,1}, \hat{\omega}_{i,j,2}) X_{i,j,\tau-k}\right]\right),
\end{aligned}
$$

where the simulated standardized innovation $\epsilon_{it}^{(s)}$ is obtained as the empirical inverse quantile function of the GAS MIDAS Copula, $\epsilon_{it}^{(s)} = F_i^{-1}(u_{it}^{(s)})$, for $s = 1, \dots, S$ such that,

$$
\begin{aligned}
&(u_{1t}^{(s)}, u_{2t}^{(s)}) \sim c_t(u_{1t}, u_{2t}|\theta_t), \\
&\qquad \theta_t = \Lambda(\lambda_t), \\
&\qquad \lambda_t = \lambda_\tau(1 - \hat{\beta}) + \hat{\alpha} \frac{\partial \log c_{t-1}(u_{1t-1}, u_{2t-1}|\lambda_{t-1})}{\partial \lambda_{t-1}} + \hat{\beta}\lambda_{t-1} + \hat{\gamma}\left(v_{t-1} - \bar{v}\right), \\
&\qquad \lambda_\tau = \hat{\lambda}_0 + \sum_{j=1}^{N} \hat{\delta}_j \left[\sum_{k=1}^{K_j} \phi_k(\hat{\omega}_{j,1}, \hat{\omega}_{j,2}) X_{j,\tau-k}\right],
\end{aligned}
$$

where $\hat{\Theta}_c = (\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\lambda}_0, \hat{\delta}, \hat{\omega})$ are the maximum likelihood estimation of the parameters in the GAS MIDAS copula model. We update the explanatory variables X_j when it is available. In the next section, we use the simulated return to quantify the associated risk of the stock and bond portfolio.

5.2 Value at Risk

Based on the simulated returns, we construct the simulated portfolio of stock and bond at time t as,

$$r_t^{(s)} = w_{1t} r_{1t}^{(s)} + (1 - w_{1t}) r_{2t}^{(s)}$$

where w_{1t} is the weight of stock in the portfolio at time t. The q% VaR is the threshold loss at the q quantile of the simulated portfolio and the ES is the associated expected loss above q% VaR such that,

$$q = Pr\left(r_t \leq \mathrm{VaR}_{q,t}\right),$$

$$\mathrm{ES}_{q,t} = E\left(r_t \,|r_t \leq \mathrm{VaR}_{q,t}\right).$$

We choose $q = 1\%$ and $q = 0.5\%$ and estimate the one-step-ahead $\mathrm{VaR}_{q,t}$ and $\mathrm{ES}_{q,t}$ for the portfolio of equal weight. As we can simulate the returns of the portfolio, it is straightforward to obtain the predictive $\mathrm{VaR}_{q,t}$ and $\mathrm{ES}_{q,t}$ for each out-out-of sample forecast $t = 1, \ldots, T$. Next, we compare the loss functions based on VaR forecasts following Caporin (2008) and Taylor (2019),

$$\begin{aligned}
\mathrm{IF} &= \sum_{t=1}^{T} \mathbf{I}\left(r_t < \mathrm{VaR}_{q,t}\right), \\
\mathrm{AD}_t &= ||r_t| - |\,\mathrm{VaR}_{q,t}\,||\,\mathbf{I}\left(r_t < \mathrm{VaR}_{q,t}\right), \\
\mathrm{SD}_t &= (|r_t| - |\,\mathrm{VaR}_{q,t}\,|)^2\,\mathbf{I}\left(r_t < \mathrm{VaR}_{q,t}\right), \\
\mathrm{QS}_t &= (r_t - \mathrm{VaR}_{q,t})(q - \mathbf{I}\left(r_t < \mathrm{VaR}_{q,t}\right)), \\
\mathrm{ALS}_t &= -\log\left(\frac{q}{\mathrm{ES}_{q,t}}\right) - \frac{(r_t - \mathrm{VaR}_{q,t})(q - \mathbf{I}\left(r_t < \mathrm{VaR}_{q,t}\right))}{q\,\mathrm{ES}_{q,t}}.
\end{aligned}$$

Note that the indicator loss function (IF) counts the number of exceptions where the portfolio return goes below the VaR threshold. The absolute deviation loss function (AD) and the squared deviation loss function (SD) measures the first order and the second order loss of the exceptions. As the VaR and (VaR, ES) are elicitable risk measures (see, for example, Nolde et al. (2017)), the quantile score (QS) loss function and the Asymmetric Laplace log score (ALS) loss function are employed for backtesting. Both of these measures are strictly consistent in the sense that the QS and the ALS are minima at the true quantile series (Gneiting, 2011).

Table 9 reports the average VaR and ES together with the aggregate of the loss functions using models: the DCC, the DCC MIDAS, the GAS MIDAS RCor, the GAS MIDAS RCor-PC IL, the GAS MIDAS PC II, the GAS MIDAS PC II-IL, for the equally weighted portfolio. As we apply the same marginal models for stock returns and bond returns, the differences in risk measures are due to the use of the dynamic copula models. In general, the GAS MIDAS copula models yield a lower level of VaR and ES than the DCC and DCC MIDAS models. Despite that the GAS MIDAS RCor model has the same number of exceptions at the 0.5% and 1% quantiles to the DCC and DCC MIDAS models, the AD, SD, QS and ALS loss function are smaller for the GAS MIDAS RCor model. The GAS MIDAS RCor-PC IL does not provide much improvement over the GAS MIDAS RCor. On the other hand, the numbers of exceptions for GAS MIDAS PC II and GAS MIDAS PC II-IL are closer to the expected exceptions of 5 and 10 at the 0.5% and 1% quantiles respectively. We also report the significant difference in loss functions between the DCC model and others using the Diebold and Mariano (1995) test. The GAS MIDAS PC II and the GAS MIDAS PC II-IL give significant improvements in risk management over the DCC model.

Table 9: Risk measures

	VaR	ES	IF	AD	SD	QS	ALS
				q = 1%			
DCC	-1.291	-1.644	14	6.149	4.212	19.383	1527.997
DCC MIDAS	-1.286	-1.641	14	6.071	4.123*	19.256*	1519.746
GAS MIDAS RCor	-1.302	-1.673	14	5.671	3.752	19.020	1503.488
GAS MIDAS RCor-PC IL	-1.305	-1.673	14	5.869	3.761	19.241	1515.204
GAS MIDAS PC II	-1.321	-1.689	12	5.461**	3.489**	18.991*	1495.735
GAS MIDAS PC II-IL	-1.325	-1.696	12	5.187**	3.241**	18.762*	1470.668
				q = 0.5 %			
DCC	-1.497	-1.905	10	4.023	2.366	11.670	1713.927
DCC MIDAS	-1.492	-1.905	10	3.745	2.092	11.365	1684.572
GAS MIDAS RCor	-1.519	-1.945	10	3.502	1.937	11.259	1669.379
GAS MIDAS RCor-PC IL	-1.522	-1.945	10	3.394	1.752	11.165	1662.926
GAS MIDAS PC II	-1.541	-1.959	8	3.043**	1.543*	10.910	1629.636
GAS MIDAS PC II-IL	-1.543	-1.970	7	2.858**	1.504**	10.733	1600.582

The table reports the average VaR and ES together with the aggregate of the loss functions using the DCC, the DCC MIDAS, the GAS MIDAS RCor, the GAS MIDAS PC II, the GAS MIDAS PC II-IL for the equally weighted portfolio. The expected exceptions at the 0.5% and 1% quantiles are 5 and 10 respectively. We report the significant difference in risk measures between the DCC model and alternatives using the Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator. ***,**,* denote that the corresponding model significantly outperforms the Gaussian VAR at 1%, 5%, 10% level.

5.3 Portfolio allocation

Next, we consider an investor who allocates wealth between stock and bond by maximizing the constant relative risk aversion (CRRA) utility function. Following Patton (2004), we assume the CRRA utility function as,

$$U(r_t, \eta) = \begin{cases} (1-\eta)^{-1} \left(P_0(1+r_t)\right)^{1-\eta} & \text{if } \eta \neq 1, \\ \log\left(P_0(1+r_t)\right) & \text{if } \eta = 1, \end{cases} \tag{6}$$
$$r_t = w_{1t} r_{1t} + (1 - w_{1t}) r_{2t},$$

where $P_0 = 1$ is the initial wealth and η is the degree of the risk aversion. The investor optimizes the CRRA utility function based on the one step ahead forecast of the conditional density function,

$$\begin{aligned} w_{1t}^* &= \arg\max E_t(U(r_t, \eta)) \\ &= \arg\max \int\int U(r_t, \eta) f_t(r_{1t}, r_{2t}) dr_{1t} dr_{2t} \\ &= \arg\max \int\int U(r_t, \eta) f_{1t}(r_{1t}) f_{2t}(r_{2t}) c_t(F_{1t}(r_{1t}), F_{2t}(r_{2t})) dr_{1t} dr_{2t}. \end{aligned} \tag{7}$$

We use a numerical method to calculate the double integral and optimize the objective function using the BFGS algorithm. In order to compare among copula models, we measure the performance fee (Fee) that the investor is willing to pay to switch from one strategy to another strategy, see more discussion in Fleming et al. (2001) and Wu and Lin (2014). Mathematically, we write as,

$$\sum_{t=1}^{T} U(r_t^B - \text{Fee}, \eta) = \sum_{t=1}^{T} U(r_t^A, \eta),$$

where r_t^A and r_t^B are the portfolio returns from two competing strategy A and B. We let the benchmark model A be a passive model where the initial weight is distributed to maximize the CRRA utility function using the historical in-sample data. We report different levels of risk aversion parameter such as $\eta = 1$, $\eta = 5$, $\eta = 10$. We also calculate the break-even transaction cost (TC) per trade that the investor is even between the passive strategy A and the dynamic optimal portfolio strategy B. The break-even TC is proportional to the portfolio turn-over in each out-of-sample period, see Han (2006),

$$\sum_{t=1}^{T} U\left(r_t^B - TC\left|w_{1t}^B - w_{1,t-1}^B \frac{1+r_{1t-1}}{1+r_{t-1}^B}\right|, \eta\right) = \sum_{t=1}^{T} U\left(r_t^A, \eta\right).$$

Table 10 reports the economic values of dynamic portfolios over a passive portfolio. The performance fees are normalized to annual basis points (bps) and the break-even transaction costs are expressed in basis points of the portfolio turn-over.

Table 10: Economic values of dynamic portfolios over a passive portfolio

	$\eta = 1$		$\eta = 5$		$\eta = 10$	
	Fee	TC	Fee	TC	Fee	TC
DCC	365.61	16.83	148.68	23.91	121.93	26.73
DCC MIDAS	381.18	17.54	157.37	25.54	134.12	30.13
GAS MIDAS RCor	393.09	17.62	161.45	26.15	137.43	30.91
GAS MIDAS RCor-PC IL	392.09	17.37	162.86	26.35	139.01	31.41
GAS MIDAS PC II	365.88	16.25	154.10	24.48	133.08	29.64
GAS MIDAS PC II-IL	370.62	15.99	161.97	25.57	140.58	31.31

The table reports the economic values of dynamic portfolios over a passive portfolio. The initial weight of the passive portfolio is chosen to maximize the CRRA utility function using the historical in-sample data. The performance fees are normalized to annual basis points (bps) and the break-even transaction costs are expressed in basis points of the proportional cost for reweighting.

We find that, under a low risk aversion, the performance fees based on the GAS MIDAS RCor model is larger than those based on the DCC and DCC-MIDAS models, indicating that the use of complete density of the copula function and the asymmetric effect of bad news on the dependence economically informative. For instance, at the portfolio return of $\eta = 1$, an investor is willing to pay roughly 30 annual basis points (bps) for switching from a DCC based models to GAS MIDAS RCor copula model. In addition, an investor would be willing to pay more break-even costs to utilize the GAS MIDAS RCor model instead of the DCC-based models to allocate his portfolio. This finding is consistent across different choices. For a relatively higher risk aversion ($\eta = 10$), the performance fees based on GAS MIDAS PC II-IL model is higher than the rest, indicating economically significance of these macroeconomic factors. In these situations, the GAS MIDAS PC II-IL model outperforms the GAS MIDAS RCor model which shows a potential improvement of using macroeconomic factors in additional to RCor to forecast the stock-bond relation. In summary, if the investor is less risk-averse, the GAS MIDAS RCor can improve over the DCC models and the improvement is consistent among different risk-averse parameters. Consistent with previous studies, a less risk-averse investor is documented to induce higher performance fees, such as, a corresponding increase in the performance fees is noticed with a decrease in risk-aversion levels.

6 Conclusion

In this study, we have proposed GAS MIDAS copula models to analyze the dynamic relationship between stock returns and bond returns. Our proposed copulas decompose the stock-bond relation into a short-term dependence and a long-term dependence. While the long-term effect is updated at a lower frequency using a MIDAS regression, the short-term effect follows a GAS process. We also incorporate the asymmetric effect of “bad news” to the dynamic dependence. The models not only perform well in-sample but also help the investors to optimize portfolio allocation and risk management in the out-of-sample forecast.

According to the results of the GAS MIDAS copula model, long-term stock–bond dependence has a positive relationship with the state of the economy factor and a negative relationship with the uncertainty/illiquidity factors. Such observation is noticed for both the one explanatory variable and two explanatory variables model (based on PC II and other macro economic factors). These results support the flight-to-quality phenomenon and indicate that when the state of the economy is poor and uncertainty in the financial markets is high, investors move their investments from stocks to bonds, which results in a negative correlation between stock and bond returns. The long-term dependence based on the two-explanatory variables model (RCor-PC IL and PC II-IL) fit the realized correlation quite well.

We also examine the out-of-sample economic values of chosen[2] GAS MIDAS copula models by comparing them with the DCC counterpart. The proposed copula models yield a lower level of VaR and ES than the DCC-based models, which is even supported by backtesting measures. The maximum expected CRRA utility strategies also indicate that the proposed copula models outperform the DCC-based model. In addition, investors can gain an extra benefit by taking into

[2]Model with significant δ parameter is used in this step.

account the asymmetry effect in the dependence and a less risk-averse investor would be willing to pay higher fees to adopt a trading strategy based on proposed copula models. In general, we find that the inflation, the interest rate and the illiquidity factors are the main drivers of the long-term stock-bond dependence. Copula models with macroeconomic explanatory variables are found to favour the high risk-averse investors than using the RCor alone. Furthermore, the models with macroeconomic explanatory variables also yield a more accurate forecast of risk measures. These findings provide a robust inference that supports the GAS MIDAS-based copula models better than the DCC-based models, and also lead us to better understand the dependence structure under extreme market conditions.

Several research directions can be extended from our proposed models, where, such as, one can analyse the dependence in high dimensional timeseries using vine copulas and factor copulas with the bivariate GAS MIDAS copula model as a building block. Secondly, the dependence between consumption and inflation is in the reverse direction with the stock-bond dependence (Li et al., 2020) and it would be interesting to see how the high-frequency stock-bond dependence can give a warning signal to the low-frequency change in the consumption-inflation dependence using the MIDAS framework. Last but not least, Bayesian inference can be provided for the GAS MIDAS copula models to obtain an efficient inference, and to counter situations when the explanatory variables are multicollinear.

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Appendix

A Weighting functions

There are several weighting scheme functions as:

(a) Restricted Beta:

$$\phi_k(1, \omega_2) = \frac{[1 - k/(K+1)]^{\omega_2 - 1}}{\sum_{l=1}^{K} [1 - l/(K+1)]^{\omega_2 - 1}}.$$

(b) Beta:

$$\phi_k(\omega_1, \omega_2) = \frac{[k/(K+1)]^{\omega_1 - 1} [1 - k/(K+1)]^{\omega_2 - 1}}{\sum_{l=1}^{K} [l/(K+1)]^{\omega_1 - 1} [1 - l/(K+1)]^{\omega_2 - 1}}.$$

(c) Exponential Almon lag:

$$\phi_k(\omega_1, \omega_2) = \frac{\exp(\omega_1 k + \omega_2 k^2)}{\sum_{l=1}^{K} \exp(\omega_1 l + \omega_2 l^2)}.$$

(d) Step function: Step function assumes that all the regression coefficients in the same quarter are the same, for example.

B Archimedean Copulas

The density of a symmetric Archimedean copula is written as an equally weighted combination of an Archemedean copula and its 180-degrees rotated Archemedean copula,

$$c_{sym}(u_1, u_2) = 0.5c(u_1, u_2) + 0.5c_{R180}(u_1, u_2).$$

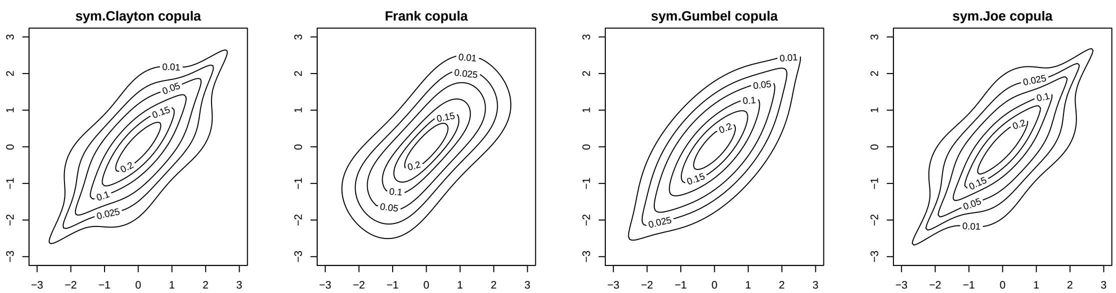

Figure 6: Contours of symmetric Archimedean copula distributions with the same marginal standard normal

C Exponentially Weighted Moving Average

The EWMA assumes a higher weight to the most recent observations, hence it is updated based on the latest observation as

$$
\begin{aligned}
Q_t &= \phi Q_{t-1} + (1-\phi) r_{t-1} r_{t-1}^{'} \text{ where } \phi = 0.96, \\
R_t &= diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}.
\end{aligned}
$$

D A DCC MIDAS Gaussian copula model

Colacito et al. (2011) and Conrad et al. (2014) extend the DCC model (Engle, 2002) such that there are N variables that can explain the long-term dependence. A DCC MIDAS Gaussian copula model can be presented as,

$$
\begin{aligned}
(u_{1t}, u_{2t}) &\sim c_{(Gauss)}(u_{1t}, u_{2t} | R_t), \\
R_t &= Q_t^{*-1/2} Q_t Q_t^{*-1/2} \text{ where } Q_t^* = diag(Q_t) \\
q_{12,t+1} &= q_{12,\tau}(1-\alpha-\beta) + \alpha \Phi^{-1}(u_{1t}) \Phi^{-1}(u_{2t}) + \beta q_{12,t}, \\
q_{12,\tau} &= \lambda_0 + \sum_{j=1}^{N} \delta_j \left[\sum_{k=1}^{K_j} \phi_k(\omega_{j,1}, \omega_{j,2}) X_{j,\tau-k} \right],
\end{aligned}
\tag{8}
$$

where $(\lambda_0, \alpha, \beta, \delta_j, \omega_j)$ are the fixed copula parameters and $\tau = \lfloor t/L \rfloor$. $(X_{1\tau}, \ldots, X_{N\tau})$ are N-dimensional vector of low-frequency variables, and $\phi_k(\omega_{j1}, \omega_{j2})$ is the weighting scheme of the variable j on its k lag, for $k = 1, \ldots, K$. The weighting scheme of each variable j depends on the regulated parameter ω_j for $j = 1, \ldots, N$. Note that $0 < \alpha + \beta < 1$.

E Robust checks

The table 11 reports the estimation results of the GAS MIDAS Student-t copula model for the dependence of stock returns and bond returns using different weighting scheme functions such as the restricted Beta, the Beta, the Exponential Almon and the Step function. Since the more current values of RCor are more important to predict the long-term change dependence, the restricted Beta functions provide a very similar result in comparison to the Beta function or the Exponential Almon function. The Step function has 8 free parameters which allow for the changes in the weighting scheme every three months, we report here two first free parameters. The BIC values show that the restricted Beta weighting function is good enough to smooth the effect of the explanatory variables. We choose the restricted Beta function as a weighting scheme of the GAS MIDAS copula for further analysis.

Table 11: Comparison of different weighting scheme functions for the GAS MIDAS Student-t Copula

Weighting function	α	β	λ_0	δ_1	$\omega_1^{(1)}$	$\omega_1^{(2)}$	ν	K	LLH	AIC	BIC
Restricted Beta	0.262***	0.933***	-0.021	1.973***	1.000	5.631***	8.4***	24	759.7	-1507.3	-1465.5
	(0.033)	(0.021)	(0.046)	(0.125)		(1.781)	(0.882)				
Beta	0.259***	0.930***	-0.006	2.001***	1.004***	6.754***	8.6***	24	759.8	-1505.5	-1456.7
	(0.032)	(0.021)	(0.045)	(0.121)	(0.010)	(2.118)	(0.931)				
Exponential Almon	0.259***	0.931***	0.009	1.992***	-0.142	-0.018	8.4***	24	759.4	-1504.8	-1456.1
	(0.033)	(0.021)	(0.045)	(0.123)	(0.202)	(0.022)	(0.885)				
Step function	0.270***	0.867***	0.022		0.480***	0.065	8.0***	24	756.2	-1488.4	-1404.8
	(0.034)	(0.035)	(0.035)		(0.054)	(0.061)	(0.817)				

The table reports the estimation results of the GAS MIDAS Student-t copula model for the dependence of stock returns and bond returns using different weighting scheme functions. We choose the RCor to explain the long-term component of the dependence. The lag length is selected such that the maximum likelihood becomes insensitive to the choice of K. The step function has 8 free parameters which allow for the changes in the weighting scheme every three months, we report here two first free parameters. ***,**,* denote significant at 1%, 5%, 10% level.

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