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Abstract

In this paper we analyze how skewness and heavy tails affect the estimated relationship between the real economy and the corporate bond-yield spread, a popular predictor of real activity. We use quarterly US data to estimate Bayesian VAR models with stochastic volatility and various distributional assumptions regarding the disturbances. In-sample, we find that – after controlling for stochastic volatility – innovations in GDP growth can be well-described by a Gaussian distribution. In contrast, both the unemployment rate and the yield spread appear to benefit from being modelled using non-Gaussian innovations. When it comes to real-time forecasting performance, we find that the yield spread is an important predictor of GDP growth, and that accounting for stochastic volatility matters, mainly for density forecasts. Incremental improvements from non-Gaussian innovations are limited to forecasts of the unemployment rate. Our results suggest that stochastic volatility is of first order importance when modelling the relationship between yield spread and real variables; allowing for non-Gaussian innovations is less important.

JEL Classification: C11, C32, C52, E44, E47, G17

Keywords: Bayesian VAR; Generalized hyperbolic skew Student's t distribution; Stochastic volatility

1 Introduction

It is a well-established finding that the corporate bond-yield spread has predictive power for the real economy.[1] Historically, the relation has been such that a higher spread is associated with lower economic activity. For a central bank, this information should be valuable in its efforts to stabilise the macroeconomy. For example, in light of an increase in the spread, it is likely that the real economy will cool down and that monetary policy therefore should become more expansive (or less contractive). With a somewhat different perspective, participants in financial markets – who want to predict the future actions of the central bank – also have interest in how the corporate bond-yield spread relates to the real economy. There is accordingly widespread interest in modelling this relation.

When modelling macro-financial or macroeconomic relations, there are a number of issues that one might want to take into account in order to specify models with good statistical properties. First, over the last fifteen years, there has been a growing literature showing that many variables are hit by disturbances with time-varying volatility; see, for example, Cogley and Sargent (2005), Fountas and Karanasos (2007), Clark (2011), Carriero et al. (2015), Chan (2017), Trypsteen (2017) and Clark et al. (2020). Second, it does not seem unusual that variables have unconditional distributions that are skewed; see, for example, Neftci (1984), Acemoglu and Scott (1997), Bekaert and Popov (2019). In addition, both the Bank of England and the IMF tend to present fan charts that are asymmetric. This raises the issue of whether disturbances should be modelled with skewed, rather than symmetric, distributions. Third, it also seems to be a common feature that unconditional distributions of variables have heavy tails; see, for example, Fagiolo et al. (2008) and Ascari et al. (2015). As pointed out already by Engle (1982), this could be caused by time-varying volatility of disturbances. However, there is also evidence that the disturbances affecting the variables can benefit from being modelled with heavy tails (Chiu et al., 2017; Liu, 2019; Karlsson and Mazur, 2020; Kiss and Österholm, 2020).

In this paper, we are concerned with the modelling of the relation between the corporate bond-yield spread and the real economy – given by GDP growth and/or the unemployment rate –

[1] See, for example, Stock and Watson (1989), Friedman and Kuttner (1998), Gilchrist et al. (2009), Gilchrist and Zakrajšek (2012), Faust et al. (2013), Prieto et al. (2016) and Karlsson and Österholm (2020) for a number of contributions to this literature.

in the United States. Our purpose is to conduct empirical analysis which can provide insights into the importance of the three issues raised above – that is, time-varying volatility, skewness and heavy tails – when modelling the relation between these variables so that more appropriate econometric models can be constructed. To conduct our analysis we apply Bayesian VAR models using quarterly real-time data and study the properties of the estimated models both within- and out-of-sample. The Bayesian VAR models employed – a setting developed by Karlsson et al. (2021) – are estimated with a number of different assumptions regarding the error terms. The most flexible specification allows for stochastic volatility and non-Gaussian error terms with both skewness and heavy tails. The non-Gaussianity is achieved by a Gaussian variance-mean mixture so that the marginal distribution is a generalized hyperbolic skew Student's t, or skew-t distribution for short (McNeil et al., 2015). We rely on low-dimensional VAR models for our analysis. This has the benefit that we can study in detail the effects of both which variables are included in the model and the specification of the error terms.

Briefly mentioning our results, we first note that the marginal (unconditional) distributions of all three variables are non-Gaussian, in line with the previous literature. For GDP growth, once we allow for time variation in volatility, modelling flexibly higher order moments does not seem to matter much either for in-sample fit or for the out-of-sample forecasts. There is some positive skewness in the innovations to the unemployment rate and the yield spread innovations; this is evidenced by both the in-sample results (mainly for the yield spread) and the finding that the density forecasts of these variables benefit (although to a limited extent) from being modelled with a skew-t distribution. Overall, our results show that stochastic volatility is the dominant factor in capturing the non-Gaussianity of the marginal distributions of the variables.

Our analysis contributes to the literature in three distinct ways. First, we provide further evidence on the relation between the corporate bond-yield spread and the US real economy. While reasonably well studied, it is nevertheless interesting to assess how sensitive empirical findings are when assumptions are relaxed. Second, we make a general contribution to the literature on macro-financial and macroeconomic modelling when it comes to the importance of time-varying volatility, skewness and heavy tails. The issue of time-varying volatility has been studied a fair amount in the literature but the same can not be said about skewness and heavy tails, even if the latter has received an increasing amount of attention after the global financial crisis of 2008 (Acemoglu et al., 2017; Chiu et al., 2017; Liu, 2019; Kiss and Österholm, 2020).[2] Third, our final contribution is to the literature concerned with density forecasting of GDP growth; see, for example, Clark (2011), Mazzi et al. (2014), Carriero et al. (2019) and Carriero et al. (2020). In the world of economic policy making, density forecasts have become increasingly important over the last two decades. Initially, density forecasts were mainly a tool for central-bank communication, which – given the inflation-targeting framework in which these forecasts were used – accordingly primarily meant a focus on inflation.[3] However, in more recent years, the perspective has widened and density forecasts have become an important analysis tool for instance when the IMF discusses GDP growth, in particular with regards to the concept "growth-at-risk" which is a means of providing a statement concerning the probability of poor outcomes for GDP growth; see, for example, Adrian et al. (2018, 2019); Prasad et al. (2019) for work on this topic. Our results should be of interest to those aiming to build model-based fan charts both in general and more specifically for the variables studied here.

The rest of this paper is organised as follows: In Section 2, we describe the Bayesian VAR models that we rely upon for our econometric analysis. In Section 3, we describe our data and the empirical analysis, covering both in-sample and out-of-sample results. Finally, Section 4 concludes.

2 Econometric framework

We conduct our analysis with different specifications of Bayesian VAR models proposed by Karlsson et al. (2021). In particular, we relax the assumption of Gaussianity in the error term in the VAR model by employing the skew-t distribution. This distribution describes well non-Gaussian features such as skewness and heavy tails. To capture heteroscedasticity, we allow stochastic volatility in addition to the non-Gaussian disturbances.

Below we present two different VAR models with skewness and heavy tails, one with non-Gaussian disturbances for the conditional distribution of each variable and another with the or-

[2] Although the economic crisis associated with the corona pandemic in 2020 is also a clear tail event, we choose to omit 2020 from our analysis. The reason is that for many economic variables – and prominently for unemployment rate, a central variable in our analysis – the crisis in 2020 caused such large movements that are best modelled as outliers rather than observations from a heavy-tailed distribution.

[3] The Bank of England was a trailblazer in this field; see Britton et al. (1998) for an important contribution. Sveriges Riksbank was also an early adopter; their present methodology is documented in Sveriges Riksbank (2007).

thogonal non-Gaussian disturbances in each VAR equation. Bayesian inference concerning the models is described in the Appendix. Furthermore, we present different methods that are used for evaluating the forecasting performance of the VAR models.

2.1 VAR models with non-Gaussian disturbances

Using the fact that the skew-t distribution can be expressed as a Gaussian variance-mean mixture (Aas and Haff, 2006; McNeil et al., 2015), the multivariate skew-t VAR model with stochastic volatility (MST-SV) is defined as follows

$$\begin{aligned} \mathbf{y}_t &= \mathbf{c} + \mathbf{B}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{B}_p \mathbf{y}_{t-p} + \mathbf{e}_t, \\ \mathbf{e}_t &= \mathbf{W}_t \boldsymbol{\gamma} + \mathbf{W}_t^{1/2} \mathbf{A}^{-1} \mathbf{H}_t^{1/2} \boldsymbol{\epsilon}_t, \quad t = 1, \ldots, T, \end{aligned} \tag{1}$$

where $\mathbf{y}_t$ is a k-dimensional vector of time series of interest; $\mathbf{c}$ is a k-dimensional vector of intercepts; $\mathbf{B}_1, \ldots, \mathbf{B}_p$ are a $k \times k$ matrices of coefficients on the lagged dependent variables;[4] $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_k)'$ is a k-dimensional vector that contains skewness parameters; $\mathbf{W}_t = diag(\xi_{1t}, \ldots, \xi_{kt})$ is a $k \times k$ diagonal matrix of mixing variables ξ_{it} that are mutually independent and follow an inverse gamma distribution with the same shape and rate parameters equal to $\nu_i/2$, i.e. $\xi_{it} \sim \mathcal{IG}(\frac{\nu_i}{2}, \frac{\nu_i}{2})$, $i = 1, \ldots, k$; $\mathbf{A}$ is a $k \times k$ lower triangular matrix with ones on the diagonal; $\mathbf{H}_t = diag(h_{1t}, \ldots, h_{kt})$ is a $k \times k$ diagonal matrix that captures the heteroskedastic volatility; and $\boldsymbol{\epsilon}_t$ is a k-dimensional vector of error terms that has multivariate Gaussian distribution with zero mean vector and identity covariance matrix, i.e. $\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. Moreover, $\mathbf{W}_t$, $\mathbf{H}_t$, and $\boldsymbol{\epsilon}_t$ are mutually independent. We also assume a random walk process of the stochastic volatility,

$$\log h_{it} = \log h_{it-1} + \sigma_i \eta_{it}, \quad i = 1, \ldots, k, \; t = 1, \ldots, T, \tag{2}$$

where $\eta_{it} \sim \mathcal{N}(0, 1)$ and let $\boldsymbol{\sigma}^2 = (\sigma_1^2, \ldots, \sigma_k^2)'$.

[4] It can be noted that we do not allow for time variation in the coefficients describing the dynamic relations between the variables, that is, the matrices $\mathbf{B}_j$, $j = 1, \ldots, p$ are constant over time. As shown by Karlsson and Österholm (2020), an assumption of drifting parameters is not supported by the data when estimating models on US data ranging from 1953 to 2018; stochastic volatility, on the other hand, has very strong support in the data.

It is worth noting that given $\mathbf{W}_t$ and $\mathbf{H}_t$ it holds that

$$\mathbf{e}_t|\mathbf{W}_t, \mathbf{H}_t \sim \mathcal{N}\left(\boldsymbol{\mu}_t = \mathbf{W}_t\boldsymbol{\gamma}, \boldsymbol{\Sigma}_t = \mathbf{W}_t^{1/2}\mathbf{A}^{-1}\mathbf{H}_t\mathbf{A}^{-1\prime}\mathbf{W}_t^{1/2}\right).$$

By integrating out the mixing matrix $\mathbf{W}_t$, the conditional distribution of y_{it} is a skew-t distribution with skewness parameter γ_i and shape parameter ν_i for $i = 1, \ldots, k$; see Aas and Haff (2006) and Karlsson et al. (2021). Note that the conditional distribution of vector $\mathbf{y}_t$ reduces to a multivariate Student's t (MT-SV) distribution when $\gamma_1 = \ldots = \gamma_k = 0$. Additionally assuming that $\nu_i \to \infty$ for $i = 1, \ldots, k$, we will arrive at the Gaussian VAR model with stochastic volatility (G-SV). Moreover, the VAR models without stochastic volatility can be obtained by setting $\boldsymbol{\sigma}^2 = \mathbf{0}$ and $\log h_{it} = \log h_{i0}$ for $i = 1, \ldots k$ and $t = 1, \ldots, T$.

2.2 VAR models with orthogonal non-Gaussian disturbances

Another way of imposing skewness and heavy tails in the VAR model is to consider the shocks to the VAR equations as linear combinations of orthogonal disturbances. In what follows, this VAR model is called the orthogonal skew-t with stochastic volatility (OST-SV) and is expressed as

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{c} + \mathbf{B}_1\mathbf{y}_{t-1} + \ldots + \mathbf{B}_p\mathbf{y}_{t-p} + \mathbf{A}^{-1}\tilde{\mathbf{e}}_t, \\
\tilde{\mathbf{e}}_t &= \mathbf{W}_t\boldsymbol{\gamma} + \mathbf{W}_t^{1/2}\mathbf{H}_t^{1/2}\boldsymbol{\epsilon}_t, \quad t = 1, \ldots, T.
\end{aligned} \tag{3}$$

Since $\mathbf{W}_t$ and $\mathbf{H}_t$ are diagonal matrices, and $\mathbf{W}_t$, $\mathbf{H}_t$ and $\boldsymbol{\epsilon}_t$ are mutually independent, $\tilde{\mathbf{e}}_t$ is a vector of independent univariate skew-t distributed random variables. Therefore, in the OST-SV model, the shock to each VAR equation is a linear combination of orthogonal skew-t disturbances. On the other hand in the MST-SV model, each VAR equation has one skew-t disturbance that correlates with other disturbances in other VAR equations. Note that when the skewness parameters $\gamma_1 = \ldots = \gamma_k = 0$, the OST-SV model reduces to the orthogonal Student's t VAR model with stochastic volatility (OT-SV) proposed by Cúrdia et al. (2014), and Chiu et al. (2017). Furthermore, if $\gamma_1 = \ldots = \gamma_k = 0$, and $\nu_1 = \ldots = \nu_k \to \infty$, we get a VAR model with Gaussian disturbances and stochastic volatility. Finally, the VAR models without stochastic volatility can again be obtained by setting $\boldsymbol{\sigma}^2 = \mathbf{0}$ and $\log h_{it} = \log h_{i0}$ for $i = 1, \ldots k$ and $t = 1, \ldots, T$.

2.3 Forecast evaluation

To compare the forecasting performance of different VAR models, we conduct a recursive out-of-sample forecast exercise. We use the periods $1, \ldots, T_0 - 1$ as an (initial) training sample and calculate h-period ahead forecasts for periods from T_0 to $T - h$. The mean square forecast error (MSFE) is utilized to evaluate the point forecasts, while the log predictive score (LPS) and the continuous rank probability score (CRPS) are used for the density forecasts.

The MSFE at h-step ahead, for $h = 1, \ldots, H$, is calculated as,

$$\text{MSFE}_{ith} = \frac{1}{T - T_0 - h + 1} \sum_{t=T_0}^{T-h} \left[\left(\bar{y}_{i,t+h|t} - y^o_{i,t+h} \right)^2 \right],$$

where $\bar{y}_{i,t+h|t}$ is the point forecast – captured by the mean of posterior predictive samples – using all the data at time t and $y^o_{i,t+h}$ is the realization of the variable i, h-steps ahead. A smaller MSFE implies better point forecasting performance.

The LPS of the posterior predictive distribution is evaluated as,

$$\begin{aligned} \text{LPS}_{ith} &= \frac{1}{T - T_0 - h + 1} \sum_{t=T_0}^{T-h} \left[\log p(y^o_{i,t+h} | \mathbf{y}_{1:t-1}) \right] \\ &= \frac{1}{T - T_0 - h + 1} \sum_{t=T_0}^{T-h} \left[\log \int_{\boldsymbol{\theta}} p(y^o_{i,t+h} | \boldsymbol{\theta}, \mathbf{y}_{1:t-1}) p(\boldsymbol{\theta} | \mathbf{y}_{1:t-1}) d\boldsymbol{\theta} \right], \end{aligned}$$

where $p(y^o_{i,t+h} | \mathbf{y}_{1:t-1})$ is the h-step ahead posterior predictive density function (the score) evaluated at the realization of the variable. We can observe that LPS_{ith} depends on the high-dimensional integral over intermediate observations and it can be approximated by using the Rao-Blackwellization idea; see Andersson and Karlsson (2008) and Karlsson et al. (2021). That is, LPS_{ith} is calculated as follows

$$\text{LPS}_{ith} \quad = \quad \frac{1}{T - T_0 - h + 1} \sum_{t=T_0}^{T-h} \left[\log \frac{1}{R} \sum_{r=1}^{R} p(y^o_{i,t+h} | \boldsymbol{\theta}^{(r)}, \mathbf{y}_{1:t-1}) \right]$$

with $\boldsymbol{\theta}^{(1)}, \ldots, \boldsymbol{\theta}^{(R)}$ being the posterior samples of the corresponding VAR model. In other words, we calculate the LPS_{ith} as the average predictive score over the posterior samples. A higher score value indicates a better density forecasting performance of the model.

The CRPS is another measure of density forecast accuracy that is derived from the (quadratic) difference between the predictive cumulative distribution function and the empirical distribution of the variable. The CRPS is given by

$$\text{CRPS}_{ith} = \frac{1}{T - T_0 - h + 1} \sum_{t=T_0}^{T-h} \left[E_f \left| y_{i,t+h|t} - y^o_{i,t+h} \right| - 0.5 E_f \left| y_{i,t+h|t} - y'_{i,t+h|t} \right| \right],$$

where f is the predictive density of $y_{i,t+h|t}$, and $(y_{i,t+h|t}, y'_{i,t+h|t})$ are independent copies of the variable from the predictive density f; see Gneiting and Raftery (2007). To evaluate the expectations, we use the Monte Carlo method where we simulate 20000 draws from the predictive density f.

In order to compare the out-of-sample forecast among models, we apply the one-sided test of equal mean predictive accuracy based on Diebold and Mariano (1995), where the standard errors of the test statistics are computed with the Newey-West estimator. This approach was used by Clark (2011) and Clark and Ravazzolo (2015) in a similar context, evaluating both point and density forecasting performance.

3 Empirical analysis

3.1 Data

We analyse quarterly data from the United States between 1953Q2 and 2019Q4. Real GDP growth is the annual (year-on-year) percentage change in real GDP. The unemployment rate is the seasonally adjusted civilian unemployment rate of individuals 16 years of age and older. Finally, the corporate bond-yield spread is given by the yield on BAA corporate bonds minus the yield on a ten-year Treasury security (henceforth referred to as the BAA spread).

Figure 1 shows the evolution of GDP growth, the unemployment rate and the BAA spread. The shaded areas highlight the recession periods, typically associated with a decrease in output growth and increases in the unemployment rate and the BAA spread. The unconditional distribution of GDP growth is negatively skewed while that of unemployment and bond spread are positively skewed. The skewness of the marginal distribution might be caused by the mixture of the heteroskedastic economic shocks and/or skewness of the innovations.

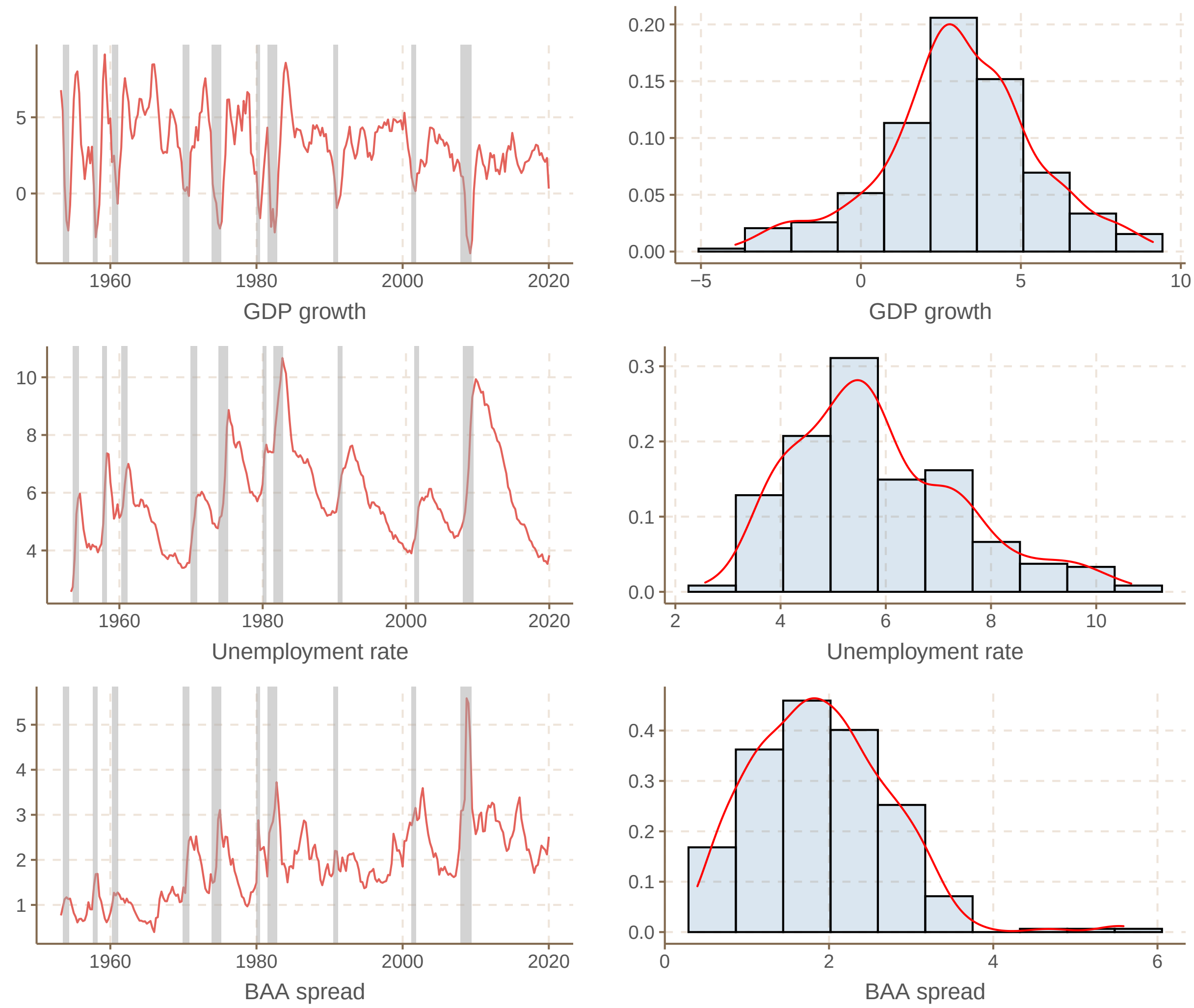

Figure 1: Time series plots and marginal distributions of the data.

The figures on the left hand side show the level of the variables. The shaded areas highlight the recession periods according to the NBER classification. The figures on the right-hand side draw the histogram and the kernel density of GDP growth, the unemployment rate and the BAA spread.

We conduct our empirical analysis in two parts. First, we estimate the VAR models to establish the importance of stochastic volatility and non-Gaussianity of the innovations in-sample. Second, we assess the out-of-sample forecasting ability of the models. In doing that, we consider specifications with stochastic volatility and allow for heavy tails and skewness in the error term sequentially. We evaluate both point and density forecasting performance of the models.

Throughout the empirical analysis, we consider three sets of models: univariate models for the three variables separately, bivariate models for each pair of variables and a trivariate specification with all the variables included.

3.2 In-sample analysis

The in-sample analysis is based on autoregressive specifications with $p = 4$ lags in the mean equation, and error structures described by equations (1) and (3). Results for the non-Gaussianity parameters are reported in Table 1 for the univariate models and Table 2 for multivariate specifications. In particular, the posterior mean of the degrees of freedom and the skewness parameter are presented, along with their 80 percent credible interval. Additionally, the log marginal likelihood (LML) is provided for each estimated model.

First, we note that in the univariate case, stochastic volatility is important for the models with Gaussian disturbances. The improvements in terms of LML are sizeable for all variables by taking stochastic volatility into account. These results suggest that there is non-negligible time variation in the second moment of the variables. To further elaborate on this, the dashed lines in Figure 2 show the estimated stochastic volatility for all three variables in the analysis that are obtained via the univariate Gaussian models. Looking at the figures, there is a clear pattern of decreasing volatility for GDP growth after the 1980s, which is in line with the idea of "Great Moderation" (Stock and Watson, 2002). A secular decline in volatility can also be observed for the unemployment rate, with the exception of the financial crisis in 2008-2009, when the volatility of the unemployment spiked again. Volatility in the BAA spread also spikes in association with economic distress. However, in contrast to the real variables where a secular decline in volatility is visible, the most turbulent period for the yield spread took place during the financial crisis.

The in-sample results in Table 1 show that accounting for non-Gaussian innovations is less important if stochastic volatility is present. The log marginal likelihoods do not show an improvement for t and skew-t specifications over and above stochastic volatility. This is also reflected in the the posterior mean of degrees of freedom of the t distribution, which is around twenty for the real variables (GDP growth and the unemployment rate); this suggests that controlling for stochastic volatility is sufficient to address the heavy tails of the unconditional distribution of these variables. In contrast, the degrees of freedom for the BAA spread is much lower (around ten), and the credible

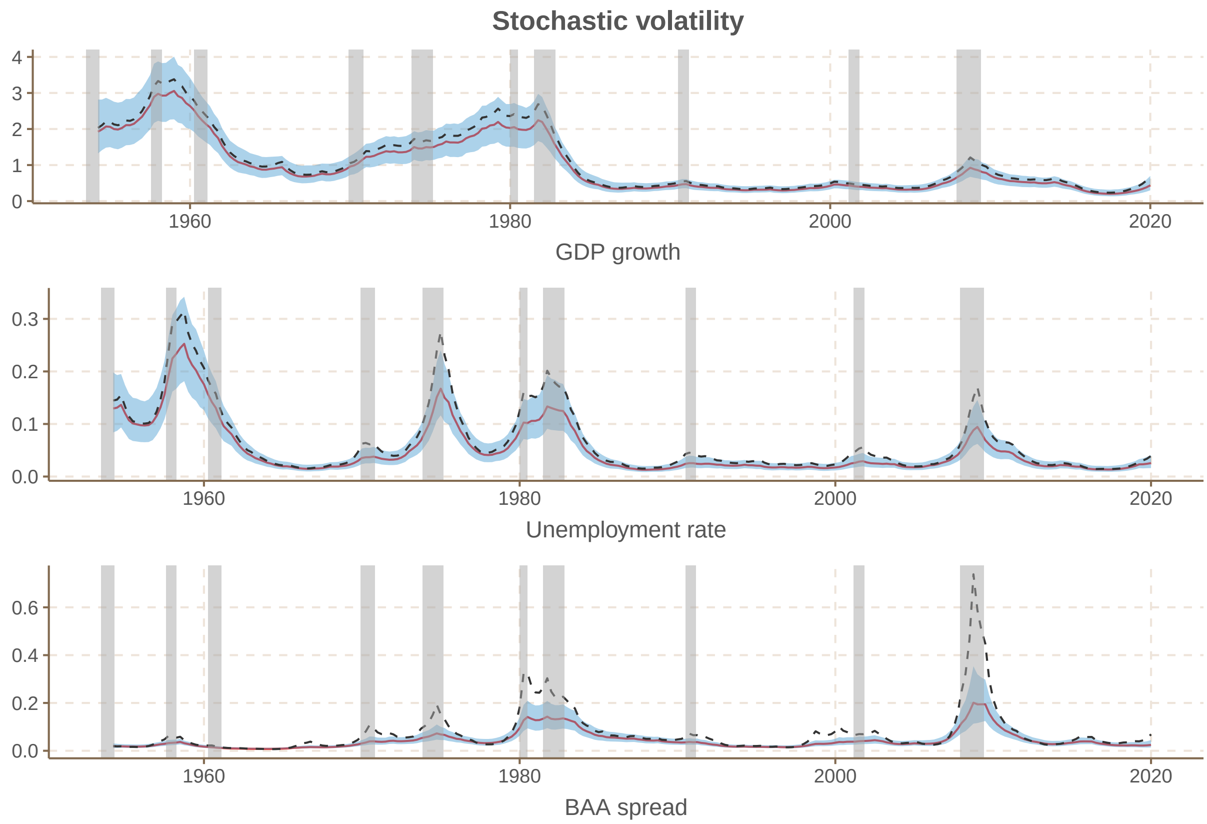

Figure 2: Stochastic volatility of GDP growth, the unemployment rate and BAA spread.

The dashed line shows the estimated mean volatility from the univariate G-SV model. The estimated mean volatility of the univariate OST-SV model is given by the red solid line with the 50% credible interval. The shaded areas highlight the recession periods according to the NBER classification.

interval includes values that are consistent with a leptokurtic innovation distribution. Skewness in the innovations has limited support for the univariate specifications for GDP growth, while the unemployment rate and the BAA spread appear slightly positively skewed.

Comparing the volatility paths based on Gaussian innovations (dashed line in Figure 2) and non-Gaussian innovations (specifically, the univariate specification with skew-t distributed innovations, solid line), it appears that estimates of stochastic volatility are not sensitive to allowing for non-Gaussianity, at least in case of the real variables. The path from both specifications follow each other closely such that the volatility implied by the Gaussian innovations always lies within the 50 percent credible interval for the volatility using non-Gaussian innovations. In contrast, the difference is visible for the BAA-spread, where the stochastic volatility based on the Gaussian model is outside the credible interval of the non-Gaussian model for a large part of the sample. The sizeable shocks created a stronger impact on the volatility movements of the Gaussian VAR models which is also similar with the findings in Cúrdia et al. (2014) and Chiu et al. (2017). These large shocks can also be generated by a heavy-tailed innovation process, which then implies a smaller shock to the variance.

Results for the multivariate specifications (Table 2) are fairly similar. First, the importance of stochastic volatility is made clear by the LML. The improvements are substantial regardless of which combinations of variables we consider. The LMLs also suggest that innovations to GDP growth and the unemployment rate are close to Gaussian. This is confirmed by the high degrees of freedom and the skewness parameter which is close to zero. Despite no improvements in the LMLs, the estimates of the heavy tail and skewness parameters suggest that the spread variable can potentially benefit from being modelled as having non-Gaussian innovations.

Figure 3 shows the posterior distribution of the degrees of freedom and skewness parameters from the trivariate specification where the innovations are from either a multivariate skew-t or an orthogonal skew-t distribution (both with stochastic volatility) as defined in equations (1) and (3). First, we can see that the two distributional assumptions lead to very similar histograms. The only exception is the skewness parameter for GDP growth, where the MST-SV model puts considerably larger weight to positive outcomes (resulting in a positive posterior mean) whereas there is a larger mass of negative posterior values for the OST-SV model (resulting in a negative posterior mean). However, as Table 2 shows, zero is included in the 80 percent credible interval for both specifications. Further, we can note that for GDP growth and the unemployment rate, there is a substantial proportion of heavy tail parameters over thirty and the posterior densities of the skewness parameter cluster around zero. These altogether mean that real economic variables can be modelled with Gaussian innovations in-sample. In contrast, the posterior densities signal heavier tails and positive skewness for the BAA spread.

Overall, based on our results, the evidence for non-Gaussianity in the innovations of the real economic variables seems limited. Our results provide a refinement of the knowledge in the literature which finds that the unconditional distribution of macroeconomic variables (most prominently, GDP growth) is characterized by heavy tails (Fagiolo et al., 2008; Ascari et al., 2015; Liu, 2019). We find that stochastic volatility captures a large fraction of the heavy-tailed behavior of the innovations and once it is controlled for, the scope of non-Gaussianity is limited, in line with the recent literature on the interaction between heavy tails and time-varying volatility (Chiu et al., 2017; Karlsson and Mazur, 2020; Karlsson et al., 2021). For the BAA spread (which is mostly used as a leading indicator in this context, that is, as a predictor for the real economy) we find that even after controlling for time variation in the second moment of the innovations, there is some scope for modelling non-Gaussian behaviour (c.f. Kiss and Österholm, 2020). Having established the results in-sample, in the next section we investigate whether heavy tails and skewness impacts the predictive relationship between the yield spread and the real economy in case of real time forecasting.

Table 1: Univariate AR(4) models for GDP, Unemployment and BAA spread

	GDP			Unemployment (U)			BAA (S)		
	Gaussian	t	Skew-t	Gaussian	t	Skew-t	Gaussian	t	Skew-t
ν	-	21.8	25.3	-	26.8	23.2	-	13.4	12.7
	-	(8.3;40.4)	(11;43.5)	-	(12.4;46.4)	(11.7;38.2)	-	(4.4;27.2)	(6;22.3)
γ	-	-	0	-	-	0.2	-	-	0.2
	-	-	(-0.7;0.7)	-	-	(0.1;0.3)	-	-	(0.1;0.3)
LML (SV)	-378.2	-377.9	-379.1	-7.5	-8.2	-9.9	-15.8	-15.4	-15.4
LML (NonSV)	-401.3	-394.8	-397	-45.1	-30.2	-29.3	-71.1	-26.9	-26.4

The table shows the posterior means of the degree of freedom and the skewness parameters in the univariate AR(4) models with SV. The numbers in the bracket are the [0.1;0.9] credible intervals. The table also presents log marginal likelihoods (LML) for the models that are evaluated as in Karlsson et al. (2021). For comparison, the last panel of the table reports the LML for the same models, without allowing for stochastic volatility.

Table 2: VAR(4) models for GDP, Unemployment (U) and BAA spread (S)

		G-SV	OT-SV	MT-SV	OST-SV	MST-SV
GDP-U	ν_{GDP}	-	19.9	18.4	24.6	24.7
		-	(7.9;35.7)	(7.6;32.6)	(10.9;43.2)	(10.9;42.1)
	ν_U	-	25.2	17.5	27.3	19.6
		-	(11.2;43.1)	(7.5;31.4)	(12.7;45.3)	(8.9;33.9)
	γ_{GDP}	-	-	-	-0.1	0.5
		-	-	-	(-0.7;0.6)	(-0.2;1.2)
	γ_U	-	-	-	0.1	0.1
		-	-	-	(0;0.3)	(0;0.3)
	LML	-355.1	-354.6	-353.7	-355.8	-354.1
GDP-S	ν_{GDP}	-	17.9	16.8	23.4	25.4
		-	(6.9;33.6)	(7;30.9)	(8.5;41.9)	(10.3;44.3)
	ν_S	-	15.1	12	12.5	11
		-	(4.6;31.3)	(4.2;25.6)	(5.4;22.8)	(5.1;19.4)
	γ_{GDP}	-	-	-	0	0.2
		-	-	-	(-0.5;0.6)	(-0.3;0.9)
	γ_S	-	-	-	0.1	0.1
		-	-	-	(0;0.2)	(0;0.2)
	LML	-387.5	-386.0	-385.9	-386.5	-386.5
U-S	ν_U	-	27.7	26.1	27.8	31.2
		-	(12.4;46.1)	(12.1;43.6)	(12.4;47.2)	(14.9;50.8)
	ν_S	-	18.1	11.7	14.1	11.3
		-	(5.9;35.2)	(4.5;22.5)	(6.1;25.8)	(5.7;18.6)
	γ_U	-	-	-	0.2	0.1
		-	-	-	(0;0.3)	(0;0.3)
	γ_S	-	-	-	0.1	0.1
		-	-	-	(0;0.3)	(0.1;0.2)
	LML	-16.4	-16.7	-16.0	-17.4	-16.8
GDP-U-S	ν_{GDP}	-	14.9	15.9	22	23.4
		-	(5.9;27.5)	(7;28)	(8;41.4)	(10.3;39.4)
	ν_U	-	24.8	20.7	29.8	26.5
		-	(10.3;43.1)	(8.7;36.8)	(14.1;48.5)	(12.5;45)
	ν_S	-	11.8	6.8	11.7	8.5
		-	(3.8;25.6)	(3.4;12.5)	(5;22.3)	(4.6;13.4)
	γ_{GDP}	-	-	-	-0.1	0.5
		-	-	-	(-0.7;0.7)	(-0.1;1.4)
	γ_U	-	-	-	0.1	0.1
		-	-	-	(-0.1;0.2)	(0;0.2)
	γ_S	-	-	-	0.1	0.1
		-	-	-	(0;0.2)	(0;0.2)
	LML	-366.7	-365.2	-362.7	-366.5	-364.2
LML (nonSV)	GDP-U	-394.6	-384.3	-381.6	-388	-385.4
	GDP-S	-461.6	-413.5	-412.1	-416.2	-415.3
	U-S	-96.6	-49.1	-46.5	-48.1	-47.6
	GDP-U-S	-450.3	-400.6	-397.3	-403.5	-401.7

The table shows the posterior means of the degree of freedom and the skewness parameters in the VAR(4) models. The numbers in the bracket are the [0.1;0.9] credible intervals. The table also presents log marginal likelihoods (LML) for models with and without stochastic volatility that are evaluated as in Karlsson et al. (2021).

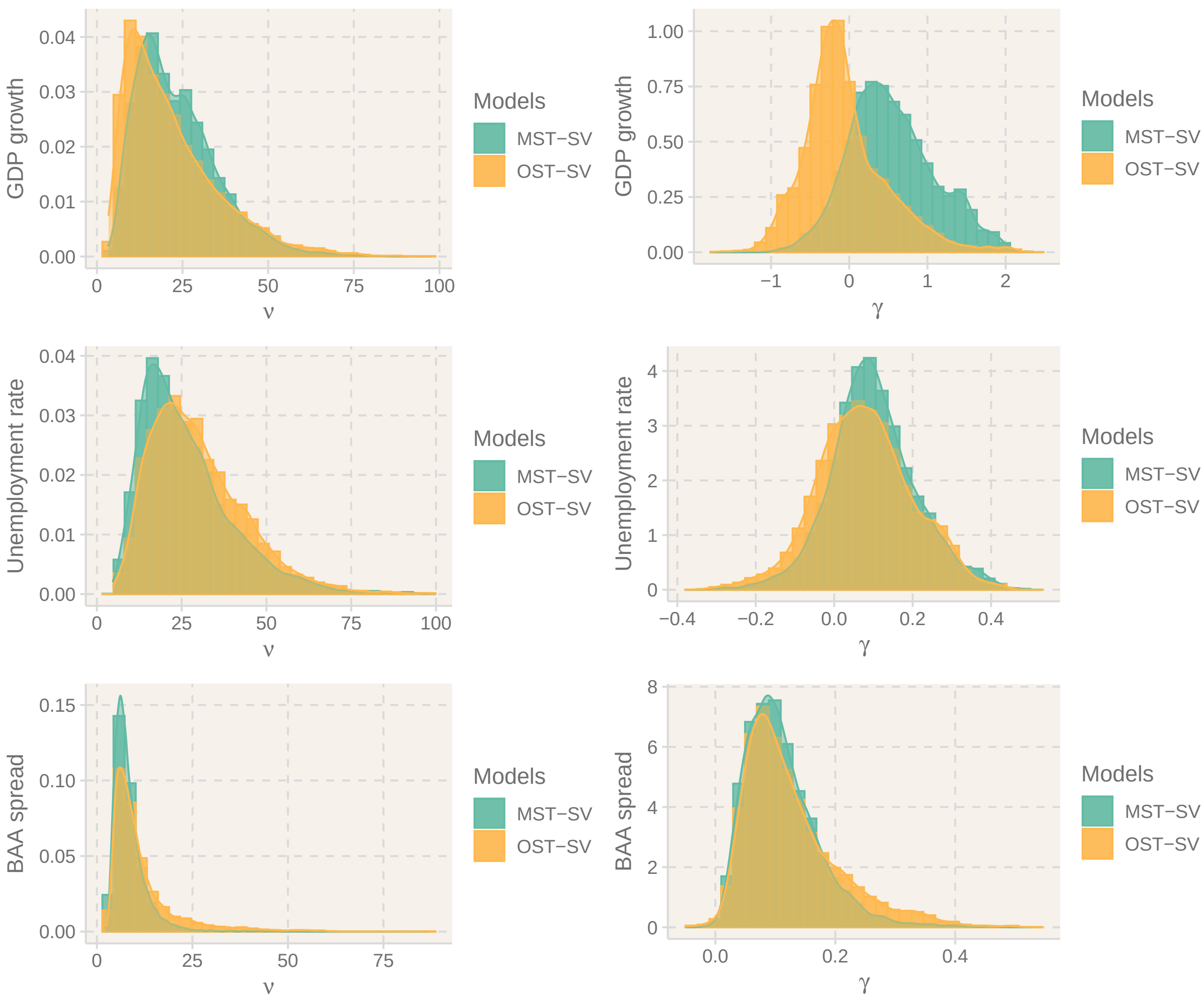

Figure 3: Posterior samples of the heavy tails and skewness parameters of the trivariate MST-SV and OST-SV VAR models.

3.3 Forecasting performance

We conduct an out-of-sample forecast exercise to assess whether accounting for skewness and heavy tails in the innovations helps to improve real-time forecasting performance of the VARs. Our focus in on the predictive relationship which goes from the BAA spread to the real economic variables. However, for completeness, we show predictive performance for all variables and models.

Our benchmark model (for each variable) is the univariate specification with homoscedastic and Gaussian disturbances. This is then compared to specifications where we sequentially relax the assumptions regarding the error term by allowing for stochastic volatility and non-Gaussian distribution in the innovations. We also increase the dimension of the model – to bi- and tri-variate models – and again assess the importance of stochastic volatility and non-Gaussianity.

The forecast evaluation is performed on an expanding window; the parameters are re-estimated each time the sample is expanded. In order to mimic the actual forecasting situation as closely as possible, we use real-time data from the Philadelphia Fed (see, for example, Croushore and Stark, 2001 and Clements and Galvão, 2013). That is, estimation is always made on the version of the data available to the forecaster at the given time point. The forecast is then evaluated based on the first vintage where the realizations of the series are available. It can be noted that the publication delay is different for the three variables. In particular, the BAA spread for a given quarter is available at the end of that quarter's last trading day; this stands in contrast with the more substantial delays in the other two variables, particularly GDP growth. However, we always use information up to the same quarter for all three variables, avoiding the so-called ragged-edge problem associated with asynchronous data publication. Note that data revisions are substantial for the GDP series (Croushore and Stark, 2001) and for the unemployment rate, but the BAA-spread is a financial variable which is not revised.

The first forecast is made using the information up to the last quarter of 1965; this means that a sample ranging from 1953Q2 to 1965Q4 is used. We consider the forecasting horizons one to four quarters and two years, and the last forecast is made based on data in 2017Q4, for periods from 2018Q1 up to 2018Q4 and for 2019Q4. This ensures that we have 211 forecasts for all horizons, so that forecast accuracy measures are comparable across horizons.

3.3.1 Point forecasts

First, we focus on the point forecasting results. Tables 3, 4 and 5 present mean squared forecasting errors for GDP growth, the unemployment rate and the BAA spread, respectively. Absolute numbers are shown for the benchmark univariate model with homoscedastic, Gaussian innovations, and relative MSFEs are given for the rest of the specifications. The relative MSFE is given as the MSFE for the model of interest divided by the MSFE of the benchmark, and a relative MSFE smaller than one accordingly indicates that the model in question is better than the benchmark model. Models that outperform the benchmark using the Diebold and Mariano (1995) test are marked with asterisks.[5]

For the point forecasts of GDP growth, the results show that neither stochastic volatility nor non-Gaussian innovations generally improve forecasting performance significantly. The MSFE increases relative to the benchmark for all horizons, when using t or skew-t distributed innovations. The only feature that really appears to matter is the inclusion of the BAA spread as a predictor variable. Table 3 indicates that the bivariate VAR with the BAA spread and GDP growth outperforms the benchmark significantly at the shortest horizon. This outperformance somewhat prevails for two quarters ahead, and for the trivariate VAR, but loses significance at conventional significance levels. These findings are in line with the fact that leading indicators (i.e. variables like the BAA spread) tend to work best at shorter horizons. Additionally, the predictive ability of the BAA spread for GDP growth seems to be unaffected by how we model the innovations: there is no improvement in terms of MSFE for the specifications with stochastic volatility and non-Gaussian innovations.

Turning to Table 4 and the unemployment rate, non-Gaussianity, in particular skewness, tends to improve point forecasting performance at the one- to four-quarter horizons. Looking at the univariate specifications, skew-t distributed innovations have the lowest MSFE at the four shortest horizons; the improvements are admittedly modest though. The VAR specifications show that inclusion of GDP growth and/or the BAA spread lowers the MSFE relative to the univariate forecast (again at the one- to four-quarter horizons). The inclusion of GDP growth produces statistically

[5] To alleviate the concerns related to the standard Diebold and Mariano (1995) test, we reproduced our results using the Clark and McCracken (2015) test, which relaxes some of the strict assumptions of the original Diebold-Mariano test. The results (in terms of the significance of improvements) are almost identical.

significant improvements. Stochastic volatility, heavy tails and skewness do not (or only marginally) change the results in the bivariate VAR with GDP growth and the unemployment rate. In contrast, for the bivariate VAR with the unemployment rate and the BAA spread, modelling innovations flexibly seems to matter: the homoscedastic model performs worse than the univariate benchmark, a result which is overturned once we account for time varying volatility and non-Gaussian innovations. The improvements are largest (although typically still not statistically significant) when accounting for skewness, where the relative MSFE is lower than one at the one- to four-quarter horizons. Lastly, we note that for the trivariate specification, the Gaussian model with stochastic volatility has a lower MSFE than the Gaussian model with homoscedastic disturbances. Allowing for heavy tails and/or skewness brings no additional improvements in general.

Even though the BAA spread benefits somewhat from being modelled with heavy-tailed and skewed innovations based on the in-sample results, this does not generally generate better forecasts. For the univariate specifications, only accounting for skewness brings some marginal (although insignificant) reductions in the MSFE at the shorter horizons. Including real variables as predictors in general barely seems to help improve forecasts (which is in line with the general notion that BAA spread itself usually leads real variables). However, models allowing for a skewed innovation distribution tend to lower the MSFE relative to the benchmark model at the one- to four-quarter horizons even for the bi- and tri-variate models. Stochastic volatility also seems relevant once GDP growth is included. In fact, for the bivariate model with GDP growth and the BAA spread, significant improvement is achieved when both stochastic volatility and skewness are allowed for. This significance partly survives for the trivariate model, where the Gaussian model with stochastic volatility and the multi skew-t model shows significant improvements.

Generally, we find that point forecasts are affected by flexible modelling of innovation to a limited extent in our context. However, this is not particularly surprising given the bias-variance trade-off of forecasting with more elaborate models. The flexible distributional assumptions mostly affect the entire shape (all moments) of the predictive distribution, and therefore the first moment (point forecasts) are less sensitive to distributional assumptions. However, the flexibility comes with a cost of increased complexity and hence reduced estimation precision, which in turn leads to worse point forecasting performance in several cases, as we have seen above.

Table 3: MSFEs and relative MSFEs [relative to the Gaussian AR(4) model] for GDP growth

	GDP growth				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - GDP					
G-nonSV	0.927	2.418	4.109	5.320	5.713
G-SV	0.999	1.005	1.023	1.047	1.079
T-SV	1.006	1.019	1.041	1.065	1.085
ST-SV	1.006	1.018	1.039	1.062	1.117
(b) Bivariate VAR(4) - GDP - Unemployment(U)					
G-nonSV	1.054	1.089	1.109	1.117	1.076
G-SV	1.080	1.138	1.185	1.218	1.166
ST-SV	1.091	1.160	1.217	1.256	1.187
MT-SV	1.095	1.164	1.215	1.254	1.185
OST-SV	1.106	1.165	1.214	1.247	1.149
MST-SV	1.110	1.189	1.247	1.288	1.210
(c) Bivariate VAR(4) - GDP - BAA spread(S)					
G-nonSV	0.895**	0.897*	0.950	1.012	1.000
G-SV	0.897**	0.915	0.995	1.093	1.143
ST-SV	0.898**	0.922	1.007	1.113	1.152
MT-SV	0.896**	0.920	1.006	1.115	1.160
OST-SV	0.899**	0.925	1.006	1.104	1.124
MST-SV	0.895**	0.924	1.006	1.105	1.124
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.986	1.034	1.101	1.156	1.059
G-SV	0.973	1.028	1.125	1.227	1.221
ST-SV	0.976	1.036	1.138	1.241	1.230
MT-SV	0.981	1.040	1.140	1.242	1.237
OST-SV	1.032	1.087	1.163	1.238	1.152
MST-SV	0.996	1.054	1.151	1.243	1.200

The first line reports the MSFE of the benchmark Gaussian Univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The relative performance is computed as the ratio of the MSFE of alternative specifications over the benchmark. The entries less than 1 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 4: MSFEs and relative MSFEs [relative to the Gaussian AR(4) model] for Unemployment (U)

	Unemployment rate				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - Unemployment(U)					
G-nonSV	0.098	0.308	0.619	0.979	2.313
G-SV	0.990	1.000	1.018	1.039	1.152
T-SV	0.991	1.004	1.022	1.043	1.157
ST-SV	0.978	0.975	0.978	0.983	1.038
(b) Bivariate VAR(4) - GDP - Unemployment(U)					
G-nonSV	0.875**	0.914*	0.941	0.953	1.013
G-SV	0.865***	0.894**	0.918**	0.935*	1.044
ST-SV	0.866***	0.901**	0.932*	0.953	1.069
MT-SV	0.864***	0.903**	0.934*	0.955	1.074
OST-SV	0.868***	0.902**	0.933*	0.951	1.040
MST-SV	0.871***	0.912**	0.944*	0.966	1.077
(c) Bivariate VAR(4) - Unemployment(U) - BAA spread(S)					
G-nonSV	1.016	1.031	1.038	1.047	1.079
G-SV	0.938*	0.929	0.957	0.996	1.170
ST-SV	0.939	0.933	0.961	1.000	1.171
MT-SV	0.938*	0.934	0.962	1.005	1.182
OST-SV	0.942*	0.931*	0.943	0.966	1.059
MST-SV	0.939*	0.929*	0.947	0.976	1.092
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.888	0.931	0.965	0.992	1.078
G-SV	0.858**	0.871**	0.903*	0.937	1.084
ST-SV	0.861**	0.878**	0.910*	0.946	1.091
MT-SV	0.852**	0.870**	0.905*	0.943	1.097
OST-SV	0.871**	0.905	0.932	0.961	1.068
MST-SV	0.854**	0.879**	0.915*	0.949	1.089

The first line reports the MSFE of the benchmark Gaussian Univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The relative performance is computed as the ratio of the MSFE of alternative specifications over the benchmark. The entries less than 1 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 5: MSFEs and relative MSFEs [relative to the Gaussian AR(4) model] for BAA spread(S)

	BAA spread(S)				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - BAA spread(S)					
G-nonSV	0.113	0.273	0.406	0.507	0.732
G-SV	1.006	1.040	1.083	1.149	1.410
T-SV	1.008	1.042	1.086	1.152	1.414
ST-SV	0.975	0.974	0.985	1.015	1.143
(b) Bivariate VAR(4) - GDP - BAA spread(S)					
G-nonSV	0.998	0.991	0.984	0.980	0.977
G-SV	0.974	0.997	1.021	1.062	1.234
ST-SV	0.975	1.001	1.029	1.069	1.236
MT-SV	0.979	1.002	1.029	1.069	1.237
OST-SV	0.959**	0.963	0.970	0.988	1.073
MST-SV	0.960*	0.961	0.963	0.979	1.058
(c) Bivariate VAR(4) - Unemployment(U) - BAA spread(S)					
G-nonSV	1.025	1.019	1.021	1.026	1.029
G-SV	1.014	1.027	1.054	1.098	1.254
ST-SV	1.018	1.033	1.060	1.103	1.259
MT-SV	1.022	1.038	1.066	1.112	1.274
OST-SV	0.996	0.980	0.978	0.994	1.047
MST-SV	0.999	0.986	0.983	1.001	1.062
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.994	1.001	1.007	1.015	1.032
G-SV	0.959*	0.985	1.014	1.057	1.216
ST-SV	0.969	0.992	1.014	1.051	1.204
MT-SV	0.968	0.993	1.018	1.058	1.214
OST-SV	0.956	0.953	0.954	0.969	1.028
MST-SV	0.951*	0.952	0.955	0.971	1.025

The first line reports the MSFE of the benchmark Gaussian Univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The relative performance is computed as the ratio of the MSFE of alternative specifications over the benchmark. The entries less than 1 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

3.3.2 Density forecasts

In this section we consider how density forecasts are affected by modelling the innovations of the VAR model flexibly. We consider two frequently used measures of density forecast evaluation, the LPS and the CRPS, as described in Section 2. We discuss both measures, since even though they mostly agree in terms of the direction of the findings (which specification improves upon the benchmark and which does not), the strength of the results change somewhat depending on which measure we use. Tables 6 to 11 show results for both measures and all three variables in a similar structure as for the point forecasts. That is, the benchmark model is the univariate model with homoscedastic, Gaussian innovations, for which the values of the LPS and CRPS are presented. For the other specifications, the improvement over the benchmark is given in units of LPS or CRPS.

Tables 6 and 7 show results for density forecasts of GDP growth. First, we can note that even if GDP growth is modelled using only its own lags, accounting for stochastic volatility produces much better density forecasts in the short run (one to two quarters), and the effect is still there, although muted, for longer horizons; further flexibility in the specification does not seem warranted though. There appears to be gains to be made by moving beyond the univariate framework, in particular by adding the BAA spread, for which even the Gaussian and homoscedastic specification outperforms the baseline model. Adding stochastic volatility to the bivariate VAR with GDP growth and BAA spread extends its advantage over the benchmark to three quarters (according to the LPS). Non-Gaussian innovations do not seem to be an important feature even in the multivariate specifications (once the volatility is allowed to vary over time). At longer horizons – in particular, two years – flexible modelling of the error term does not improve density forecasts.

Forecasts of the unemployment rate – which is our other variable capturing real activity – appear slightly better when accounting for skewness and heavy tails, but improvements are typically small over and above stochastic volatility. Results in Tables 8 and 9 show an improvement at shorter horizons (results are significant up to four quarters) for the univariate specifications when we allow for stochastic volatility. Heavy tails do not contribute additionally to the forecasting performance, but allowing for skewness seems to be a relevant feature which is not completely subsumed by time-varying volatility, at least according to the LPS measure. Moving to the VARs, we see the importance of GDP growth in forecasting unemployment - a not particularly surprising finding.

However, GDP growth itself cannot really capture all the tail behaviour of the forecasts of the unemployment rate, so time-varying volatility appears useful for the horizons up to four quarters (and skewness at the shortest horizon, according to the LPS). The BAA spread does not seem to improve the density forecast of the unemployment rate, unless the error terms are modelled flexibly, with stochastic volatility again playing a key role in improved density forecasts. Finally, for the unemployment rate, the trivariate VAR does not materially produce better results than the VAR with GDP growth and unemployment.

Finally, density forecasts of the BAA spread (Tables 10 and 11) benefit from both stochastic volatility and skewness, in particular for the shorter horizons (one to three quarters). Stochastic volatility seems to be sufficient to capture heavy tails in the distribution of the innovations, since the specifications with the orthogonal or multivariate t-distributions do not improve over the one with time-varying volatility. Interestingly, adding GDP growth appears to help forecasting yield spread density, even in the homoscedastic, Gaussian model for several horizons according to the LPS measure. Note however, that improvements are small in absolute value.[6] Stochastic volatility improves the density forecasts over the short horizon, but its effect is weaker for longer horizons. For three and four quarters ahead, skewness also plays a role in improving forecasts (with stronger results provided by the CRPS measure). Adding the unemployment rate does not help in density forecasting the BAA spread, which is evidenced by both the bivariate unemployment-BAA spread results, and the incremental changes when moving to the trivariate VAR. Finally, longer horizon (two year) density forecasts do not benefit from the flexible modelling either, similar to the other variables.

Overall, we find mixed evidence in terms of whether it is worth going for flexible modelling of error terms. The homoscedastic Gaussian model works relatively the worst at shorter horizons for all three variables. However, in most cases, accounting for stochastic volatility appears flexible enough to capture the deviations from the baseline. This is in line with the unconditional non-Gaussianity of the variables mostly being associated with time-variation in the second moment.

[6] In fact, the forecast errors implied by the univatiare model and the GDP growth-BAA spread VAR are very close to each other. Hence, both the difference in the LPS and the standard error is small in magnitude. This makes the test indicate significance even in case the difference in LPS is small.

Table 6: LPS and difference in LPS relative to the Gaussian AR(4) model for GDP growth

	GDP growth				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - GDP					
G-nonSV	-1.384	-1.844	-2.099	-2.229	-2.268
G-SV	0.104***	0.124***	0.102*	0.059	-0.000
T-SV	0.099***	0.121***	0.097*	0.056	0.001
ST-SV	0.098***	0.118***	0.100*	0.061	0.005
(b) Bivariate VAR(4) - GDP - Unemployment(U)					
G-nonSV	-0.016	-0.039	-0.063	-0.081	-0.027
G-SV	0.082***	0.084**	0.043	-0.003	-0.024
ST-SV	0.076**	0.078*	0.036	-0.015	-0.020
MT-SV	0.077**	0.077*	0.038	-0.017	-0.025
OST-SV	0.075**	0.074*	0.035	-0.012	0.003
MST-SV	0.075**	0.064	0.012	-0.045	-0.057
(c) Bivariate VAR(4) - GDP - BAA spread(S)					
G-nonSV	0.042**	0.034	0.010	-0.017	-0.018
G-SV	0.166***	0.190***	0.136**	0.060	-0.053
ST-SV	0.163***	0.189***	0.131**	0.054	-0.049
MT-SV	0.165***	0.189***	0.130**	0.048	-0.052
OST-SV	0.169***	0.188***	0.136**	0.062	-0.018
MST-SV	0.166***	0.187***	0.132**	0.057	-0.019
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.010	-0.023	-0.066	-0.098	-0.069
G-SV	0.129***	0.125**	0.061	-0.010	-0.046
ST-SV	0.128***	0.122**	0.054	-0.020	-0.036
MT-SV	0.127***	0.120**	0.051	-0.026	-0.041
OST-SV	0.126***	0.118**	0.053	-0.017	0.004
MST-SV	0.121***	0.109**	0.039	-0.033	-0.021

The first line reports the LPS of the benchmark Gaussian univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The difference in LPS is computed the LPS of the alternative specifications minus the LPS of the benchmark model. The entries greater than 0 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 7: CRPS and difference in CRPS relative to the Gaussian AR(4) model for GDP growth

	GDP growth				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - GDP					
G-nonSV	-0.533	-0.833	-1.076	-1.230	-1.274
G-SV	0.021***	0.031**	0.026	0.006	-0.036
T-SV	0.019***	0.026*	0.018	-0.003	-0.037
ST-SV	0.019***	0.026*	0.018	-0.002	-0.025
(b) Bivariate VAR(4) - GDP - Unemployment(U)					
G-nonSV	-0.008	-0.029	-0.057	-0.068	-0.044
G-SV	0.009	-0.006	-0.043	-0.080	-0.089
ST-SV	0.007	-0.014	-0.060	-0.100	-0.091
MT-SV	0.006	-0.015	-0.058	-0.099	-0.094
OST-SV	0.005	-0.026	-0.064	-0.110	-0.072
MST-SV	0.004	-0.026	-0.080	-0.127	-0.118
(c) Bivariate VAR(4) - GDP - BAA spread(S)					
G-nonSV	0.027**	0.041**	0.025	-0.010	-0.011
G-SV	0.051***	0.076***	0.050*	-0.014	-0.086
ST-SV	0.050***	0.074***	0.044	-0.025	-0.084
MT-SV	0.050***	0.074***	0.044	-0.026	-0.088
OST-SV	0.050***	0.073***	0.045*	-0.019	-0.061
MST-SV	0.051***	0.073***	0.045*	-0.019	-0.060
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.007	-0.015	-0.069	-0.113	-0.065
G-SV	0.033**	0.029	-0.028	-0.105	-0.118
ST-SV	0.032**	0.025	-0.036	-0.113	-0.117
MT-SV	0.031**	0.023	-0.040	-0.119	-0.123
OST-SV	0.027*	0.015	-0.044	-0.121	-0.074
MST-SV	0.029**	0.017	-0.045	-0.122	-0.105

The first line reports the CRPS of the benchmark Gaussian univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The difference in CRPS is computed the CRPS of the alternative specifications minus the CRPS of the benchmark model. The entries greater than 0 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 8: LPS and difference in LPS relative to the Gaussian AR(4) model for Unemployment (U)

	Unemployment rate				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - Unemployment(U)					
G-nonSV	-0.492	-1.082	-1.417	-1.625	-1.989
G-SV	0.374***	0.393***	0.332***	0.230*	-0.113
T-SV	0.375***	0.393***	0.335***	0.228*	-0.126
ST-SV	0.391***	0.428***	0.369***	0.280**	-0.023
(b) Bivariate VAR(4) - GDP - Unemployment(U)					
G-nonSV	0.073***	0.134***	0.156***	0.163***	0.101**
G-SV	0.409***	0.425***	0.375***	0.301**	-0.003
ST-SV	0.410***	0.427***	0.375***	0.287**	-0.022
MT-SV	0.416***	0.433***	0.371***	0.298**	-0.033
OST-SV	0.413***	0.434***	0.376***	0.292***	0.003
MST-SV	0.421***	0.437***	0.375***	0.294***	-0.046
(c) Bivariate VAR(4) - Unemployment(U) - BAA spread(S)					
G-nonSV	0.010	-0.013	-0.028	-0.043	-0.068
G-SV	0.370***	0.377***	0.311***	0.200	-0.128
ST-SV	0.371***	0.380***	0.315***	0.202*	-0.125
MT-SV	0.372***	0.380***	0.318***	0.209*	-0.133
OST-SV	0.381***	0.405***	0.348***	0.258**	-0.035
MST-SV	0.385***	0.399***	0.337***	0.242**	-0.069
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.077***	0.114***	0.117***	0.112***	0.041*
G-SV	0.398***	0.421***	0.372***	0.297***	-0.006
ST-SV	0.397***	0.417***	0.373***	0.287***	-0.012
MT-SV	0.406***	0.422***	0.378***	0.291***	-0.023
OST-SV	0.394***	0.412***	0.362***	0.262**	-0.023
MST-SV	0.407***	0.418***	0.367***	0.288***	-0.021

The first line reports the LPS of the benchmark Gaussian univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The difference in LPS is computed the LPS of the alternative specifications minus the LPS of the benchmark model.The entries greater than 0 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 9: CRPS and difference in CRPS relative to the Gaussian AR(4) model for Unemployment (U)

	Unemployment rate				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - Unemployment(U)					
G-nonSV	-0.192	-0.342	-0.481	-0.600	-0.900
G-SV	0.034***	0.064***	0.081***	0.080**	-0.014
T-SV	0.034***	0.064***	0.080***	0.078**	-0.016
ST-SV	0.034***	0.065***	0.084***	0.087***	0.026
(b) Bivariate VAR(4) - GDP - Unemployment(U)					
G-nonSV	0.013***	0.030***	0.047***	0.058***	0.042**
G-SV	0.040***	0.074***	0.094***	0.099***	0.027
ST-SV	0.040***	0.074***	0.092***	0.095***	0.022
MT-SV	0.041***	0.074***	0.093***	0.096***	0.018
OST-SV	0.040***	0.072***	0.089***	0.092***	0.021
MST-SV	0.040***	0.073***	0.089***	0.090***	0.006
(c) Bivariate VAR(4) - Unemployment(U) - BAA spread(S)					
G-nonSV	0.000	-0.007	-0.015	-0.026	-0.057
G-SV	0.035***	0.067***	0.083***	0.078***	-0.033
ST-SV	0.035***	0.066***	0.083***	0.078***	-0.031
MT-SV	0.035***	0.067***	0.083***	0.078**	-0.033
OST-SV	0.034***	0.065***	0.083***	0.081***	0.009
MST-SV	0.035***	0.066***	0.084***	0.082***	-0.001
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	0.012***	0.022***	0.031***	0.032**	-0.007
G-SV	0.039***	0.072***	0.092***	0.093***	0.008
ST-SV	0.039***	0.071***	0.091***	0.091***	0.005
MT-SV	0.040***	0.072***	0.092***	0.093***	0.007
OST-SV	0.038***	0.069***	0.083***	0.083***	0.006
MST-SV	0.040***	0.071***	0.089***	0.089***	0.001

The first line reports the CRPS of the benchmark Gaussian univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The difference in CRPS is computed the CRPS of the alternative specifications minus the CRPS of the benchmark model. The entries greater than 0 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 10: LPS and difference in LPS relative to the Gaussian AR(4) model for BAA spread(S)

	BAA spread(S)				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - BAA spread(S)					
G-nonSV	-0.478	-0.886	-1.076	-1.194	-1.400
G-SV	0.303***	0.158**	0.072	-0.043	-0.312
T-SV	0.306***	0.155**	0.070	-0.042	-0.291
ST-SV	0.329***	0.201***	0.136*	0.065	-0.085
(b) Bivariate VAR(4) - GDP - BAA spread(S)					
G-nonSV	0.010**	0.040***	0.055***	0.067***	0.065***
G-SV	0.329***	0.185**	0.113	0.014	-0.203
ST-SV	0.333***	0.182**	0.109	0.014	-0.193
MT-SV	0.333***	0.183**	0.105	0.013	-0.198
OST-SV	0.345***	0.215***	0.149*	0.068	-0.065
MST-SV	0.347***	0.213***	0.147*	0.086	-0.037
(c) Bivariate VAR(4) - Unemployment(U) - BAA spread(S)					
G-nonSV	-0.038	-0.021	-0.017	-0.013	-0.022
G-SV	0.292***	0.165**	0.099	0.013	-0.173
ST-SV	0.300***	0.164**	0.088	-0.003	-0.167
MT-SV	0.298***	0.152**	0.073	-0.022	-0.171
OST-SV	0.322***	0.197**	0.128	0.068	0.003
MST-SV	0.324***	0.196***	0.114	0.060	-0.020
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	-0.018	0.016**	0.029***	0.035***	0.013
G-SV	0.331***	0.192**	0.130	0.044	-0.125
ST-SV	0.334***	0.196***	0.130	0.059	-0.098
MT-SV	0.334***	0.187***	0.122	0.047	-0.106
OST-SV	0.353***	0.224***	0.162*	0.110	0.033
MST-SV	0.354***	0.212***	0.146*	0.097	0.039

The first line reports the LPS of the benchmark Gaussian univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The difference in LPS is computed the LPS of the alternative specifications minus the LPS of the benchmark model.The entries greater than 0 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

Table 11: CRPS and difference in CRPS relative to the Gaussian AR(4) model for BAA spread(S)

	BAA spread(S)				
	1Q	2Q	3Q	4Q	8Q
(a) Univariate AR(4) - BAA spread(S)					
G-nonSV	-0.189	-0.295	-0.361	-0.407	-0.498
G-SV	0.023***	0.018**	0.007	-0.015	-0.104
T-SV	0.024***	0.018**	0.006	-0.015	-0.099
ST-SV	0.026***	0.025***	0.021*	0.011	-0.026
(b) Bivariate VAR(4) - GDP - BAA spread(S)					
G-nonSV	0.002*	0.007***	0.013***	0.017***	0.019***
G-SV	0.027***	0.027***	0.022*	0.008	-0.051
ST-SV	0.028***	0.025***	0.020*	0.007	-0.048
MT-SV	0.027***	0.025***	0.020*	0.007	-0.046
OST-SV	0.028***	0.029***	0.028**	0.021*	-0.006
MST-SV	0.028***	0.029***	0.028**	0.023*	-0.000
(c) Bivariate VAR(4) - Unemployment(U) - BAA spread(S)					
G-nonSV	-0.006	-0.005	-0.005	-0.006	-0.012
G-SV	0.024***	0.022***	0.017	0.004	-0.049
ST-SV	0.024***	0.021**	0.015	0.002	-0.047
MT-SV	0.024***	0.019**	0.011	-0.003	-0.052
OST-SV	0.025***	0.026***	0.024**	0.020	0.007
MST-SV	0.024***	0.025***	0.022*	0.017	0.001
(d) Trivariate VAR(4) - GDP - Unemployment(U) - BAA spread(S)					
G-nonSV	-0.002	0.003	0.004	0.006	-0.002
G-SV	0.029***	0.028***	0.024**	0.013	-0.038
ST-SV	0.029***	0.027***	0.024**	0.013	-0.031
MT-SV	0.029***	0.026***	0.021*	0.010	-0.033
OST-SV	0.029***	0.031***	0.031***	0.027**	0.013
MST-SV	0.029***	0.030***	0.028**	0.024*	0.012

The first line reports the CRPS of the benchmark Gaussian univariate AR(4) model without stochastic volatility during 1965:Q4-2017:Q1 (211 recursive estimations). The difference in CRPS is computed the CRPS of the alternative specifications minus the CRPS of the benchmark model. The entries greater than 0 indicate that the given model is better. ***,**,* denote that the corresponding model significantly outperforms the benchmark at 1%, 5%, 10% level based on the one-sided Diebold and Mariano (1995) test where the standard errors of the test statistics are computed with the Newey–West estimator (Clark, 2011).

4 Conclusions

Financial conditions are tightly connected to real economic activity and information from financial markets are easy to collect in a timely manner. These features make financial variables great candidates to be used in nowcasting and forecasting macroeconomic activity. In this paper, we examine closely a prime example – the nexus between the bond-yield spread and GDP growth and the unemployment rate. We contribute to the previous knowledge by scrutinizing whether allowing for flexible modelling of the innovation distribution helps when forecasting real economic activity using the BAA spread.

Our results show that the variables – in particular the unemployment rate and the yield spread – are characterized by heavy-tailed unconditional distributions. However, once the dynamics of the variables are modelled appropriately and stochastic volatility is accounted for, there is little room for modelling non-Gaussianity in the innovations. This also translates into its limited usefulness in terms of real-time forecasting performance. While stochastic volatility is crucial in capturing the distribution of the real-economic variables more precisely, hence impacting density forecasting performance substantially, allowing for skewness and heavy tails in the innovations yields little incremental improvement.

The findings are important from an applied modelling perspective. Accurate and timely forecasts of real economic activity have always been on demand and there is a continuous effort to find leading indicators of GDP growth, for example. However, it is also important how we use the information from these variables. Our results suggest that relying on the normal distribution, which is the workhorse approach, might be sufficient when using the yield spread as a forecasting variable, once stochastic volatility is accounted for.

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Appendix

A Bayesian Inference

Following Karlsson et al. (2021), we employ the Bayesian approach to make inferences for the set of the model parameters $\boldsymbol{\theta} = \{vec(\mathbf{B})^{'}, \mathbf{a}^{'}, \boldsymbol{\gamma}^{'}, \boldsymbol{\nu}^{'}, \boldsymbol{\sigma}^{2'}, \xi^{'}_{1:K,1:T}, \mathbf{h}^{'}_{1:K,0:T}\}^{'}$, where $\mathbf{B} = (\mathbf{c}, \mathbf{B}_1, \ldots, \mathbf{B}_p)$ is a $k \times (1 + kp)$ variate matrix, $\mathbf{a}$ is the stack vector of the elements in the lower triangular matrix $\mathbf{A}$. The prior distribution of $vec(\mathbf{B})$ is assumed to follow the Minnesota prior distribution, $\pi(vec(\mathbf{B})) \sim \mathcal{N}(\mathbf{b}_0, \mathbf{V}_{\mathbf{b}_0})$, with the overall shrinkage $l_1 = 0.2$ and the cross-variable shrinkage $l_2 = 0.5$, see Koop and Korobilis (2010). The prior distribution of $\mathbf{a}$ imposes a weak assumption of no interaction among endogenous variables, $\mathbf{a} \sim \mathcal{N}(0, 10I_{0.5k(k-1)})$. We assume that the degree of freedom for each variable follows a gamma distribution and the skewness parameter follows a standard normal distribution, i.e. $\nu_i \sim \mathcal{G}(2, 0.1)$ and $\boldsymbol{\gamma}_i \sim \mathcal{N}(0, 1)$. Due to the model settings, we let the mixing variables to be distributed as follows $\xi_{it}|\nu_i \sim \mathcal{IG}(\frac{\nu_i}{2}, \frac{\nu_i}{2})$. Lastly, the prior for the variance of shock to the volatility is $\sigma_i^2 \sim \mathcal{G}(\frac{1}{2}, \frac{1}{2})$, see Kastner and Frühwirth-Schnatter (2014).

We extend the Gibbs sampler to make inferences on model parameters. Let $\boldsymbol{\Psi}$ be a set of conditional parameters except the one that we sample from. We describe the Bayesian inference for the OST-SV model using a seven-step Metropolis-within-Gibbs Markov chain Monte Carlo (MCMC) algorithm as follows,

1. We sample $\pi(vec(\mathbf{B})|\boldsymbol{\Psi})$ from a conjugate normal distribution by rewriting Equation (3) as a multivariate linear regression,

$$\mathbf{y}_t - \mathbf{A}^{-1}\mathbf{W}_t\boldsymbol{\gamma} = \mathbf{B}\mathbf{x}_t + \boldsymbol{\Sigma}_t^{1/2}\boldsymbol{\epsilon}_t,$$

where $\mathbf{x}_t = (1, \mathbf{y}^{'}_{t-1}, \ldots, \mathbf{y}^{'}_{t-p})^{'}$ is $(1 + kp)$-dimensional vector and $\boldsymbol{\Sigma}_t^{1/2} = \mathbf{A}^{-1}\mathbf{W}_t^{1/2}\mathbf{H}_t^{1/2}$.

Then, $\pi(vec(\mathbf{B})|\mathbf{\Psi}) \sim \mathcal{N}(\mathbf{b}^*, \mathbf{V}^*_{\mathbf{b}})$ with

$$\begin{aligned}
\mathbf{V}^{*-1}_{\mathbf{b}} =& \mathbf{V}^{-1}_{\mathbf{b}_0} + \sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t^{'} \otimes \mathbf{\Sigma}_t^{-1}, \\
\mathbf{b}^* =& \mathbf{V}^*_{\mathbf{b}} \left[\mathbf{V}^{-1}_{\mathbf{b}_0} \mathbf{b}_0 + \sum_{t=1}^{T} (\mathbf{x}_t \otimes \mathbf{\Sigma}_t^{-1} (\mathbf{y}_t - \mathbf{A}^{-1} \mathbf{W}_t \gamma)) \right].
\end{aligned}$$

2. Similarly, the conditional posterior distribution of $\boldsymbol{\gamma}$ is a conjugate normal distribution. We rewrite Equation (3) as,

$$\begin{aligned}
\mathbf{A}(\mathbf{y}_t - \mathbf{B}\mathbf{x}_t) = \mathbf{W}_t \boldsymbol{\gamma} + \mathbf{W}_t^{1/2} \mathbf{H}_t^{1/2} \epsilon_t, \\
\pi(\boldsymbol{\gamma}|\mathbf{\Psi}) \sim \mathcal{N}(\boldsymbol{\gamma}^*, \mathbf{V}^*_{\boldsymbol{\gamma}}),
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{V}^{*-1}_{\boldsymbol{\gamma}} &= \mathbf{I} + \sum_{t=1}^{T} \mathbf{W}_t^{0.5} \mathbf{H}_t^{-1} \mathbf{W}_t^{0.5}, \\
\boldsymbol{\gamma}^* &= \mathbf{V}^*_{\boldsymbol{\gamma}} \left(\sum_{t=1}^{T} \mathbf{W}_t^{0.5} \mathbf{H}_t^{-1} \mathbf{W}_t^{-0.5} \mathbf{A}(\mathbf{y}_t - \mathbf{B}\mathbf{x}_t) \right).
\end{aligned}$$

3. We sample $\pi(\mathbf{a}|\mathbf{\Psi})$ based on Cogley and Sargent (2005). The conditional posterior of elements in vector $\mathbf{a}$ can be drawn equations by equations using a conjugate normal posterior distribution.

4. We sample $\pi(\mathbf{h}^{'}_{1:K,0:T}|\mathbf{\Psi})$ based on Kim et al. (1998); Primiceri (2005); Del Negro and Primiceri (2015). Let $\tilde{\tilde{\mathbf{u}}}_t = \mathbf{W}_t^{-1}(\mathbf{A}(\mathbf{y}_t - \mathbf{B}\mathbf{x}_t) - \mathbf{W}_t \boldsymbol{\gamma})$, for each series $i = 1, \ldots, k$, we have that $\log \tilde{\tilde{u}}^2_{it} = \log h_{it} + \log \epsilon_t^2$. Kim et al. (1998) approximated the distribution χ^2 of ϵ_t^2 using a mixture of 7 normal components. Then $\log h_{it}$ could be sampled using the forward filter backward smoothing algorithm in Carter and Kohn (1994).

5. We sample $\pi(\boldsymbol{\sigma}^2|\mathbf{\Psi})$ to using the independent Metropolis-Hastings algorithm (Kastner and Frühwirth-Schnatter, 2014) . We draw the proposed value $\sigma_i^{2(*)} \sim \mathcal{IG}(\alpha_i, \beta_i)$ with $\alpha_i = \frac{1}{2}T$ and $\beta_i = \frac{1}{2} \left(\sum_{t=1}^{T} (\log h_{i,t} - \log h_{i,t-1})^2 \right)$ and accept with the probability

$$\min \left\{ 1, \frac{\sigma_i^{(*)}}{\sigma_i} \exp \left(\frac{\sigma_i^2 - \sigma_i^{2(*)}}{2} \right) \right\}$$

6. We sample $\pi(\nu_i|\mathbf{\Psi})$ for $i = 1, \ldots, k$, using an adaptive random walk Metropolis-Hastings algorithm with the proposal value $\nu_i^{(*)} = \nu_i + \eta_i \exp(c_i)$, where $\eta_i \sim \mathcal{N}(0, 1)$ and the adaptive variance c_i is adjusted automatically such that the acceptance rate is around 0.25, see Roberts and Rosenthal (2009).

7. We sample $\pi(\xi_{it}|\mathbf{\Psi})$ for $i = 1, \ldots, k, t = 1, \ldots, T$ using an independent Metropolis-Hastings algorithm. We find the mode and inverse Hessian at the mode of the full conditional log-posterior of $\pi(\xi_{it}|\mathbf{\Psi})$. We draw a proposal value $\log(\xi_{it})$ from a t distribution with four degrees of freedom with mean equal to the mode and scale equal to the inverse Hessian at the mode, see Creal and Tsay (2015) and Nguyen et al. (2019).

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