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The Strategic Jump – The Order Effect on Winning "The Final Three" in Long Jump Competitions

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Abstract

The tournament rules for long jump competitions have changed in recent years. Today, only the three athletes with the best jumps from the five initial attempts are qualified to make an additional sixth jump – a format called The Final Three. In the first implemented version of The Final Three, the top athletes sequentially make one final jump, starting with the athlete ranked third place from the initial attempts. The athlete with the longest jump in this sixth attempt wins the competition, irrespective of achieved results in previous attempts. In this study, we analyze the effect of the athletes' jump order on the probability of winning the competition within this first implemented version of The Final Three. We derive the final's symmetric subgame perfect equilibrium and compute the corresponding equilibrium winning probabilities, given the values assigned to the distributional parameters. The modelling of the game is preceded by a development of a stochastic model for the outcome in long jumping. An athlete affects the distribution of the outcome by choosing where to start her approach run. Our results indicate a last mover advantage, albeit small. The athlete jumping last, wins the final with a probability 0.35, followed by the athlete jumping second with a probability 0.33 to win the final.

JEL Classifications: C49, C72, D81, Z20

Keywords: OR in sports, sequential game, order effect

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1. Introduction

A common argument for changing rules within a given sport is that the change will make the sport event more exiting or increase fairness. Wright (2014) and Kendall and Lenten (2017) provide surveys of changes in rules from various sports and their impact upon competitors' behaviour. Both papers present examples of when rule changes, aiming to increase excitement, have led to unexpected – and in some cases – unfair consequences. In this paper we add an additional sport to their list of examples: long jumping. To make the long jumping international competitions more exiting, a new tournament format, "The Final Three", was introduced in 2020. Given the number of prequalified contestants in a competition, the three highest placed athletes, after a sequence of attempts (in general five attempts, henceforth rounds), were qualified for the sixth round (the final henceforth). The medals among these three athletes were then distributed according to their achieved result in the final, irrespective of what distance they jumped in previous rounds. This rule differed from the former applied rule, where the winner was the one who made the single longest jump over all rounds in the competition. A consequence, albeit not unforeseen, of this change in tournament rule, is that the winner may not be the athlete who had the best overall result achieved in the competition. According to the organizer Diamond League, the motive for changing the rule was to award athletes who has the ability to perform under the most intense pressure, bringing more drama to the competition as nothing would be decided until the last jump.[1] Alongside the criticism that the new rule was unfair because the first five rounds by the finalists become void in the final, objections were also raised because the rule weakened the athlete's incentive to perform her best in the first five rounds, since ending up in third place was as good as ending up in first place in order to reach the final. Following the consultation with athletes, coaches and meeting organizers, the Wanda Diamond League General Assembly decided in December 2021 to revise The Final Three format.[2] The properties of this revised Final Three format and its expected impact on fairness and excitement are discussed in the last section in this paper.

Even though the format of The Final Three, as described above, likely is to be put aside in the future, the format's properties are nevertheless interesting to analyse. There will probably always be a debate about how to build drama and excitement into the long jumping competition, which in its traditional format may be perceived by some as an endlessly repetitive jumping. After all, the format of the The Final Three bears a resemblance of the format applied today in running tournaments at track and field, where runners qualify to a final in a sequence of elimination heats. In this paper we try to shed some light on the question how beneficial it is for an athlete for having achieved the best jump in the five first rounds to win The Final Three. By analysing the strategic interaction between the finalists, we try to predict an athlete's probability of winning the final, given her ranking from the initial five rounds.

[1] See https://athleticsweekly.com/athletics-news

[2] See https://world-track.org/2021/12/wanda-diamond-league-final-3-format-for-horizontal-jumps-revised-2/

The design of The Final Three suggests the athlete's placement after five rounds could have a direct impact on the probability of winning the competition. There are two reasons for this assertion. First, in case of a tie in the final, the best performance from the previous five rounds separates the athletes. Second, the mutual placement of the finalists after five rounds determines the order in which the finalists jump in the final. The athlete who is placed third after five rounds begins, the second-best performer is next, while the leading athlete after five rounds finishes the final. This order suggests an advantage for the leader after five rounds, since it is possible to adapt the risk of making a foul, an illegal result, depending on the other two finalists' performance in the final. Likewise, it could be argued that the third placed athlete has a disadvantage of going first in that no results of the rivals yet exist.

We believe the jumper adapts this risk by the position of the distance marker, from where the jumper starts the approach run. By moving the marker backwards, the risk of making a foul will obviously decline. However, this comes with the cost of lowering the probability of a good result. The reason for that is that the takeoff foot, most likely, will be far from the official take-off line, from which the distance of the jump is measured. Thus, there is a trade-off between having a low risk of making a foul and a high probability of making a good result.

The way the Final Three format was designed, in combination with the elements of strategic thinking in long jump, i.e., deciding on where to place the distance marker, enables a game theoretic approach to analyse the impact of the finalist's position after five rounds - first, second or third - on the probability to win the final. Such an approach could also highlight the importance of the dimension of strategic thinking in this type of format.

In the paper we first present a stochastic model for the outcome of a long jump, where the derived distribution of the outcome depends on the position of the distance marker. Second, based on this stochastic model, a sub-game perfect equilibrium is characterized, and equilibrium win probabilities for the three finalists in The Final Three are derived. Here, the competition is viewed as a non-cooperative rational strategic sequential-move game where the three finalists are players with asymmetric information about the achieved outcomes of other players' moves.[3] Their decision making is interdependently related and all of them aim to maximize the probability of winning the final by making a strategic choice of the position of the distance marker. Third, the model is used to analyse

[3] The old rule also implies the use of backward induction to solve for an optimal jumping strategy. However, the presence of more attempts for a jumper to ensure a legal jump, and the possibilities to improve the distance achieved in previous attempts, creates a very cumbersome sequential decision structure which is hard to capture in a model.

the importance of the strategic dimension within this format, in that a poor strategic choice might decline the win probability to a certain amount.

The rest of the paper is organized as follows: In Section 2 a stochastic model for the result of a long jump is presented. Section 3 provides a literature overview. In Section 4 equilibrium win probabilities are derived, while the findings in the paper are discussed in Section 5.

2. Literature

The first part of our work is related to the literature on modeling the jumper's problem in the long jump competition to decide her optimal aimed take-off line, an issue analyzed in some early papers. In Ladany et al. (1975), a general model is developed to determine the aimed optimal take-off line which maximizes the jumper's expected distance in a competition with three rounds. In their model, the jumper's choice of take-off line is not conditioned on the jumps of her competitors. Given their collected data from ten consecutive training sessions of an athlete, they solve the problem by simulation of a specific case. Assuming that both the real jumping distance, and the distance between the tip of the take-off foot and the take-off line aimed for, are normally distributed, that is, neglecting that the later distribution is truncated, the authors derive an analytical solution, which provides a valid approximation of the simulated result. A similar model is applied in Ladany and Singh (1978) in which the jumper's objective is to maximize the probability of jumping a given distance. This policy is then compared with the case when the objective is to maximize the expected distance jumped. Using the results from the training sessions obtained in the previous study by Ladany et al. (1975), the finding is that the later policy – to jump the longest jump – is inconsistent with the desire to maximize the probability to jump a given distance. Ladany and Mehrez (1987) extend the problem of maximizing the probability of jumping a given distance in a single jump to a 3-trial competition case. They show that the optimal location of the aimed take-off line, maximizing the probability to jump a given distance in a single jump, also maximizes the same probability in the 3-trial case. However, to prevent erroneous values of the optimal location of the aimed take-off line due to the aforementioned truncation of the distribution of the distance between the tip of the take-off foot and the aimed take-off line, they solve the maximization problem by simulation, leaning on the estimates from Ladany et al. (1975). Ladany and Mehrez (1987) find that when the targeted distance is relatively short in relation to the jumper's ability, the jumper can select an aimed take-off line more far behind the official take-off line and still enjoy a probability close to 1 of success. Conversely, when the targeted distance is relatively long, a deviation from optimal aimed take-off line, closer to the official take-off line, has a little effect upon the very low probability of success.

The contribution of our work to this literature is an extension of the model by Ladany and Mehrez (1987). We derive a probability distribution of the jumper's score which captures the fact that one of the underlying distributions – the distance between the tip of the take-off foot and (in our model) the foul line – is truncated.

The second part of our paper relates to the relatively large body of literature on the design of sporting contests and competitors' performance (for a survey, see Szymanski 2003). More precisely, the structure of The Final Three in the long jumping final, directs our interest towards the literature on the effect of the order of actions on performances in sports with sequential moves. Several factors are put forward to explain the observed differences in competitors' performances, e.g., psychological pressure, learning effects, cognitive effects, and strategic interaction. The observed performance in penalty shootouts in soccer, where the kicks are taken alternately by the two teams - *A* and *B* - according to *ABAB*, has attracted attention in a row of papers. A survey of empirical studies on penalty shootouts, provided in Csató (2021), indicates that there exists a first-mover advantage of kicking first due to the mental pressure the player taking the second kick is put under. Hence, even if this order of kicks is ex ante fair by randomly determine which team starts the shootout, the order is not ex post fair. Albeit not largely tested in soccer, the Prouhet-Thue-Morse sequence of order, *ABBA,* has been considered being a fairer alternative to the standard shootout format (see, e.g., Palacios-Huerta 2012, Brams and Ismail 2018, Vandebroek et al. 2018, Anbarcı et al. 2021, Csató 2021, Lambers and Spieksma 2021). Penalty shootouts are also applied in many ice-hockey leagues to determine the winner if a match is still drawn following overtime. The shooting is in general ordered *ABAB*, where the home team decides who will start the shootout. Kolev et al. (2015) analyse data from shootouts from the National Hockey League and find a lower winning frequency for the team shooting first. The probability of scoring a goal from a penalty shot in hockey is much lower than it is in soccer. Therefore, it is more likely that the team taking the first shot will be lagging when the second team shoots, giving the second team an advantage. The serves in tennis tiebreak have an order close to the Prohuet-Thue-Morse sequence. Analysing data from larger tennis tournaments, Cohen-Zada et al. (2018) find no systematic advantage to any player serving first or second in tiebreak.

Brady and Insler (2019) analyse recorded data from the PGA tours on playing partners in golf to examine whether there is an order effect when both players' golf balls are positioned relatively close to each other when playing from fairway or putting on green. They find evidence of a second-mover advantage, which they motivate is consistent with a learning effect. Krumer et al. (2017) analytically show that competing in the first round and in the last round in a round-robin tournament with three symmetric competitors is more advantageous than any other ordering. Winning in the first round affects the continuation values of the competitors, providing the winner a higher continuation value of winning than her opponents. Krumer and Lechner (2017) find this prediction to be in line with their analysis of the observed outcome in Olympic wrestling and in UEFA/FIFA soccer cups, in which the group stages are arranged as round-robin tournaments. González-Díaz and Palacios-Huerta (2017) show that most chess matches - where two players play an even number of games against each other, altering the colour of pieces – is won by the player playing with the white pieces in the first game. Playing with the white pieces in chess confers a strategic advantage. Therefore, the player playing with the black pieces in the first game is more likely to lag in the match and thus being at a psychological disadvantage, affecting the player's cognitive performance.

In the sports referred to above, a competitor's best response to an outcome, caused by her opponent's move in the sequential game, is always to try to shoot a goal, hit the ball in the hole, win a new match, etc., irrespectively of her opponent's outcome. However, in the long jumping final, a competitor will condition her best response on the information she has about her opponents' performances, by optimally positioning her distance marker.

A similar situation can be found in many running tournaments at track and field events, where runners first compete in a sequence of qualification heats prior to the final. A runner advances to the final either by being among the top placed runners in her heat or having one of the fastest times of those who did not advance by place, regardless of the heat ("a lucky looser"). Therefore, in a sequence of qualification heats, runners in later qualification heats are better informed about times required to reach the final than are runners in earlier heats. Hill (2012) analyses data on runners' performance in International Association of Athletics Federations (IAAF) 5,000-meters events, held 2001-2011, where runners compete in two separate qualification heats. Given the runners' asymmetric information, the analysis does not support the hypothesis that there is an advantage or a disadvantage of competing in either of the two qualification heats.

Facing a similar situation with asymmetric information in long jumping competitions, we in this study adopt a game theoretical approach to examine how the ordering of the three jumps in the final affects a competitor's probability of winning. By deriving each jumper's choice of expected distance from the tip of the take-off foot to the official take-off line, given her conjecture/information about the competitors' scores, we characterize a sub-game perfect equilibrium of this game.

3. A Stochastic Model for the Result of a Long Jump

In this section a stochastic model for the outcome of a long jump will be presented, taking the effect of the athlete's attitude towards risk of making a foul jump into account. The athlete masters the risk by changing the position of the distance marker indicating the start of the approach run towards the official take-off line (henceforth the foul line), from which the athlete tries to make as long a jump as possible before landing inside a sandpit.

For a jump to be legal the toe of the athlete's shoe needs to be behind the foul line while launching. Otherwise, if the toe crosses the foul line, the athlete has made a foul jump that doesn't count. Long jumps are measured from the foul line to the impression in the sandpit, closest to the take-off board, made by any part of the athlete's body, while landing.

For a certain athlete, X is defined to be the distance of the approach run, where X is assumed to follow a $N(\mu_X, \sigma_X^2)$. We define $\gamma + \mu_X$ to be the distance from the marker to the front edge of the foul line. Thus, the distance from the point of take-off to the foul line, denoted by T, can be written as $T = \gamma + \mu_X - X$, where a fair jump requires $T \geq 0$, while $T < 0$ defines a foul jump. It follows that $T \sim N(\gamma, \sigma_X^2)$.

Furthermore, we define Y to be the entire distance of the jump, from the point of take-off to the first landing point of the athlete. Y is assumed to follow a $N(\mu_Y, \sigma_Y^2)$. In addition, define $V^* = Y - T$. Now, the score V is defined by

$$V = \begin{cases} V^* & if \;\; T \geq 0 \\ foul & otherwise. \end{cases}$$

Thus, V can be considered a mixed random variable with a continuum of positive[4] real values as possible outcomes as well as a qualitative outcome.

Assuming Y and T to be independent, the pdf of $V^* | \, T \geq 0$, denoted by $f_{V^*|T\geq 0}(v^*)$, can be shown to be

$$f_{V^*|T\geq 0}(v^*) = \frac{\exp\left(-\frac{(\mu_Y - v^* - \gamma)^2}{2(\sigma_X^2 + \sigma_Y^2)}\left(1 + \operatorname{erf}\left(\frac{(\mu_Y - v^*)\sigma_X^2 + \gamma\sigma_Y^2}{\sqrt{2}\sigma_X\sigma_Y\sqrt{\sigma_X^2 + \sigma_Y^2}}\right)\right)\right)}{\sqrt{2\pi}\left(1 + \operatorname{erf}\left(\frac{\gamma}{\sqrt{2}\sigma_X}\right)\right)\sqrt{\sigma_X^2 + \sigma_Y^2}},$$

where erf $(\cdot)$ is the error function.

The probability of making a foul is given by $1 - \Phi(\frac{\gamma}{\sigma_X})$, where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal. Consider the function

$$\mathrm{g}(v^*) = \Phi\left(\frac{\gamma}{\sigma_X}\right) f_{V^*|T\geq 0}(v^*).$$

The probability of having a score better than or equal to v_0^* is

$$P(V \geq v_0^*) = \int_{v_0^*}^{\infty} g(v^*)dv^*.$$

In Figure 1 the function $\mathrm{g}(v^*)$ is illustrated graphically for $\mu_Y = 8.00$, $\sigma_X = \sigma_Y = 0.1$ and for two different values of γ ($\gamma = 0$ thin line; $\gamma = 0.20$ thick line), where metre is the unit of measurement. Note that the function $\mathrm{g}(v^*)$ is not a probability distribution function and the area under the graph is

[4] Theoretically, using our model it is possible for V to take on negative values. However, for parameter values representing real-world athletes the probability for that to occur is negligible. Henceforth, we will therefore think of v^* as a value always larger than zero. Also, we define a score of v^* to be larger than a foul.

smaller than 1. For the thin curve the area is smaller than for the thick curve, resulting from a larger probability of making a foul.

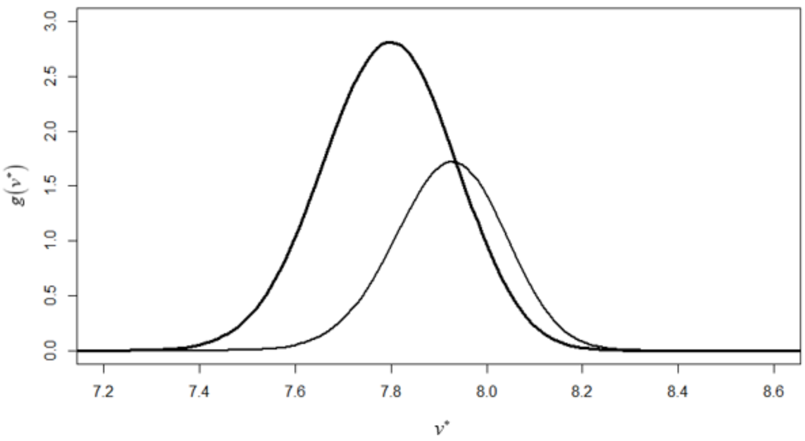

Figure 1. Graphical illustration of $g(v^*)$, *the continuous part of the distribution for the mixed random variable V for* $\mu_Y = 8.00$, $\sigma_X = \sigma_Y = 0.1$ *and for two different values of* γ *(*$\gamma = 0$ *thin line;* $\gamma = 0.20$ *thick line).*

An athlete choosing a value of $\gamma = 0$ is willing to take a large risk of making a foul, here $P(foul) = 0.5$, to have a reasonable chance of making a good score, say better than or equal to 8.00. A value of $\gamma = 0.20$ means the athlete's risk of making a foul is small, here 0.023, and from Figure 1 it should be evident that the probability of making a descent result, say 7.70 or better, is quite large compared to the case where $\gamma = 0$. However, this comes with the cost of lowering the chance of reaching a very good result, since for $\gamma = 0.20$ the probability of getting a result better than or equal to 8.00 is smaller than for $\gamma = 0$. Thus, by choosing the value of γ, i.e., by choosing the position of the distance marker, the athlete balances the trade-off between the risk of making a foul and the chance of a long legal jump.

4. Deriving Equilibrium Win probabilities

This section takes the model presented in Section 3 as a benchmark when analysing the sequential game and calculating equilibrium win probabilities for each of the three competitors.

4.1 Modelling the Sequential Game

The competitors (henceforth players) are denoted by A, B and C, where the order in which they jump is descending. We assume they all aim to maximize the probability of winning the competition by making a strategic choice of the value of γ. It is assumed that each player has full knowledge about his own capacity as well as the two opponents' capacity in terms of individual specific distributions of X and Y along with V, being introduced above. [5] In addition, we assume all players to be aware of each other's goal to maximize the probability of winning.

Figure 2 presents the ordering of players' jumps as a game tree. Player C starts jumping and makes either a foul, f_C, or scores $\overline{V}_C$. Then, at each of the nodes B_2, A_1, A_2, and A_3 there exist three possible outcomes of a player's jump: (*i*) a foul (*f*); (*ii*) a legal jump scored lower than the leading score $(\underline{V})$; (*iii*) a legal jump scored higher than the leading score $(\overline{V})$. At node B_1, the jump by player B will either be a foul or a legal jump with a leading score. The game contains seven subgames of which one has a trivial solution. If player C and player B make fouls, player A automatically wins the final irrespectively of her achieved result (A_0). The last row indicates the winning player.

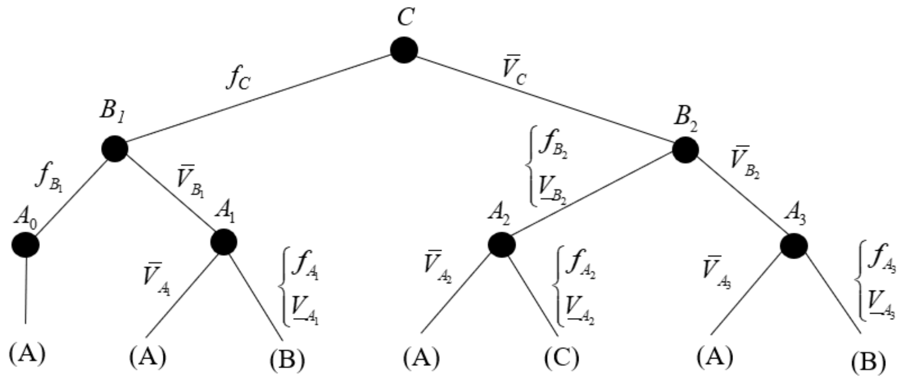

Figure 2. The ordering of jumps and possible outcomes in the long jumping final

To solve for the combination of the three players' strategies that are a best response to each other, i.e., the equilibrium in such a game, the method of backward induction will be used.

Therefore, a stagewise solution of the equilibrium probabilities is performed. A rough description is as follows. First, the strategy of A is obtained, given the best score after the first two jumps by C and B, where at least one of these jumps is legal corresponding to the nodes A_1, A_2 and A_3 in Figure 2.

[5] The assumption of all players having full information about the opponents' capacity is just a sufficient condition for the validity of the derivation of win probabilities. It is not necessary for the player jumping last to have this kind of information about the other two players, and for the player who jumps second only information about the capacity of the player who finishes the final is crucial.

Second, the strategy of B is derived. Two separate cases are examined depending on C has a legal jump or not (nodes B_1 and B_2). For both cases information about A's strategy from the first stage is considered as well. Third, we obtain the strategy and calculate the equilibrium win probability of C considering the response from B and A described in stage 2 and 1, respectively (node C). Fourth, the equilibrium win probability of B is derived, by considering the two separate cases described in stage 2. Finally, in the fifth stage, A's equilibrium win probability is derived.

In parallel to the theoretical description of how to obtain the equilibrium win probabilities, we provide a numerical example to illustrate. In this example all three finalists are assumed to have identical capacities, with parameters $\mu_Y = 8.00$ and $\sigma_X = \sigma_Y = 0.1$, all measured in metres. The parameter values are set, in discussion with a long jump coach, to represent the capacity of a male world class long jumper. The choice of having identical capacities is the primarily interesting case, since we would like to isolate the effect of the order in which the three finalists jump.

4.2 Jumping Last - The Strategy of Player A

Player A observes the achieved result by player C and player B and will condition her strategy on the best score achieved among these two players. As mentioned above, no strategy is needed for player A to win the final, if the other two players make fouls. Denote the best score from the previous jumps by v_0^*. Now, the probability for A to end up as the winner can be written as

$$p_{A|\gamma_A} = P(V_A \geq v_0^*|\gamma_A) = \int_{v_0^*}^{\infty} g(v_A^*|\gamma_A)dv_A^*$$

where the probability depends on three fixed parameters; μ_Y, σ_X, and σ_Y, and a parameter γ_A, whose value can be affected by A by changing the position of the distance marker. Facing v_0^*, A chooses the value of γ_A that maximizes the probability of getting a score better than or equal to v_0^*. That is, the maximization problem

$$\max_{\gamma_A} \int_{v_0^*}^{\infty} g(v_A^*|\gamma_A)dv_A^*$$

is solved for given values of μ_Y, σ_X, and σ_Y, and for different values of v_0^*. For $\mu_Y = 8.00$ and $\sigma_X = \sigma_Y = 0.1$, the solution to the problem in terms of γ_{Aopt}, the optimal choice of γ_A for different values of

v_0^* is represented by small dots in Figure 3 below, and the corresponding win probabilities, denoted by p_{Aopt}, are shown as large dots[6].

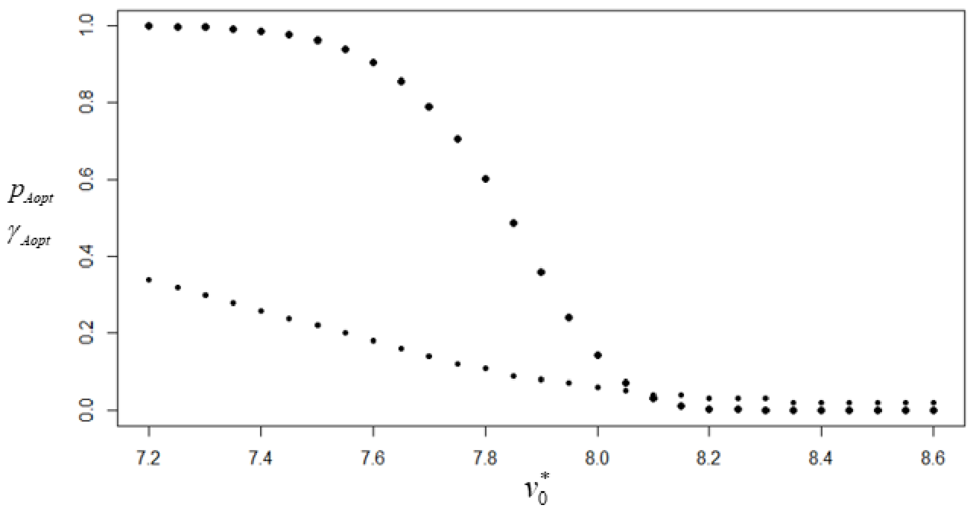

Figure 3. The relation between γ_{Aopt} *and* v_0^* *(small dots) and corresponding win probabilities (large dots)* for $\mu_Y = 8.00$ and $\sigma_X = \sigma_Y = 0.1$. Note that the scale on the y-axis represents units of measurements for metres as well as for probabilities.

Figure 3 reveals that if A is to beat 7.60, for example, his best choice of γ is 0.18, resulting in a win probability slightly above 0.90. Instead, if the best score to beat is as good as 8.00, he needs to gamble in the sense that the expected distance from the point of take-off to the foul line, i.e., γ_A, be small. Here, $\gamma_A = 0.06$ does the trick to maximize the win probability, yielding a value for p_{Aopt} of about 0.14.

4.3 Jumping Second - The Strategy of Player B

Player B needs to be prepared to be able to solve two maximization problems, depending on C makes a foul jump or not.

Define the implicitly given function $p_{Aopt} = h(v_0^*)$, presented as the solution to player A's maximization problem, given in the previous subsection. Starting with the case of a foul jump made by player C, i.e., player B's optimal choice is conditioned only on her conjecture about player A's choice of strategy, the probability that player B wins the final given γ_B, denoted by $p'_{B|\gamma_B}$, can be written as

[6] Some comments: The midpoint rule with a rectangular width of 0.01 has been applied to solve definite integrals numerically. Calculations are done for scores in between 6.90 and 8.90, since a score outside that interval is almost impossible to get. Also, we have considered measurements in the long jump are rounded below to the nearest centimeter. Therefore, our midpoints are in the middle of two integer centimeters. Finally, we restrict choices of γ to be integer centimetre values, explaining the stepwise pattern in the graph. Determining γ_{Aopt} with increasing accuracy would finally result in a strictly negative pattern.

$$p'_{B|\gamma_B} = \int_0^{\infty} (1 - h(v_B^*))g(v_B^*|\gamma_B)dv_B^*.$$

For our numerical example the integral above is solved numerically and in Figure 4 the win probability for B for different choices of γ_B is presented.

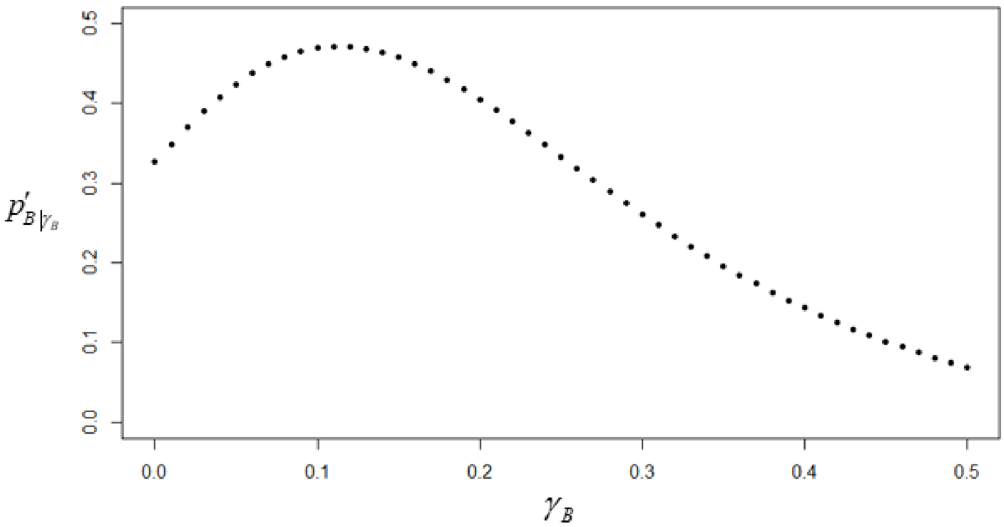

Figure 4. The probability of player B winning the final for different choices of γ_B, conditioning on player C makes a foul jump and player A maximizes his chance of winning.

The maximum probability for *B* to win is about 0.472 attained at $\gamma_B = 0.12$. These values are the solution to the maximization problem

$$\max_{\gamma_B} \int_0^{\infty} (1 - h(v_B^*))g(v_B^*|\gamma_B)dv_B^*$$

and are denoted by p'_B and γ'_B, respectively. The figure also reveals the importance of being strategic to increase the chance of winning. For example, a poor choice of γ_B, say $\gamma_B = 0.01$, reduces the chance of winning by 0.12, compared to a perfectly strategic choice, since the win probability drops to 0.35.

Now, we turn to the case when player *B* also must respond to the score achieved by player *C*, besides conjecturing the strategic behaviour by player *A*. Let V_{Aopt} be the score made by player *A*. Then, the probability that player B is the winner, given v_C^* and γ_B, denoted by $p_{B|\gamma_B,v_C^*}$, can be written as

$$p_{B|\gamma_B,v_C^*}$$

$$= P(V_B \geq v_C^*, V_B > V_{Aopt}|\gamma_B)$$

$$= P(V_B \geq v_C^*|\gamma_B)P(V_B > V_{Aopt}|\gamma_B, V_B \geq v_C^*)$$

$$= P(V_B \geq v_C^* | \gamma_B) P\left(V_B^* > V_{Aopt} \middle| \gamma_B, V_B^* \geq v_C^*\right)$$

$$= \int_{v_C^*}^{\infty} g(v_B^* | \gamma_B) dv_B^* \frac{1}{1 - F_{V_B^*}(v_C^* | \gamma_B)} \int_{v_C^*}^{\infty} (1 - h(v_B^*)) f(v_B^* | \gamma_B) dv_B^*,$$

where $F_{V_B^*}(\cdot)$ is the distribution function of V_B^*.

Figure 5 shows the solution to the maximization problem

$$\max_{\gamma_b} p_{B|\gamma_B, v_C^*}$$

for our numerical example.

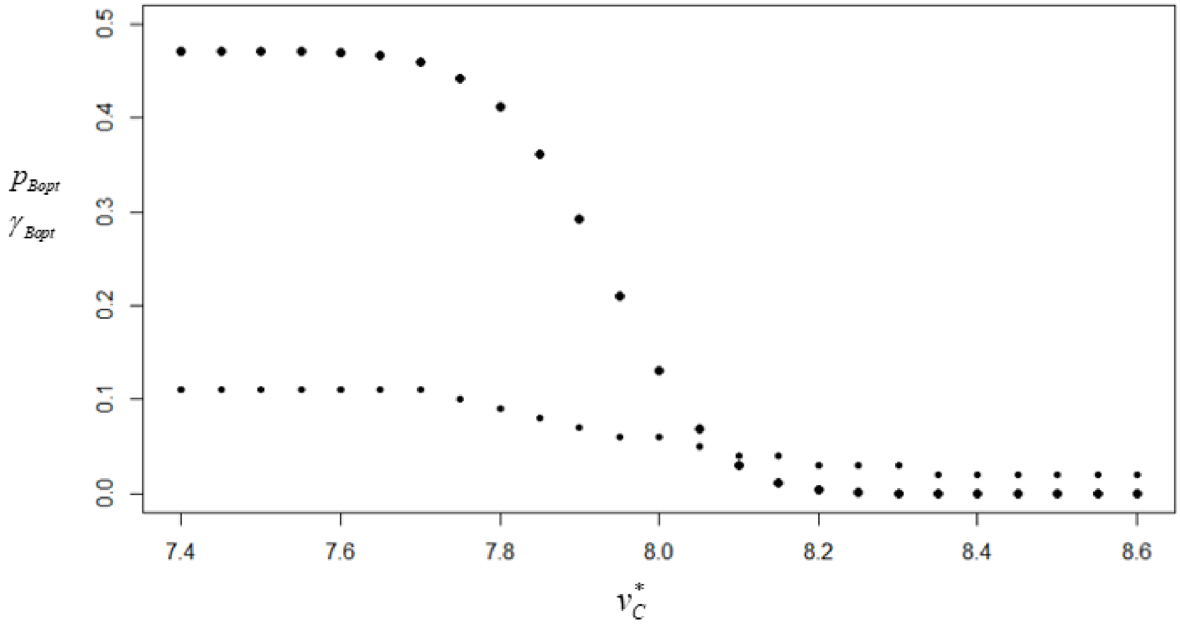

Figure 5. The relation between γ_{Bopt} and v_C^* (small dots) and corresponding win probabilities (large dots) for $\mu_Y = 8.00$ and $\sigma_X = \sigma_Y = 0.1$. Note, as in figure 3, that the scale on the y-axis represents units of measurements for metres as well as for probabilities.

We represent γ_{Bopt}, the optimal choice of γ_B, for different values of v_C^*, with small dots. The corresponding win probabilities, denoted by p_{Bopt}, are shown as large dots.

Not surprisingly, the better score made by C, the larger risk of making a foul jump is player B willing to take, to maximize the probability of winning, as γ_{Bopt} is decreasing in v_C^*.

4.4 Jumping First - The Strategy of Player C

Let V_{Aopt} and V_{Bopt} be the scores made by player A and player B, respectively. In addition, define the implicit function $s(v_C^*)$, the relation between the probability that player B beats player C and the score by player C, v_C^*. Then, the probability of player C being the winner given γ_C, denoted by $p_{C|\gamma_C}$, can be written as

$$p_{C|\gamma_C}$$

$$= P\left(V_C > \max\left(V_{Aopt}, V_{Bopt}\right) |\gamma_C\right)$$

$$= \int_0^\infty (1 - s(v_C^*))(1 - h(v_C^*))g(v_C^*|\gamma_C)dv_C^*$$

The integrals are solved numerically for our numerical example and in Figure 6 the win probability for player C for different choices of γ_C is shown.

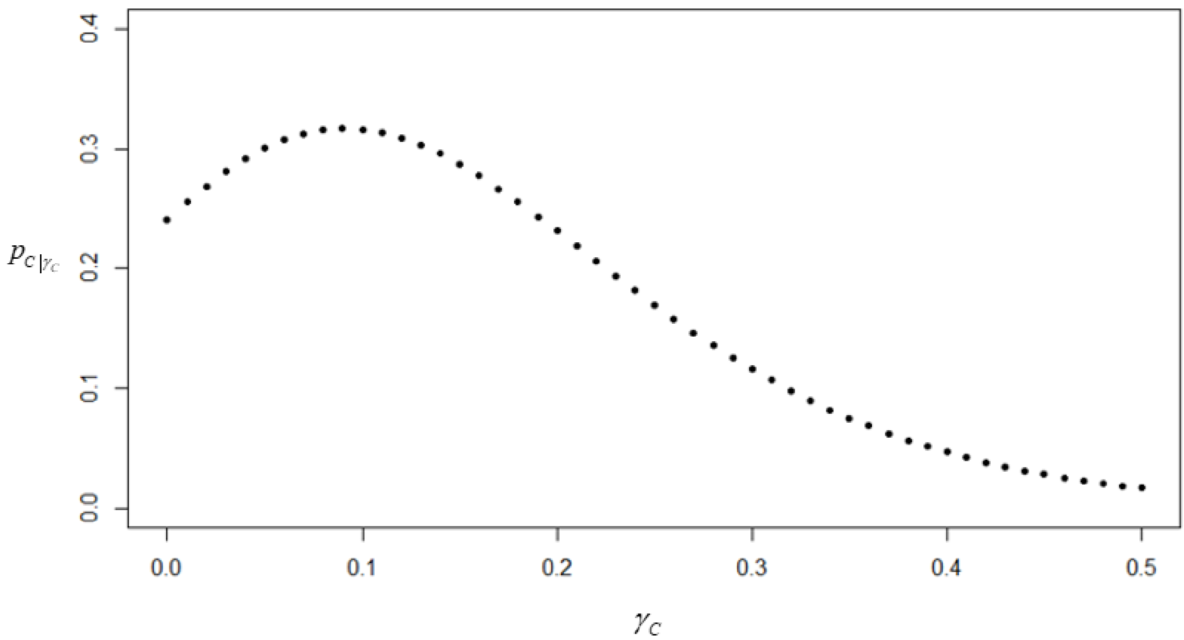

Figure 6. The probability of C winning the final for different choices of γ_C.

The maximum probability for player C to win, the equilibrium win probability p_C, is about 0.317 attained at $\gamma_C = 0.09$, and denoted by γ_{Copt}. Thus, there is a minor disadvantage of going first, about 1.5 percentage points lower probability of winning compared to the case where the probability of winning is the same for all three finalists. Figure 6 also highlights the importance of player C acting strategically. A deviation from $\gamma_C = 0.09$ might substantially lower her win probability.

4.5 The Equilibrium Win Probability of Player B

As illustrated in Figure 2 B is the winner if either C makes a foul jump and B has a better score than A, or if C makes a legal jump and B has at least as good a score as C and a better score than A. Thus p_B, the equilibrium win probability for player B, can be written as the sum of two probabilities. By defining the implicitly given function $p_{Bopt} = r(v_C^*)$ presented in section 4.3 as the solution to player B's maximization problem when player C has scored, we get

$$p_B = \left(1 - \Phi\left(\frac{\gamma_{Copt}}{\sigma_{X,C}}\right)\right) p_B' + \int_0^\infty r(v_C^*) g\left(v_C^* \middle| \gamma_{Copt}\right) dv_C^*.$$

For our numerical example the first term is 0.0868 (0.184×0.472), while the second term is 0.244. This means that $p_B = 0.331$, a somewhat larger equilibrium win probability than we got for player C.

4.6 The Equilibrium Win Probability of Player A

Finally, the win equilibrium probability for player A is given by $p_A = 1 - p_B - p_C$. For our numerical example we get $p_A = 1 - 0.331 - 0.317 = 0.352$. Thus, there is a last mover advantage in this play.[7]

The event that player A is the winner can occur in four mutually exclusive ways as illustrated in Figure 2, meaning that p_A can be expressed as the sum of four probabilities. Therefore, even though all equilibrium probabilities are derived, for completeness, we split p_A into these four different parts.

First, A is the winner if player C and player B both make a foul jump. Denote the probability for that event by p_0. We get

$$p_0 = \left(1 - \Phi\left(\frac{\gamma_{Copt}}{\sigma_{X,C}}\right)\right)\left(1 - \Phi\left(\frac{\gamma_B'}{\sigma_{X,B}}\right)\right).$$

Another possibility for player A to win is that player C makes a foul jump and player B has a legal score no better than player A. The probability for that event to occur, denoted by p_1, is

$$p_1 = \left(1 - \Phi\left(\frac{\gamma_{Copt}}{\sigma_{X,C}}\right)\right) \int_0^\infty h(v_B^*) g(v_B^* | \gamma_B') dv_B^*.$$

[7] The equilibrium probabilities are quite robust to deviations from the chosen parameter values. For example, for different combinations of σ_X and σ_Y, where both parameters are in between 0.05 and 0.2, the equilibrium probabilities do not differ much from those presented in the text, at most less than 0.01 units for a single probability. The last mover advantage increases slightly with σ_X, while there is a small decrease associated with a larger value of σ_Y. Using the results from the training sessions obtained in the study by Ladany et al. (1975), where $\sigma_X = 0.075$ and $\sigma_Y = .2044$ yields $p_A = 0.325$, $p_B = 0.333$ and $p_C = 0.343$, there still exists a last mover advantage.

A third possibility for player A to win is that player B makes a foul jump and player C has a legal score no better than player A. We get, by denoting the probability for that event by p_2,

$$p_2 = \left(1 - \Phi\left(\frac{\gamma_{Bopt}}{\sigma_{X,B}}\right)\right)\int_0^\infty h(v_C^*)g\left(v_C^*\middle|\gamma_{Copt}\right)dv_C^*.$$

Finally, player A wins if both player B and player C make legal jumps no longer than player A. The probability for that event, denoted by p_3, is given by

$$p_3 = \int_0^\infty\int_0^\infty h(\max(v_C^*, v_B^*))g\left(v_C^*\middle|\gamma_{Copt}\right)g(v_B^*|\gamma_{Bopt})dv_C^*dv_B^*.$$

For our example we get $p_0 = 0.025$, $p_1 = 0.072$, $p_2 = 0.065$, and $p_3 = 0.190$.

4.7 The Importance of having a Low Variation in the Length of the Approach Run

Apart from calculating equilibrium win probabilities, this modelling approach also enables us to examine the necessity in this type of tournament of having a well-adapted approach run, in terms of low variation in its length.

To shed light on this issue, suppose that $\sigma_{X,A}$ is increased to 0.2, while $\sigma_{X,B}$ and $\sigma_{X,C}$ are kept to 0.1. This results in a drop of the equilibrium win probability for A by 0.135 units, from 0.352 to 0.217. Not surprisingly we see an increase in the win probabilities for B and C, from 0.331 and 0.317 to 0.404 and 0.379, respectively. Corresponding changes of $\sigma_{X,B}$ and $\sigma_{X,C}$, one at the time, result in similar effects.

The intuition behind this effect is that a player, with a high standard deviation in the length of the approach run, will compensate for this shortcoming by moving the distance marker backwards, so that the probability of a foul does not become too great. This action will inevitably result in a shorter expected legal jump.

5. Discussion

To what extent a long jumping competition has become more exciting after the introduction of a new tournament format is a not a matter of question in this study. Yet, the original design of The Final Three has features of a tournament with a play-off: after a series of rounds – in which competitors may perform very mixed results – the competitors' achieved scores are compared, and the contestants with the highest scores then battle for winning in a one-shot game. Unless the two first jumpers in the final have not fouled, the outcome of the final hinges on the last jump.

The single decisive jump in the final can be said to amplify two features in a long jump competition. First, the one-shot round generates a more transparent strategic interaction between the athletes than was the case in the old tournament format, where winning the competition was conditioned on the competitor’s best achievement in six rounds. Second, athletes may experience more psychological pressure being aware of that they now only have one chance to beat a rival’s prior or conjectured performance. Pending access to larger datasets, covering results from long jumping competitions, we in this study leave the second issue on psychological pressure in long jumping finals. Instead, we have focused on the question what effect the sequential order of jumps has upon each athlete’s probability of winning the final. For this purpose, we first extend previous probability models on long jumping, e.g., Ladany (1987), by developing a stochastic model for the outcome in long jumping which captures the truncation of one of the underlying probability density functions. Assuming athletes having equal capacities, i.e., symmetric players, we make use of backward induction to analyze the subgame perfect equilibrium in this sequential game. Given the values we assigned to the distributional parameters, we find the equilibrium probabilities of winning the final when jumping first, second and last to be 0.317, 0.331, and 0.352 respectively. Hence, the athlete who achieves the best score in the initial rounds prior to the final, will have a relatively small advantage over the other athletes of jumping last.

In the light of the competitor’s small difference in favor of being the best jumper in five rounds, it is easy to understand the claim that this new tournament format is not as fair as the older tournament format. Since the advantage of jumping last in the final rests on an ordinal ranking of each competitor’s best achieved result from the earlier rounds, our relatively small, estimated reward of jumping last in the final may appear insufficient to compensate an athlete for having made an outstanding performance in one or in several of the five rounds prior to the final.

However, it should be emphasized, that the sequential one-shot jumping in the final implicates that the component of strategic interaction is lifted out of the last jumper’s decision process. Unlike her competitors, whose optimal strategies are based on their prior information about the capacities of the subsequent jumper(s), the last jumper’s optimal choice - where to position her distance marker - does not require such information. Her decision problem will only be to maximize the probability to score at least a given distance. If her competitors’ priors are wrong, our derived result of the last jumper's probability of winning the final, likely forms a lower estimate of the probability to win. As indicated by our numerical analysis, small deviations from the optimal choice of the parameter γ set by the other two finalists, e.g., due to misjudgments of the prior of the last jumper’s capacity, significantly reduce their probabilities of winning the final.

To facilitate our analysis, it is assumed that each athlete's objective is to win the final, that is, we have not incorporated athletes' placing considerations when formulating their strategies. It goes without saying, that the presence of prize money in long jumping competitions likely affects the athletes' choice of strategies. In such case, an analysis would be based on modelling athletes' expected utility and alternative attitudes towards risk, which is a challenge for future research.

Our prediction of a last mover advantage in The Final Three rests on two effects. First, in case of a tie in the final, the best performance from the previous five rounds separates the athletes. Second, the athlete jumping last will, unlike her competitors, condition her jump solely on the other two finalists' actual performance in the final. However, we have not separated the two effects in our analysis. Developing approaches to split up these two effects is a theme for further research.

As mentioned in Section 1, The Final Three format has been revised as this paper is written and the new format is planned to be implemented in upcoming seasons. In its revised format, two major changes are to be seen. It remains that only the top three jumpers after five rounds will get a sixth attempt, but a jumper's achieved results from all her sixth jumps will be counted when distributing the medals among the three finalists. Also, the order in which the three remaining athletes jumps in the sixth round is redrawn, where the best-placed athlete after round five goes first, followed by the athlete placed second and third, respectively. These changes are interesting, and they invite us to reflect on how they may affect fairness and excitement in the long jumping final.

The first change – the inclusion of the best result from all sixth attempts when ranking the athletes - is likely motivated by fairness considerations. Increasing the number of attempts, will increase the probability that the jumper with the highest capacity wins the final. The change also places less emphasis on the importance of strategic behavior in the final, i.e., the positioning of the distance marker. Given the achieved score after the initial five rounds, the athlete's objective in her last attempt will now be trying to improve her standing score rather than to avoid making a foul and – which was the case in the original Final Three - facing the risk of ending up at third place. In other words, the expected distance from the take-off foot to the foul-line is likely to be smaller for the finalists in this revised form of The Final Three than was the case in its previous form. Hence, in addition to a down-size of the strategic part, this change also comes with an expectation of an increasing number of extraordinary results in the final round, albeit at the prize of more fouls.

The second change – reversing the order of the jumps of the three finalists – is a natural consequence of the first change, bringing excitement to the final. Letting the third placed jumper from the five initial attempts to jump last in the sixth attempt, will keep the uncertainty of the distribution of the medals to the very last jump in the competition. It is true that there may exist an advantage of jumping last also in the revised form of The Final Three, which thus would harm fairness. However, in the light of the results we have obtained in this work on equilibrium probabilities to win The Final Three, as it originally was designed, the reduction in fairness, due to the reversed order of jumps, seems to be a low price to pay to maintain the excitement throughout the competition.

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