The Evolution of the Natural Rate of Interest – Evidence from the Scandinavian Countries

Hanna Armelius, Martin Solberger, Erik Spånberg and Pär Österholm

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Hanna Armelius[#]
Confederation of Swedish Enterprise

Martin Solberger[∀]
Department of Statistics, Uppsala University

Erik Spånberg[♣]
Department of Statistics, Stockholm University

Pär Österholm[◊]
School of Business, Örebro University
National Institute of Economic Research

Abstract

In this paper, the natural rate of interest in Denmark, Norway and Sweden are estimated. This is done by augmenting the Laubach and Williams (2003) framework with a dynamic factor model linked to economic indicators – a modelling choice which allows us to better identify business cycle fluctuations. We estimate the model using Bayesian methods on data ranging from 1990Q1 to 2022Q4. The results indicate that the natural rate has declined substantially and in all countries is at a low level at the end of the sample.

[#] Confederation of Swedish Enterprise, 114 82 Stockholm, Sweden
e-mail: hanna.armelius@ekonomifakta.se

[∀] Department of Statistics, Uppsala University, Box 513, 751 20 Uppsala, Sweden
e-mail: martin.solberger@statistik.uu.se

[♣] Department of Statistics, Stockholm University, 106 91 Stockholm, Sweden
e-mail: erik.spanberg@stat.su.se

[◊] School of Business, 701 82 Örebro, Sweden
e-mail: par.osterholm@oru.se

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1. Introduction

It is now a stylized fact that both nominal and real interest rates in OECD countries had been on a downward trend for several decades prior to the covid pandemic. The causes behind this development are disputed but lower global growth, increased income inequality, demographics, loose monetary policy, deleveraging, changes to government bond markets and shifts in saving and investment preferences are examples of factors put forward.[1] An additional explanation that is commonly suggested – and which clearly is related to some of the previously mentioned factors – is that the natural rate of interest has declined; see, for example, Holston *et al.* (2017), Rachel and Smith (2017) and Benati (2023).[2]

From a monetary-policy perspective, a potential decline in the natural rate of interest is of particular relevance since it could indicate that the environment in which central banks are acting today is different to that of the 1990s when the inflation-targeting framework was adopted in many countries. For example, with a low natural rate, one can expect monetary policy to be constrained at the zero/effective lower bound more frequently and for longer periods of time. This means that central banks will have to rely on unconventional measures to a large extent if they want to be active in their conduct of monetary policy. Several aspects of unconventional measures can be considered problematic though. For example, Cecchetti and Schoenholtz (2020) pointed out that central banks' purchases of corporate bonds should be considered fiscal policy and therefore undermines democratic accountability. Other aspects include that the central banks might induce excessive risk taking (IMF, 2013; Altunbas, 2014; Beckmann *et al.*, 2020; Adrian, 2020; Sveriges Riksbank, 2021). In light of the recent covid pandemic, these issues have only become more relevant as many central banks expanded their asset-purchase programs. The post-pandemic surge in inflation in many countries also highlights the importance of having an estimate of the natural rate, since it will provide an indication of where the future level of policy rates might be.

In this paper, we add to the discussion regarding the evolution of the natural rate of interest by providing new empirical evidence on this topic. More specifically, our purpose is to estimate the natural rate of interest in the Scandinavian countries – that is, Denmark, Norway and Sweden. This is done by augmenting the well-known Laubach and Williams (2003) framework with a small dynamic factor model that uses key economic

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[1] Discussions and empirical analyses can be found in, for example, Caballero *et al.* (2008), Broadbent (2014), Teulings and Baldwin (2014), Summers (2014), Gourinchas *et al.* (2016), Marx *et al.* (2019), Auclert *et al.* (2021) and Milan *et al.* (2021).

[2] The natural rate of interest as a concept was first introduced by Wicksell in 1896 as the equilibrium rate of interest that is consistent with stable prices. We follow Laubach and Williams (2003) – and others, including Holston *et al.* (2017) – and define the natural rate of interest as the real interest rate consistent with output equal to its natural rate and stable inflation. Seeing that we use an extended version of the Laubach and Williams (2003) model, it is also reasonable to use the same terminology. For a further discussion regarding terminology, see Platzer *et al.* (2022).

indicators to help identify business-cycle fluctuations. We estimate the model using Bayesian methods on data ranging from 1990Q1 to 2022Q4.

In conducting our analysis, we make two specific contributions. First, we add empirical evidence on the evolution of the natural rate of interest. As pointed out above, falling interest rates have been an international phenomenon and it is accordingly relevant to study economies beyond those that typically receive the focus (such as the United States and the euro area). While being similar in several ways, the three Scandinavian countries studied here also come with some interesting variation; for example, while Sweden can be seen as a fairly traditional small open economy, Denmark has a fixed exchange rate vis-à-vis the Euro, and the Norwegian economy is highly influenced by the country's oil production.

Second, we improve the popular Laubach and Williams (2003) framework, which has been used in a number of studies; see, for example, Garnier and Wilhelmsen (2009), Belke and Klose (2017), Holston *et al.* (2017) and Armelius *et al.* (2018). In Laubach and Williams' model, the business cycle is explained by an output gap, which in turn is a key determinant for price inflation. However, in most studies based on this framework, the information used to identify the output gap – and thereby also the natural rate of interest – has been quite limited; only a few variables and relationships have been employed. In this paper, we use a larger number of variables to identify the business cycle by linking a small dynamic factor model to Laubach and Williams' model. In employing a larger information set to improve identification, we follow a line of research similar to that of Aastveit and Trovik (2014), Jarociński and Lenza (2018) and Barigozzi and Luciano (2019), who all used more variables in order to improve output-gap measures. Through our approach, we should produce better estimates of both the natural rate of interest and the output gap.

Concerning our results, we find a marked decline in the natural rate of interest in all three countries. Our estimates also indicate that most of this decline can be attributed to factors other than a decline in potential growth. During the Nordic banking crisis in the early 1990s, the natural rate seems to have been pushed up by exogenous factors. During the last couple of decades this has reversed, and these factors are now exerting a downward pressure on Scandinavian natural rates. Our results also point to a more active use of monetary policy in Sweden and Norway as compared to Denmark.

The covid pandemic introduces some difficulties in the estimation and the results for the post-pandemic years should be interpreted with some caution. The pandemic and related disruptions in global trade and value chains, as well as the war in Ukraine, are responsible for large swings in most of the variables included in our model near the end of the sample. Since these events are exogenous it is hard to interpret them within the model framework. However, we have still chosen to include all the available observations in the estimations since we think it is of interest to get an indication of where the natural rates are heading according to

this framework. In the beginning of 2022, real interest rates dropped substantially in Denmark and Sweden (and somewhat in Norway) due to a rapid increase in inflation. Consequently, our natural rate estimates also drop. Our estimates of the real interest rate gap at the end of the sample are negative for Denmark and Sweden. While these as previously mentioned should be interpreted with caution, it can nevertheless be noted that this indicates that increasing policy interest rates should – at least in these two countries – contribute to a move towards a neutral monetary policy stance.

The remainder of this paper is organised as follows: In Section 2, we describe the model that we utilize. The data set is described in Section 3. In Section 4, we present the results from our empirical analysis. Section 5 concludes.

2. The model

Economic stabilization policies, as carried out by central banks and fiscal authorities, are based on the notion that economic variables can deviate from their "natural" levels, which gives rise to "gaps" such as the output gap. The task of economic policy is to implement the adequate economic measures to smooth the economic cycle and close the inflation and output gaps. This implies that it is an important task for economists to find reliable measures of the potential levels of key macroeconomic variables, so that gaps can be calculated. It is, however, a difficult task, and a lot of research has been produced refining the methods for estimating the cyclical position of the economy.

In the original Laubach and Williams' (2003) model, the estimation of the output gap, the inflation gap and the gap of the natural rate of interest are calculated using a semi-structural model and classical (point- and stepwise) estimation methods. In essence, inflation works as a signal informing about the cyclical position of the economy. However, if the Phillips curve is flat, inflation will not necessarily be a good indicator of whether the economy is in a boom or a bust. Therefore, these methods can be problematic when the Phillips curve is flat, which many argue that it presently is; see, for example, Zhang *et al.* (2021) for a more detailed discussion.

We believe that an improvement over traditional Laubach-Williams types of models can be made by utilizing the fact that we have more data available for making correct estimations of the economic cycle. By better pinpointing the cyclical position of the economy, we should also be able to get a more reliable estimate of the natural rate of interest. We here therefore merge a dynamic factor model that uses key economic indicators to help identify business-cycle fluctuations into a Laubach and Williams-type model. The dynamic factor model builds on Jarociński and Lenza (2018), who use a similar model to estimate the output gap for the euro area. The model uses national accounts data, such as consumption and gross fixed capital formation, and

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survey data in terms of consumer confidence and capacity utilization, which are known to be good indicators of the cyclical position of the economy.

As a practical concern, we have also found that we need to address large swings associated with the covid pandemic. For this purpose, we introduce twelve dummy variables (denoted D_1, …, D_{12}) individually indicating the quarters 2020Q1, 2020Q2, …, 2022Q4. (That is, the dummy variable takes on the value 1 for the quarter in question and 0 otherwise.) These are included in every equation relating to cycle indicators.

We assume that there exists an interest rate level that is compatible with a balanced resource utilization, and call this level the *natural* rate of interest, r_t^*. The ex-ante real rate (r_t) will be a combination of the natural rate and an interest rate gap (r_t^G) according to

$$r_t = r_t^* + r_t^G. \tag{1}$$

That is, r_t^G is the deviation from the natural level, which can be seen as the stance of monetary policy.

As in standard macroeconomic models, the natural interest rate is assumed to be related to potential growth, g_t, but as in Laubach and Williams (2003) there are also deviations from this relationship, that are modelled with another non-observable and time-varying series z_t according to

$$r^* = 4g_{t-1} + z_{t-1}, \tag{2}$$

where z_t follows a random walk,

$$z_t = z_{t-1} + \epsilon_t^z, \qquad \epsilon_t^z \sim N(0, \sigma_z^2). \tag{3}$$

The component z_t captures all other factors that affect the interest rate, such as demographics, global saving, global demand for safe assets and structural changes in fiscal policy; see Armelius *et al.* (2014), Bean *et al.* (2015), and Rachel and Smith (2015) for more detailed discussions.

The real interest rate gap is assumed to follow an autoregressive process with a zero mean,

$$r_t^G = \varphi_1 r_{t-1}^G + \varphi_2 r_{t-2}^G + \epsilon_t^{r^G}, \qquad \epsilon_t^{r^G} \sim N(0, \sigma_{r^G}^2). \tag{4}$$

We assume that the cyclical position of the economy can be described by an unobserved common factor (f_t) that is influencing a number of observed variables and economic indicators. Furthermore, we assume that (the log of) real GDP (y_t) is given by

$$y_t = y_t^* + f_t + \sum_{d=1}^{12} \delta_{y,d} D_d + \epsilon_t^y, \qquad \epsilon_t^y \sim N(0, \sigma_y^2), \tag{5}$$

where y_t^* is the trend component; the cyclical component of real GDP is accordingly given by the deviation $y_t^* - y_t$.

In accordance with Laubach and Williams (2003), we let potential GDP follow a local trend and assume that there can be random disturbances (ϵ_t^g and $\epsilon_t^{y^*}$) to the growth rate (g_t) and trend (y_t^*), respectively,

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t^{y^*}, \quad \epsilon_t^{y^*} \sim N(0, \sigma_{y^*}^2), \tag{6}$$

$$g_t = g_{t-1} + \epsilon_t^g, \qquad \epsilon_t^g \sim N(0, \sigma_g^2). \tag{7}$$

The cyclical factor is assumed to drive inflation, emulating a Phillips curve, according to

$$\pi_t = \left(1 - \sum_{\ell=2}^p \phi_{\pi,\ell}\right) \pi_{t-1} + \sum_{\ell=2}^p \phi_{\pi,\ell} \pi_{t-\ell} + \gamma_{\pi,1} f_t + \gamma_{\pi,2} f_{t-1} + \sum_{d=1}^{12} \delta_{\pi,d} D_d + \epsilon_t^\pi, \tag{8}$$

$$\epsilon_t^\pi \sim N(0, \sigma_\pi^2),$$

where π_t is core inflation (that is, excluding energy and food). Observe that the parameters for inflation lags sum to one.[3]

We link the cyclical factor to a small dynamic factor model, by adding the panel

$$x_{i,t} = w_{i,t} + \gamma_{1,i} f_t + \gamma_{2,i} f_{t-1} + \sum_{d=1}^{12} \delta_{i,d} D_d + \epsilon_{i,t}^x, \qquad \epsilon_{i,t}^x \sim (0, \sigma_{x,i}^2), \tag{9}$$

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[3] The number of lags (p) in equation (8) was for each country chosen based on the Schwarz Information Criterion (Schwarz, 1978) from a univariate autoregression for inflation; the maximum number of lags was set to eight.

where $x_{i,t}$ are additional economic indicators ($i = 1, 2, \ldots, n$) and $w_{i,t}$ are their trend-components. Thus, because the unobserved common factor f_t also enters equation (9), it is driving the cyclical position of each individual indicator, in addition to inflation. Some of the indicators are non-trending (and have a zero mean), in which case $w_{i,t}$ is set to zero. However, if $x_{i,t}$ is a variable with a trend, $w_{i,t}$ is modelled as a local trend according to

$$w_{i,t} = w_{i,t-1} + u_{i,t-1} + \epsilon_{i,t}^w, \quad \epsilon_{i,t}^w \sim N\left(0, \sigma_{w,i}^2\right), \tag{10}$$

$$u_{i,t} = u_{i,t-1} + \epsilon_{i,t}^u, \qquad \epsilon_{i,t}^u \sim N\left(0, \sigma_{u,i}^2\right). \tag{11}$$

Finally, an important assumption is that the real rate gap has an influence on the factor driving the cyclical position of the economy. This is to say that if inflation expectations are anchored, then monetary policy can be used to smooth economic cycles. We therefore assume that

$$f_t = \rho_{f,1} f_{t-1} + \rho_{f,2} f_{t-2} + \rho_{r,0} r_t^G + \rho_{r,1} r_{t-1}^G + \epsilon_t^f, \qquad \epsilon_t^f \sim N\left(0, \sigma_f^2\right), \tag{12}$$

allowing for both contemporary and dynamic effects between the real interest rate gap, r_t^G, and the cyclical factor, f_t.

We estimate the model given by equations (1) to (12) using a Bayesian filter that is outlined in Appendix A. For a given country, all equations are estimated simultaneously. This is an advantage of our approach, compared to standard Laubach-Williams-type approaches that typically use point- and stepwise estimation that tend to disregard parameter uncertainty. A necessity of Bayesian inference is to introduce prior distributions, making some subjective probabilistic statement of the parameters prior to conducting inference. In this paper, we have generally chosen very wide prior distributions. We therefore let the data "speak" more than what is typically found in similar studies that have used Bayesian methods to estimate the natural interest rate, such as Berger and Kempa (2014), Pedersen (2015) and Armelius *et al.* (2018). To our benefit, the inclusion of more data in the form of additional economic indicators enables us to produce reasonable estimates without resorting to more dogmatic prior beliefs.

3. Data

We use quarterly data from Denmark, Norway and Sweden as listed in Table 1. We define the (ex ante) real interest rate as the nominal rate minus expected inflation one year ahead. Following Laubach and Williams

(2003), we estimate inflation expectations by making forecasts four quarters ahead from a simple regression on past inflation.[4] As the nominal interest rate, we use the three-month interbank rate.

Our model utilizes ten observable time series for each country (the remaining time series in the model are non-observable). The observable time series include standard variables for natural rate estimation (core inflation, real interest rate and real GDP), and seven additional variables connected to the cyclical factor by the small dynamic factor model introduced in Section 2 (consumption, gross fixed capital formation, exports, imports, unemployment rate, capacity utilization and consumer confidence indicator). Of the latter (added) seven variables, the first five (consumption, gross capital formation, exports, imports and the unemployment rate) are treated as having a trend.[5] The last two variables (capacity utilization and consumer confidence indicator) should be mean reverting and have a zero mean by construction; they are hence modelled as not having a trend.

The sample for the estimation of the natural interest rate ranges from 1990Q1 to 2022Q4. As can be seen in Table 1, not all included variables have observations for the entire sample. The Bayesian filter handles missing values though; they are simply integrated out from the (conditional) likelihood in each sampling step.

Figure 1 shows the first three variables – that is, core inflation, real interest rate and real GDP – per country, as well as the estimated inflation expectations. A few things are worth noticing. First, core inflation was substantially higher in Sweden than the other countries in the early 1990s. Second, inflation in Norway was higher (and closer to target) during the period of low inflation in the years around 2014–2018. Third, both Danmarks Nationalbank and Sveriges Riksbank had negative policy rates during much of this episode. Finally, the Norwegian real interest rate has also more or less consistently been higher than the Danish and Swedish counterparts; given the expected positive relationship between interest rates and growth, this would be expected since growth in Norway has been substantially higher than in the other countries.

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[4] The regression is an AR(3) process with a rolling estimation window of 40 quarters.

[5] The unemployment rate obviously does not have a trend similar to that of the other four variables (seeing that those variables increase over time). However, the unemployment rate may have a unit root – at least within certain bounds – and hence a stochastic trend. This can be motivated, for example, by the work of Blanchard and Summers (1986) and the issue of a unit root in unemployment rates has been an issue of much empirical research; see, for example, Papell *et al.* (2000).

Table 1. Data description.

Variable	Sample	Transformation	Source
Denmark			
Interest rate, CIBOR, 3 Month	1990Q1–2022Q4	None	DFBF
Consumer price index (CPI), SA	1990Q1–2022Q4	400 × log difference	Statistics Denmark
Core inflation, SA	1990Q1–2022Q4	400 × log difference	OECD Economic Outlook
Gross domestic product, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Denmark
Final consumption expenditure, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Denmark
Gross fixed capital formation, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Denmark
Exports, goods & services, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Denmark
Imports, goods & services, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Denmark
Unemployment, percentage, calendar adjusted, SA	1990Q1–2022Q4	None	Eurostat
Sentiment indicators, consumer confidence indicator	1990Q1–2022Q4	Standardized	Eurostat
Current level of capacity utilization, balance, SA	1996Q1–2022Q4	Standardized	Eurostat
Norway			
Interest rate, NIBOR, 3 Month	1990Q1–2022Q4	None	NoRe
Consumer price index (CPI), SA	1990Q1–2022Q4	400 × log difference	Statistics Norway
Core inflation, SA	1990Q1–2022Q4	400 × log difference	OECD Economic Outlook
Gross domestic product, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Norway
Final consumption expenditure, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Norway
Gross fixed capital formation, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Norway
Exports, goods & services, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Norway
Imports, goods & services, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Norway
Unemployment, percentage, calendar adjusted, SA	1990Q1–2022Q4	None	Eurostat
Consumer surveys, expectations barometer	1992Q4–2022Q4	Standardized	Kantar DNS, Norway
Capacity utilization, weighted average, trend adjusted	1996Q1–2022Q4	Standardized	Statistics Norway
Sweden			
Interest rate, STIBOR, 3 Month	1990Q1–2022Q4	None	SFBF
Consumer price index (CPIF), SA	1990Q1–2022Q4	400 × log difference	Statistics Sweden
Core inflation, SA	1990Q1–2022Q4	400 × log difference	OECD Economic Outlook
Gross domestic product, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Sweden
Final consumption expenditure, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Sweden
Gross fixed capital formation, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Sweden
Exports, goods & services, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Sweden
Imports, goods & services, constant prices, SA	1990Q1–2022Q4	100 × log of index	Statistics Sweden
Unemployment, percentage, calendar adjusted, SA	1990Q1–2022Q4	None	Eurostat
Sentiment indicators, consumer confidence indicator	1995Q4–2022Q4	Standardized	Eurostat
Current level of capacity utilization, balance, SA	1996Q1–2022Q4	Standardized	Eurostat

Note: All data were downloaded from Macrobond. GDP, consumption, gross fixed capital formation, exports and imports are indexed so that 1990Q1 = 1 before transformation. Standardized series are adjusted to have mean 0 and standard deviation 1.

Figure 1. Key variables.
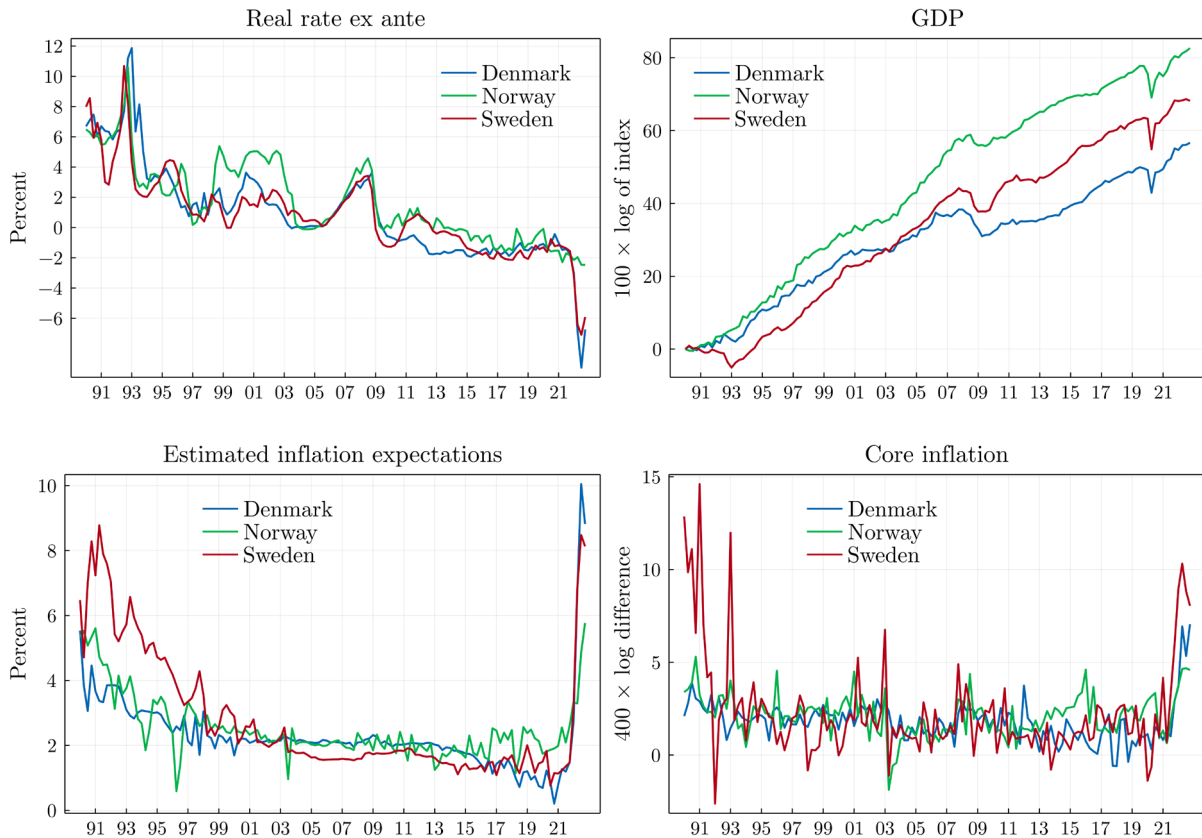
Note: For details regarding data, see Table 1.

4. Results

In Figures 2 to 4, the estimated natural rate of interest, output gap, real interest rate gap, as well as the contribution to the natural rate of interest from the processes g_t and z_t are shown for Denmark, Norway and Sweden, respectively.

The natural rate has fallen from around 5 to 6 percent in the early 1990's to negative values by the end of 2022 in all three countries. The Norwegian natural rate has been slightly higher than the natural rates in Denmark and Sweden since the mid 1990's. This is mostly explained by a lower negative contribution from the additional factors represented by the process z_t. The general pattern of a downward trend in the natural rates is in line with previous findings; see, for example, Holston *et al.* (2017).

Figure 2. Natural interest rate, output gap, real interest rate gap and contributions to the natural interest rate, Denmark.
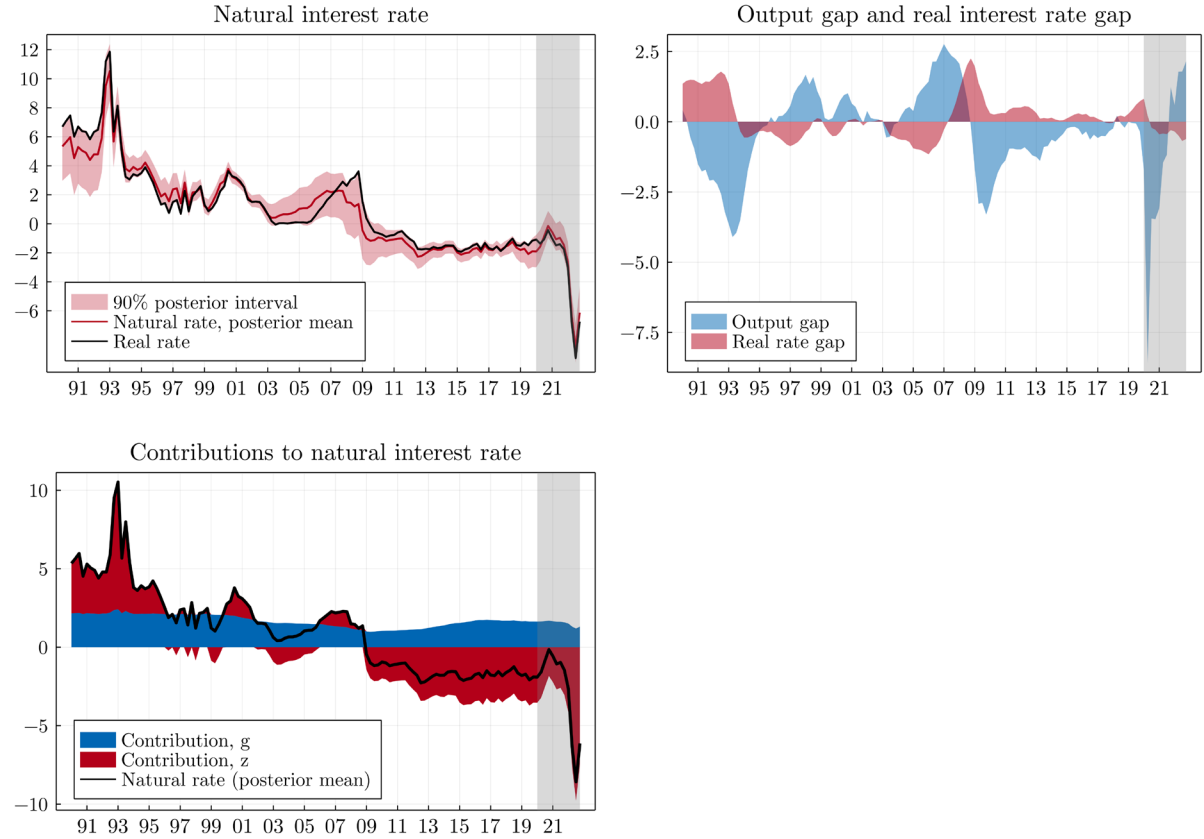
Note: Natural interest rate in percent. Output gap in percent of potential GDP. Real interest rate gap in percentage points. Contributions to natural interest rate in percentage points. Shaded areas denote the inclusion of dummy variables.

Figure 3. Natural interest rate, output gap, real interest rate gap and contributions to the natural interest rate, Norway.
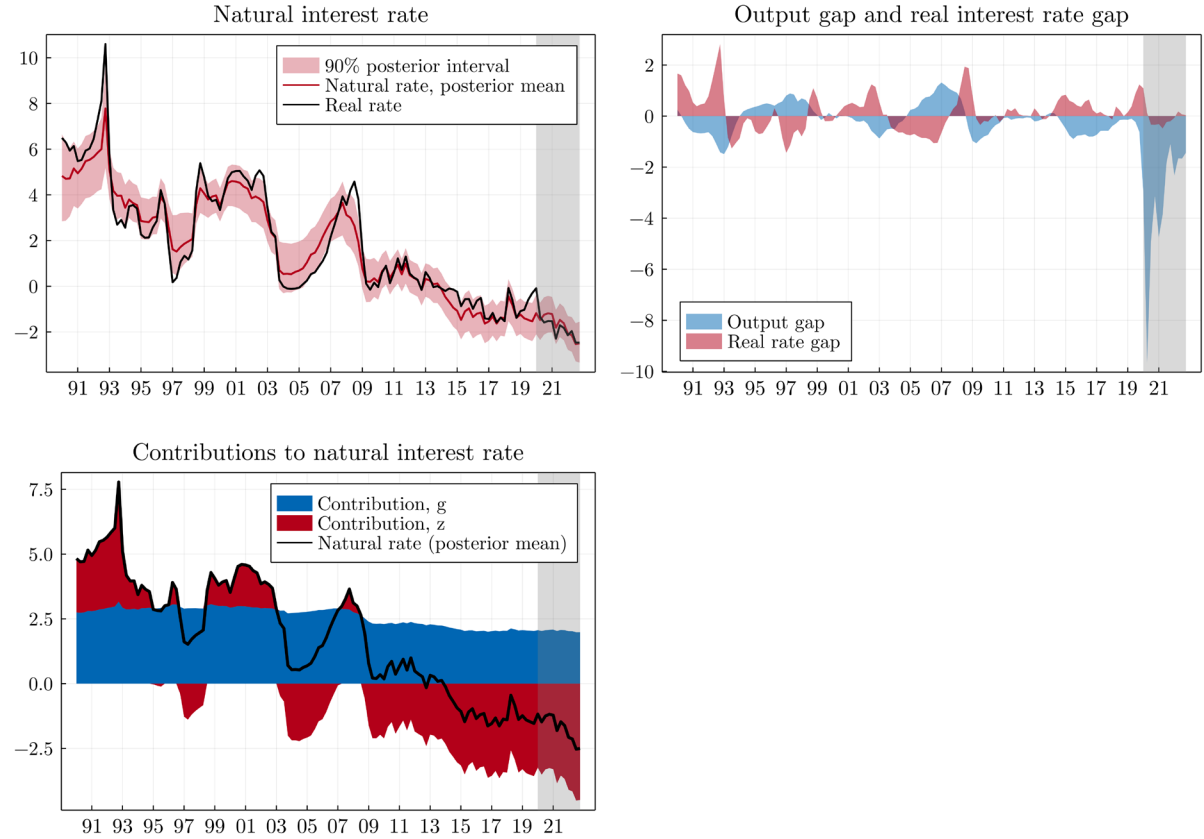
Note: Natural interest rate in percent. Output gap in percent of potential GDP. Real interest rate gap in percentage points. Contributions to natural interest rate in percentage points. Shaded areas denote the inclusion of dummy variables.

Figure 4. Natural interest rate, output gap, real interest rate gap and contributions to the natural interest rate, Sweden.

Natural interest rate

Output gap and real interest rate gap

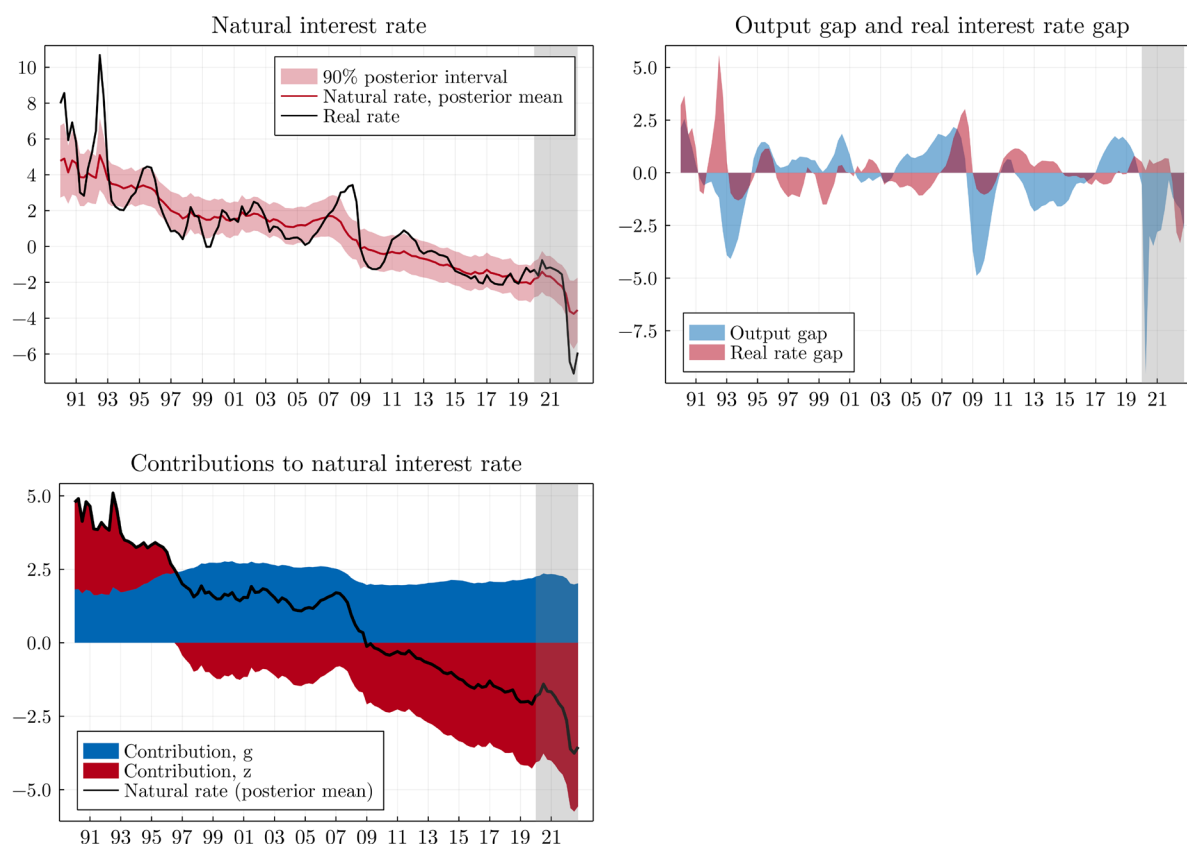Contributions to natural interest rate

Note: Natural interest rate in percent. Output gap in percent of potential GDP. Real interest rate gap in percentage points. Contributions to natural interest rate in percentage points. Shaded areas denote the inclusion of dummy variables.

Another previous finding in the literature is that foreign factors seem to have a large negative impact on natural rates in small open economies (Holston *et al.*, 2017; Zhang *et al.*, 2021). In our model this would be captured by the component z_t (which can also contain other factors such as changes in fiscal policy rules and demographics). According to our estimates, there seem to exist some common patterns in the Scandinavian countries. All three countries have a positive contribution from z_t in the early 1990's, but then have a negative contribution from z_t after the global financial crisis in 2008–2009, although the magnitudes of the contributions differ somewhat between the countries. These patterns largely explain the decline in the natural rate. We do not know what is driving z_t, but given that all countries are small open economies it does not seem unlikely that foreign developments are important.

It should also be noted that in the two first quarters of 2022, our estimates of the natural rate of interest fell substantially in Denmark and Sweden – a fall which originated from a rapid increase in inflation, resulting in historically low real interest rates. The nature behind this development is too complicated to be well described

by our model, and our estimates of both the natural rate of interest and the real interest rate gap at the end of the sample should hence be taken with a substantial pinch of salt. We nevertheless note that the negative real interest rate gaps in Denmark and Sweden at the end of the sample are consistent with increasing policy interest rates contributing to a move towards a neutral monetary policy stance.

Turning to some other results from the estimations, we find that the real interest rate gap has varied more in Norway and Sweden than in Denmark. This can be interpreted as monetary policy having been used more actively in Norway and Sweden. Since Denmark has a fixed exchange rate vis-à-vis the euro, this is not an unexpected finding. Overall, Norway has had the smallest fluctuations in the output gap over time – in particular in the aftermath of the global financial crisis.

In Table 2, the parameter posterior estimates are shown in terms of posterior means and 90 per cent credible intervals for the parameters in the main equations. The posterior estimates for the parameters in the dynamic factor model and its trend components are provided in Appendix C. If a 90 per cent credible interval does not encompass 0, then we can be relatively confident that the corresponding parameter is different from 0. This is the case for most of the parameters in Table 2. Looking at the estimated parameters for the Phillips curve (first panel of Table 2), we see no clear effect on inflation in Sweden from the cyclical factor f_t, judging by the posterior estimates of the parameters $\gamma_{\pi,1}$ and $\gamma_{\pi,2}$. We see an effect on inflation in Denmark and Norway. However, for both countries, the parameters $\gamma_{\pi,1}$ and $\gamma_{\pi,2}$ are of similar magnitude and have different signs, and the dynamic effect over time is moderate. Overall, our estimates are in line with a quite flat Phillips curve, motivating the use of additional indicators to estimate the output gap in these types of models.[6] Apart from inflation, we find that the economic cycle has a clear effect on the other economic indicators for each country, as shown by the posterior estimates provided in Appendix C.

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[6] There has been a lively discussion during the last fifteen years regarding the slope of the Phillips curve; see, for example, Gaiotti (2010), Kuttner and Robinson (2010), Blanchard *et al.* (2015) and Ball and Mazumder (2019). A Swedish perspective can be found in Karlsson and Österholm (2020).

Table 2. Parameter posterior estimates.

Parameter	Denmark Mean	Denmark 90% interval	Norway Mean	Norway 90% interval	Sweden Mean	Sweden 90% interval
Phillips curve						
$1-\sum_{\ell=2}^{p}\phi_{\pi,\ell}$	0.36	[0.17, 0.54]	0.23	[0.08, 0.39]	0.37	[0.21, 0.53]
$\phi_{\pi,2}$	0.05	[-0.06, 0.15]	0.48	[0.37, 0.58]	0.38	[0.26, 0.49]
$\phi_{\pi,3}$	0.18	[0.09, 0.27]	0.29	[0.18, 0.40]	0.25	[0.14, 0.36]
$\phi_{\pi,4}$	0.15	[0.06, 0.24]	-	-	-	-
$\phi_{\pi,5}$	0.27	[0.18, 0.35]	-	-	-	-
$\gamma_{\pi,1}$	0.34	[0.18, 0.49]	-0.90	[-1.26, -0.52]	0.06	[-0.24, 0.36]
$\gamma_{\pi,2}$	-0.25	[-0.40, -0.09]	1.19	[0.78, 1.57]	0.01	[-0.30, 0.32]
σ_{π}^2	0.45	[0.36, 0.55]	0.77	[0.62, 0.94]	3.94	[3.20, 4.81]
Real rate gap						
φ_1	1.44	[0.68, 1.79]	1.08	[0.82, 1.30]	1.19	[1.05, 1.35]
φ_2	-0.55	[-0.82, 0.06]	-0.40	[-0.61, -0.19]	-0.47	[-0.64, -0.33]
$\sigma_{r^G}^2$	0.04	[0.01, 0.09]	0.22	[0.06, 0.47]	0.42	[0.21, 0.62]
Cyclical factor						
$\rho_{f,1}$	0.54	[0.28, 0.80]	1.30	[1.04, 1.53]	1.50	[1.37, 1.61]
$\rho_{f,2}$	0.18	[0.02, 0.34]	-0.41	[-0.61, -0.19]	-0.61	[-0.72, -0.49]
$\rho_{r,0}$	0.81	[-0.37, 2.07]	-0.13	[-0.24, -0.05]	0.11	[0.03, 0.20]
$\rho_{r,1}$	-1.39	[-2.38, -0.49]	-0.02	[-0.12, 0.07]	-0.25	[-0.37, -0.15]
σ_f^2	0.03	[0.01, 0.06]	0.01	[$8\cdot10^{-3}$, 0.02]	0.10	[0.05, 0.16]
Trend						
σ_y^2	$2\cdot10^{-4}$	[10^{-4}, $5\cdot10^{-4}$]	$2\cdot10^{-4}$	[10^{-4}, $5\cdot10^{-4}$]	$2\cdot10^{-4}$	[10^{-4}, $5\cdot10^{-4}$]
$\sigma_{y^\star}^2$	0.55	[0.43, 0.68]	0.78	[0.63, 0.97]	0.38	[0.29, 0.49]
σ_g^2	$3\cdot10^{-3}$	[$5\cdot10^{-4}$, $7\cdot10^{-3}$]	$2\cdot10^{-3}$	[$4\cdot10^{-4}$, $6\cdot10^{-3}$]	$2\cdot10^{-3}$	[$5\cdot10^{-4}$, $5\cdot10^{-3}$]
σ_z^2	0.77	[0.63, 0.89]	0.35	[0.10, 0.57]	0.16	[0.04, 0.36]

Note: "90% interval" gives the 90 per cent credible interval. Estimates for dummy variables are shown in Appendix C.

5. Conclusions

We have estimated the natural rate of interest in the Scandinavian countries Denmark, Norway and Sweden from 1990Q1 to 2022Q4. In our approach, we have augmented the Laubach-Williams methodology with a dynamic factor model linked to conventional economic indicators to better identify business cycle fluctuations. This approach is particularly useful when the Philips curve is flat, since inflation is unlikely to provide

a good signal of the cyclical position of the economy. Our results indicate that this is indeed the case in the economies we study.

The estimates of the natural rate differ somewhat between the three countries. However, a common feature is that they indicate that natural rate has declined substantially during the sample. The last three years of the sample are turbulent, with both the covid pandemic and the period of high inflation (and rising interest rates) that followed. The results from this period should be interpreted with caution, but we note that the point estimates suggest that the natural rate of interest is still very low – in fact negative.

Our findings are hence in line with recent results for other economies; for example, Benati (2023) suggests that the natural rate in late 2019 in Canada, the euro area, the United Kingdom and the United States was negative in all four cases. Concerning implications for monetary policy, we note that there is an ongoing discussion regarding whether policy rates will stay high or return to lower levels – see, for example, Gopinath (2022) and International Monetary Fund (2023) – and recent evidence for the United States suggests that the short nominal rate will come down from its present (April 2023) level (Beechey *et al.*, 2023). There is obviously a substantial amount of uncertainty regarding the future development of policy rates, but our results suggest that if inflation stabilizes at the target in a not too distant future, then a low natural rate of interest could contribute to low policy rates also in the future.

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Appendix A: Estimation method

Equations (1) to (12) can be written as a Gaussian state space model,

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{A}\boldsymbol{\alpha}_t + \boldsymbol{B}\boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t \sim \mathcal{N}(0, \boldsymbol{\Sigma}), \tag{A1}$$

$$\boldsymbol{a}_t = \boldsymbol{F}\boldsymbol{a}_{t-1} + \boldsymbol{G}\boldsymbol{u}_t, \qquad \boldsymbol{u}_t \sim \mathcal{N}(0, \boldsymbol{H}), \tag{A2}$$

where $\boldsymbol{y}_t = \left(r_t, y_t, \pi_t, x_{1,t}, \dots, x_{7,t}\right)'$ is a vector (10×1) of the observed variables, and $\boldsymbol{\alpha}_t$ is a latent state vector (18×1),

$$\boldsymbol{\alpha}_t = \left(r_t^*, z_t, r_{t+1}^G, r_t^G, y_t^*, g_t, f_t, f_{t-1}, w_{1,t}, \dots, w_{5,t}, u_{1,t}, \dots, u_{5,t}\right)'.$$

Note the leading index on r_{t+1}^G, which is due to that we allow for a contemporary effect between f_t and r_t^G in equation (12). Also note that only five (and not seven) of each of the latent trend variables $w_{i,t}$ and $u_{i,t}$ from equations (10) and (11) are elements of $\boldsymbol{\alpha}_t$, since only the first five of the economic indicators in the panel $x_{i,t}$ of equation (9) have trends; see Section 3 and Appendix B.

The vector $\boldsymbol{c}$ (10×1) relates to the observable right-hand time series of the equations, including the dummy variables,

$$\boldsymbol{c} = \left(0, \ \textstyle\sum_{d=1}^{12} \delta_{y,d} D_d, \ \left(1 - \sum_{\ell=2}^{p} \phi_{\pi,\ell}\right)\pi_{t-1} + \sum_{\ell=2}^{p} \phi_{\pi,\ell}\pi_{t-\ell} + \sum_{d=1}^{12} \delta_{\pi,d} D_d, \right.$$
$$\left. \textstyle\sum_{d=1}^{12} \delta_{1,d} D_d, \dots, \sum_{d=1}^{12} \delta_{7,d} D_d\right)'.$$

The matrices $\boldsymbol{A}$ (10×18) and $\boldsymbol{F}$ (18×18) collect the parameters associated with the latent state vector,

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{\pi,1} & \gamma_{\pi,2} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{1,1} & \gamma_{2,1} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{1,2} & \gamma_{2,2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{1,7} & \gamma_{2,7} & 0 & 0 & \cdots & 1 \end{pmatrix},$$

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & \varphi_1 & \varphi_2 & 0 & 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & \rho_{r,0} & \rho_{r,1} & 0 & 0 & \rho_{f,1} & \rho_{f,2} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I}_5 & \boldsymbol{I}_5 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I}_5 \end{pmatrix},$$

where $\boldsymbol{I}_5$ denotes the 5×5 identity matrix and $\mathbf{0}$ denotes a zero matrix of appropriate size.

The matrices $\boldsymbol{B}$ (10×9) and $\boldsymbol{G}$ (18×15) are selection matrices consisting of subsets of the columns of the identity matrix, loading onto the errors $\boldsymbol{\epsilon}_t$ (9×1) and $\boldsymbol{u}_t$ (15×1),

$$\boldsymbol{\epsilon}_t = \left(\epsilon_t^y, \epsilon_t^\pi, \epsilon_{1,t}^x, \dots, \epsilon_{7,t}^x \right)',$$

$$\boldsymbol{u}_t = \left(\epsilon_t^z, \epsilon_t^{r^G}, \epsilon_t^{y^*}, \epsilon_t^g, \epsilon_t^f, \epsilon_{1,t}^w, \dots, \epsilon_{5,t}^w, \epsilon_{1,t}^u, \dots, \epsilon_{5,t}^u \right)'.$$

The matrices $\boldsymbol{\Sigma}$ and $\boldsymbol{H}$ contain the error variances on their main diagonals; the off-diagonal elements are zero.

Let $\boldsymbol{\theta}$ denote the set of all parameters and let $\boldsymbol{\alpha}_{1:T} = \{\boldsymbol{a}_t\}_{t=1}^T$ and $\boldsymbol{y}_{1:T} = \{\boldsymbol{y}_t\}_{t=1}^T$ denote the sets of latent variables and observable data, respectively. We apply a Bayesian filter with the goal to find the joint posterior distribution, $p(\boldsymbol{\theta}, \boldsymbol{\alpha}_{1:T} | \boldsymbol{y}_{1:T})$. We use Gibbs sampling – see, for example, Geman and Geman (1984) and Gelman $et\ al.$ (2013) – to draw iteratively from the full conditional posteriors, $\boldsymbol{\theta}^{(d)} \sim p(\boldsymbol{\theta} | \boldsymbol{\alpha}_{1:T}^{(d-1)}, \boldsymbol{y}_{1:T})$, $\boldsymbol{\alpha}_{1:T}^{(d)} \sim p(\boldsymbol{\alpha}_{1:T} | \boldsymbol{\theta}^{(d)}, \boldsymbol{y}_{1:T})$, over $d = 1, \dots, D$. We set $D = 100{,}000$, where the first $10{,}000$ draws are left out. Gibbs sampling is a Markov chain Monte Carlo method for posterior simulation and is often used for joint estimation of parameters and latent states in linear Gaussian state space systems; see, for example, Bai and Wang (2015) and Jarociński and Lenza (2018).

Note that, given $\boldsymbol{\alpha}_{1:T}$, equations (A1) and (A2) can be seen as a multivariate linear regression system with Gaussian errors. Furthermore, due to the diagonal properties of $\boldsymbol{\Sigma}$ and $\boldsymbol{H}$, the system is just a collection of standard simple linear regressions. By using conjugate priors, we can find $p(\boldsymbol{\theta} | \boldsymbol{\alpha}_{1:T}^{(d-1)}, \boldsymbol{y}_{1:T})$ by standard Bayesian linear regressions. Here, we use the Gaussian-scaled-inverse-chi-square prior; see, for example, Gelman $et\ al.$ (2013). Let $\boldsymbol{\beta}$ be a vector of non-variance parameters in any of these single linear regressions, with corresponding residual variance σ^2. The prior becomes

$$\boldsymbol{\beta} | \sigma^2 \sim \mathcal{N} \left(0, \frac{\sigma^2}{\omega} \boldsymbol{I} \right),$$

$$\sigma^2 \sim \textit{Scale-inverse-}\chi^2(\nu, \tau^2).$$

We choose $\omega = 1/100$ for all equations, creating a reasonably wide prior for each non-variance parameter. For all variance parameters (with exceptions for σ^2_{rG} and σ^2_f, see below) we choose $\nu = 0.01$ and $\tau^2 = 0.25$, placing around 5 percent prior probability for variances less than 1, and an almost uniform prior for variances from 1 going up the positive real line in many orders of magnitude – that is, a very wide prior, with just a little extra emphasis on small numbers. For σ^2_{rG} and σ^2_f, we choose $\nu = 1$, and $\tau^2 = \frac{s_r^2}{9}$ and $\tau^2 = \frac{s_y^2}{9}$, respectively, where s_r^2 and s_y^2 are the respective sample variances of $(r_t - r_{t-1})$ and $(y_t - y_{t-1})$; that is, a large part of the prior density is placed such that the residual variances σ^2_{rG} and σ^2_f are smaller than the sample variances of the underlying variables. The priors underbuild the scaling of the system and help to simulate gaps in sizes that make economic sense.

Lastly, the other conditional posterior, $p(\boldsymbol{\alpha}_{1:T} | \boldsymbol{\theta}^{(d)}, \boldsymbol{y}_{1:T})$, is found using the Simulation smoother of Durbin and Koopman (2002).

Appendix B: Additional data

Figure B1 shows time series of the additional seven economic indicators: consumption, gross fixed capital formation, exports, imports, unemployment rate, capacity utilization and consumer confidence indicator. The last two series have a zero mean by construction. Thus, for these series the trend components $w_{i,t}$ and $u_{i,t}$ are set to zero.

Figure B1. Additional economic indicators.
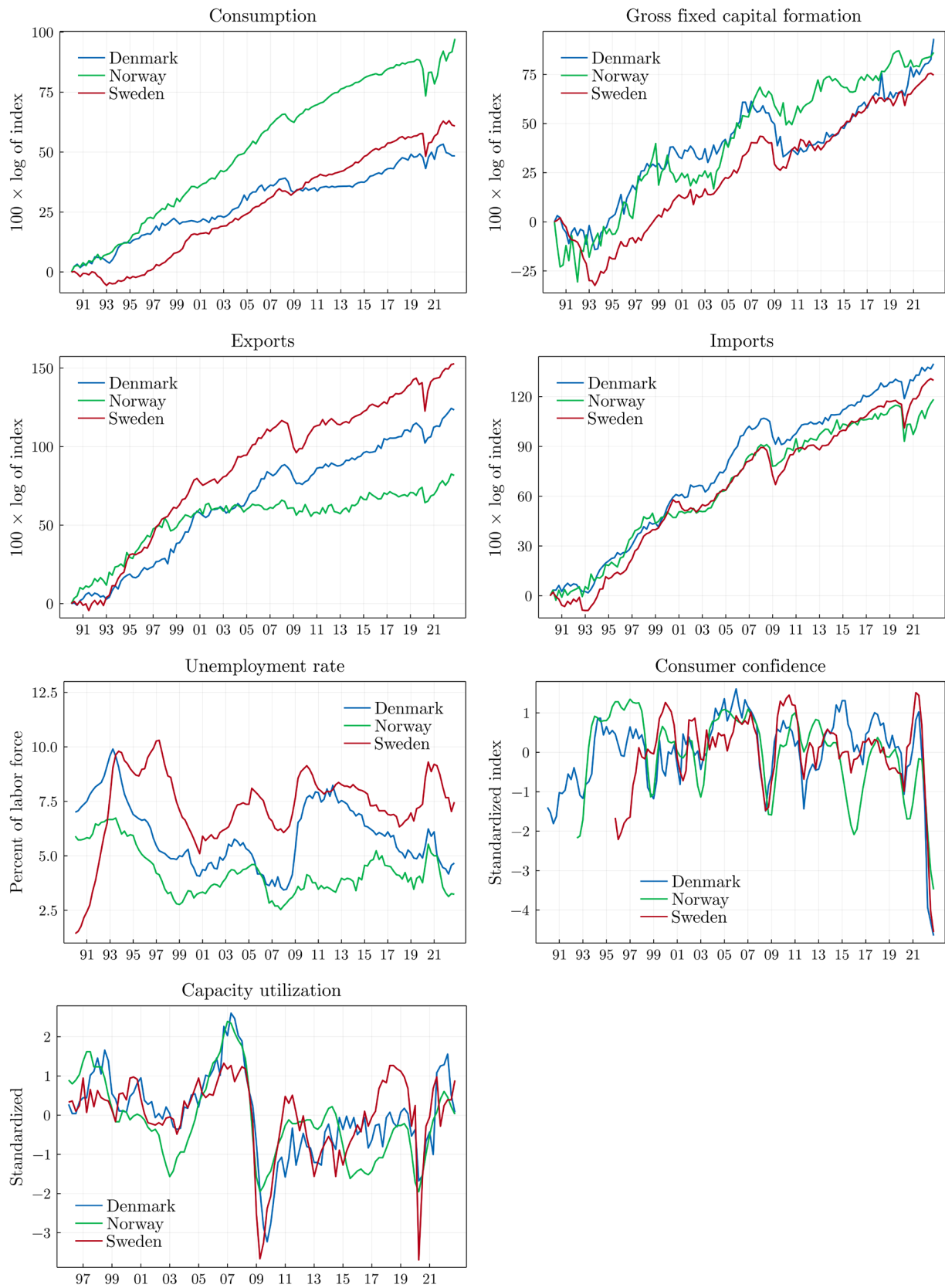
Appendix C: Additional results

Table C1. Parameter posterior estimates, economic indicator equations.

Parameter	Denmark		Norway		Sweden	
	Mean	90% interval	Mean	90% interval	Mean	90% interval
Consumption						
$\gamma_{1,1}$	-0.69	[-0.99, -0.24]	0.75	[-0.12, 1.59]	1.03	[0.76, 1.32]
$\gamma_{2,1}$	7.17	[6.72, 7.52]	0.70	[-0.13, 1.56]	-0.63	[-0.91, -0.37]
$\sigma_{x,1}^2$	0.02	$[10^{-6}, 0.08]$	0.29	[0.16, 0.44]	0.12	[0.05, 0.20]
$\sigma_{w,1}^2$	1.21	[0.77, 1.67]	0.35	[0.02, 0.66]	0.14	$[2 \cdot 10^{-3}, 0.36]$
$\sigma_{\delta,1}^2$	0.01	$[10^{-3}, 0.02]$	0.03	[0.00, 0.09]	0.03	[0.01, 0.07]
Capital formation						
$\gamma_{1,2}$	7.17	[6.72, 7.52]	-0.22	[-0.72, 0.20]	1.73	[1.62, 1.83]
$\gamma_{2,2}$	-1.26	[-1.60, -0.88]	3.50	[3.08, 4.27]	1.05	[0.96, 1.14]
$\sigma_{x,2}^2$	$2 \cdot 10^{-4}$	$[9 \cdot 10^{-6}, 6 \cdot 10^{-4}]$	$3 \cdot 10^{-5}$	$[2 \cdot 10^{-6}, 7 \cdot 10^{-5}]$	10^{-5}	$[8 \cdot 10^{-7}, 3 \cdot 10^{-5}]$
$\sigma_{w,2}^2$	13.77	[10.81, 17.26]	28.64	[23.22, 35.13]	6.05	[4.87, 7.45]
$\sigma_{\delta,2}^2$	0.01	$[8 \cdot 10^{-4}, 0.05]$	0.01	$[8 \cdot 10^{-4}, 0.06]$	0.01	$[8 \cdot 10^{-4}, 0.04]$
Exports						
$\gamma_{1,3}$	2.87	[2.62, 3.10]	2.04	[1.84, 2.30]	3.33	[2.41, 4.34]
$\gamma_{2,3}$	-0.68	[-1.03, -0.35]	1.48	[1.28, 1.73]	-0.74	[-1.58, 0.07]
$\sigma_{x,3}^2$	$2 \cdot 10^{-4}$	$[2 \cdot 10^{-6}, 8 \cdot 10^{-5}]$	10^{-5}	$[10^{-6}, 4 \cdot 10^{-5}]$	0.16	[0.06, 0.33]
$\sigma_{w,3}^2$	4.54	[3.61, 5.65]	8.12	[6.58, 9.95]	2.89	[2.07, 3.84]
$\sigma_{\delta,3}^2$	0.02	$[10^{-3}, 0.07]$	0.02	$[2 \cdot 10^{-3}, 0.05]$	0.01	$[2 \cdot 10^{-3}, 0.04]$
Imports						
$\gamma_{1,4}$	5.20	[5.02, 5.39]	2.80	[0.16, 5.37]	3.69	[2.70, 4.75]
$\gamma_{2,4}$	-1.67	[-1.89, -1.43]	2.22	[-0.33, 4.84]	-0.56	[-1.48, 0.27]
$\sigma_{x,4}^2$	$4 \cdot 10^{-5}$	$[5 \cdot 10^{-6}, 10^{-4}]$	1.10	[0.10, 2.95]	0.09	[0.03, 0.20]
$\sigma_{w,4}^2$	2.36	[1.53, 3.31]	6.25	[2.08, 10.28]	2.25	[1.53, 3.06]
$\sigma_{\delta,4}^2$	0.05	$[2 \cdot 10^{-3} \, 0.16]$	0.01	$[10^{-3}, 0.05]$	0.02	$[8 \cdot 10^{-4}, 0.07]$

Note: "90% interval" gives the 90 per cent credible interval.

Table C1 (continued). Parameter posterior estimates, economic indicator equations.

Parameter	Denmark		Norway		Sweden	
	Mean	90% interval	Mean	90% interval	Mean	90% interval
Unemployment rate						
$\gamma_{1,5}$	-0.25	[-0.31, -0.21]	0.22	[0.03, 0.38]	-0.07	[-0.13, 0.01]
$\gamma_{2,5}$	-0.19	[-0.26, -0.11]	-0.15	[-0.28, -0.04]	-0.25	[-0.36, -0.16]
$\sigma_{x,5}^2$	$2 \cdot 10^{-5}$	$[2 \cdot 10^{-7}, 7 \cdot 10^{-5}]$	$5 \cdot 10^{-6}$	$[2 \cdot 10^{-7}, 10^{-5}]$	$3 \cdot 10^{-4}$	$[2 \cdot 10^{-7}, 10^{-3}]$
$\sigma_{w,5}^2$	0.06	[0.05, 0.08]	0.07	[0.05, 0.09]	0.05	[0.03, 0.07]
$\sigma_{\delta,5}^2$	$2 \cdot 10^{-3}$	$[4 \cdot 10^{-4}, \cdot 10^{-3}]$	$2 \cdot 10^{-3}$	$[4 \cdot 10^{-4}, 6 \cdot 10^{-3}]$	$8 \cdot 10^{-3}$	$[3 \cdot 10^{-3}, 0.02]$
Capacity utilization						
$\gamma_{1,6}$	0.89	[0.76, 1.05]	2.62	[2.32, 2.91]	0.77	[0.62, 0.95]
$\gamma_{2,6}$	-0.79	[-0.94, -0.65]	-1.71	[-2.04, -1.38]	-0.86	[-1.05, -0.70]
$\sigma_{x,6}^2$	0.40	[0.32, 0.49]	0.30	[0.24, 0.38]	0.37	[0.29, 0.47]
Confidence indicator						
$\gamma_{1,7}$	0.49	[0.34, 0.64]	1.31	[1.20, 1.42]	0.59	[0.40, 0.80]
$\gamma_{2,7}$	0.37	[0.19, 0.56]	0.58	[0.45, 0.68]	0.07	[-0.12, 0.25]
$\sigma_{x,7}^2$	0.07	[0.05, 0.09]	$3 \cdot 10^{-7}$	$[5 \cdot 10^{-7}, 9 \cdot 10^{-6}]$	0.06	[0.04, 0.08]

Note: "90% interval" gives the 90 per cent credible interval.

Table C2. Parameter posterior estimates, dummy variables, Denmark.

Parameter	Q1 Mean	Q1 90% interval	Q2 Mean	Q2 90% interval	Q3 Mean	Q3 90% interval	Q4 Mean	Q4 90% interval
2020								
GDP	-1.21	[-2.18, -0.31]	-6.61	[-7.75, -5.48]	-1.68	[-2.87, -0.33]	-2.38	[-3.89, -0.56]
Core inflation	0.71	[-0.40, 1.81]	-0.06	[-1.19, 1.06]	0.97	[-0.13, 2.07]	-0.07	[-1.18, 1.04]
Consumption	-0.32	[-1.78, 0.48]	-1.48	[-4.35, 0.02]	0.11	[-0.96, 1.36]	0.46	[-0.28, 2.00]
Capital formation	-0.01	[-0.15, 0.14]	0.01	[-0.13, 0.16]	0.00	[-0.14, 0.14]	0.00	[-0.14, 0.15]
Exports	0.01	[-0.06, 0.06]	-0.02	[-0.07, 0.05]	0.00	[-0.06, 0.05]	-0.02	[-0.06, 0.05]
Imports	0.01	[-0.08, 0.10]	0.00	[-0.09, 0.08]	0.00	[-0.09, 0.09]	0.01	[-0.08, 0.10]
Unemployment rate	-0.01	[-0.06, 0.02]	-0.01	[-0.05, 0.02]	0.01	[-0.02, 0.08]	0.00	[-0.04, 0.03]
Consumer confidence	0.15	[-0.93, 1.22]	0.28	[-0.81, 1.37]	-0.33	[-1.40, 0.75]	-0.72	[-1.80, 0.37]
Capacity utilization	0.01	[-0.56, 0.57]	-0.54	[-1.21, 0.12]	0.04	[-0.69, 0.76]	0.51	[-0.26, 1.27]
2021								
GDP	-2.07	[-3.86, -0.12]	-1.10	[-2.91, 0.99]	-0.92	[-2.49, 0.84]	0.92	[-0.55, 2.49]
Core inflation	-0.11	[-1.22, 0.99]	0.22	[-0.88, 1.33]	-0.03	[-1.14, 1.07]	1.57	[0.46, 2.69]
Consumption	-1.06	[-3.24, 0.03]	0.58	[-0.18, 2.41]	0.91	[-0.05, 3.17]	0.83	[-0.06, 3.02]
Capital formation	-0.01	[-0.16, 0.12]	0.00	[-0.15, 0.15]	-0.01	[-0.17, 0.13]	-0.01	[-0.17, 0.12]
Exports	0.03	[-0.05, 0.08]	0.00	[-0.06, 0.05]	0.00	[-0.06, 0.06]	0.00	[-0.05, 0.07]
Imports	0.00	[-0.10, 0.09]	0.00	[-0.09, 0.09]	0.00	[-0.10, 0.10]	0.00	[-0.09, 0.10]
Unemployment rate	0.02	[-0.01, 0.11]	-0.01	[-0.06, 0.02]	0.00	[-0.04, 0.03]	0.00	[-0.04, 0.03]
Consumer confidence	0.16	[-0.92, 1.23]	0.34	[-0.74, 1.42]	0.96	[-0.12, 2.05]	-0.51	[-1.59, 0.56]
Capacity utilization	0.45	[-0.35, 1.24]	-0.46	[-1.31, 0.38]	1.32	[0.43, 2.20]	1.18	[0.23, 2.12]
2022								
GDP	0.42	[-0.88, 1.79]	1.54	[0.13, 3.09]	1.68	[-0.11, 3.51]	1.67	[-0.85, 4.09]
Core inflation	1.95	[0.84, 3.06]	4.79	[3.68, 5.89]	1.67	[0.55, 2.78]	3.27	[2.16, 4.39]
Consumption	-0.28	[-1.76, 0.71]	-0.26	[-1.84, 0.81]	-0.37	[-2.30, 0.70]	-0.58	[-2.97, 0.58]
Capital formation	0.01	[-0.12, 0.18]	0.00	[-0.15, 0.14]	0.00	[-0.15, 0.13]	0.02	[-0.11, 0.18]
Exports	0.00	[-0.06, 0.06]	0.00	[-0.06, 0.06]	0.02	[-0.04, 0.07]	0.00	[-0.06, 0.05]
Imports	0.00	[-0.10, 0.09]	0.00	[-0.09, 0.10]	-0.01	[-0.10, 0.09]	0.00	[-0.09, 0.10]
Unemployment rate	0.00	[-0.03, 0.04]	-0.01	[-0.07, 0.02]	0.00	[-0.03, 0.04]	0.00	[-0.03, 0.04]
Consumer confidence	-1.21	[-2.29, -0.13]	-3.98	[-5.07, -2.90]	-4.16	[-5.25, -3.07]	-4.94	[-6.04, -3.84]
Capacity utilization	1.08	[0.08, 2.08]	1.36	[0.30, 2.41]	0.36	[-0.77, 1.49]	-0.20	[-1.41, 0.99]

Note: "90% interval" gives the 90 per cent credible interval.

Table C3. Parameter posterior estimates, dummy variables, Norway.

Parameter	Q1 Mean	Q1 90% interval	Q2 Mean	Q2 90% interval	Q3 Mean	Q3 90% interval	Q4 Mean	Q4 90% interval
2020								
GDP	-1.91	[-3.23, -0.57]	-8.62	[-10.93, -7.13]	-4.19	[-5.59, -2.87]	-2.69	[-3.68, -1.49]
Core inflation	0.69	[-0.75, 2.14]	1.03	[-0.40, 2.47]	1.06	[-0.38, 2.50]	-1.77	[-3.21, -0.33]
Consumption	-2.29	[-3.80, -0.76]	-13.10	[-15.02, -11.11]	-3.66	[-5.84, -1.40]	-3.94	[-6.29, -1.51]
Capital formation	0.00	[-0.06, 0.06]	0.00	[-0.06, 0.06]	0.00	[-0.07, 0.06]	0.00	[-0.06, 0.06]
Exports	0.00	[-0.04, 0.04]	0.00	[-0.05, 0.05]	0.00	[-0.05, 0.04]	0.00	[-0.05, 0.05]
Imports	3.22	[-1.01, 7.22]	-12.75	[-19.61, -6.05]	-2.92	[-10.51, 3.61]	-3.30	[-11.76, 3.70]
Unemployment rate	0.00	[-0.04, 0.02]	0.00	[-0.03, 0.02]	0.01	[-0.02, 0.04]	0.00	[-0.03, 0.02]
Consumer confidence	0.92	[0.01, 1.83]	-0.35	[-1.26, 0.56]	-1.60	[-2.51, -0.69]	-1.72	[-2.63, -0.81]
Capacity utilization	0.00	[-0.03, 0.02]	0.00	[-0.03, 0.02]	0.00	[-0.02, 0.03]	0.00	[-0.02, 0.02]
2021								
GDP	-4.55	[-5.84, -3.15]	-3.85	[-5.73, -2.36]	-1.89	[-3.90, -0.38]	-1.38	[-3.29, 0.26]
Core inflation	-1.03	[-2.46, 0.40]	-0.85	[-2.28, 0.59]	0.83	[-0.61, 2.26]	1.99	[0.55, 3.44]
Consumption	-9.27	[-11.78, -6.71]	-6.49	[-9.11, -3.87]	0.03	[-2.70, 2.70]	2.37	[-0.53, 5.16]
Capital formation	0.00	[-0.06, 0.06]	0.00	[-0.07, 0.06]	0.00	[-0.07, 0.06]	0.00	[-0.06, 0.07]
Exports	0.00	[-0.04, 0.04]	0.00	[-0.05, 0.05]	0.00	[-0.05, 0.05]	0.00	[-0.05, 0.05]
Imports	-9.84	[-19.35, -1.92]	-7.16	[-16.58, 0.31]	-2.01	[-10.89, 4.67]	-0.27	[-8.89, 6.04]
Unemployment rate	0.00	[-0.03, 0.02]	0.01	[-0.02, 0.04]	0.00	[-0.02, 0.03]	0.00	[-0.03, 0.02]
Consumer confidence	-1.52	[-2.43, -0.62]	-0.96	[-1.87, -0.05]	-0.35	[-1.25, 0.56]	-0.81	[-1.70, 0.10]
Capacity utilization	0.00	[-0.02, 0.02]	0.01	[-0.02, 0.03]	0.00	[-0.03, 0.02]	0.00	[-0.02, 0.03]
2022								
GDP	-2.66	[-4.44, -0.90]	-1.86	[-3.80, 0.07]	-1.74	[-3.91, 0.51]	-1.42	[-4.21, 1.50]
Core inflation	1.96	[0.52, 3.40]	1.61	[0.18, 3.05]	0.77	[-0.67, 2.21]	0.08	[-1.34, 1.52]
Consumption	-2.24	[-5.32, 0.74]	0.48	[-2.84, 3.75]	0.78	[-2.87, 4.44]	5.86	[1.70, 9.99]
Capital formation	0.00	[-0.06, 0.06]	-0.01	[-0.07, 0.06]	0.00	[-0.06, 0.06]	0.00	[-0.06, 0.06]
Exports	0.00	[-0.05, 0.05]	0.00	[-0.05, 0.05]	0.00	[-0.05, 0.05]	0.00	[-0.04, 0.05]
Imports	-5.96	[-14.81, 0.34]	-1.61	[-9.92, 4.46]	0.90	[-6.83, 7.57]	2.82	[-5.11, 10.55]
Unemployment rate	0.00	[-0.03, 0.02]	0.00	[-0.03, 0.02]	0.00	[-0.02, 0.03]	0.00	[-0.02, 0.03]
Consumer confidence	-1.30	[-2.21, -0.40]	-1.97	[-2.88, -1.06]	-2.81	[-3.72, -1.91]	-3.27	[-4.18, -2.36]
Capacity utilization	0.00	[-0.02, 0.03]	0.00	[-0.03, 0.02]	0.00	[-0.03, 0.02]	0.00	[-0.03, 0.03]

Note: "90% interval" gives the 90 per cent credible interval.

Table C4. Parameter posterior estimates, dummy variables, Sweden.

Parameter	Q1 Mean	Q1 90% interval	Q2 Mean	Q2 90% interval	Q3 Mean	Q3 90% interval	Q4 Mean	Q4 90% interval
2020								
GDP	0.49	[-0.48, 1.50]	-7.21	[-8.87, -5.70]	-0.68	[-1.86, 0.63]	-1.48	[-2.69, -0.46]
Core inflation	-2.88	[-6.14, 0.39]	-0.87	[-4.10, 2.41]	2.61	[-0.60, 5.87]	0.89	[-2.35, 4.15]
Consumption	1.14	[0.05, 2.24]	-7.88	[-9.33, -6.32]	-3.39	[-5.03, -1.66]	-3.70	[-5.46, -1.87]
Capital formation	0.00	[-0.04, 0.04]	0.00	[-0.04, 0.04]	0.00	[-0.04, 0.05]	0.00	[-0.04, 0.04]
Exports	4.04	[1.47, 6.63]	-10.11	[-13.74, -6.63]	-0.05	[-3.27, 3.03]	2.83	[-0.22, 5.99]
Imports	2.84	[0.65, 5.01]	-6.82	[-10.25, -3.65]	-1.01	[-3.88, 1.66]	1.62	[-1.02, 4.37]
Unemployment rate	-0.12	[-0.44, 0.01]	0.07	[-0.05, 0.33]	0.11	[-0.02, 0.45]	-0.09	[-0.38, 0.02]
Consumer confidence	0.39	[-0.65, 1.44]	-0.13	[-1.18, 0.93]	-0.07	[-1.14, 0.98]	-0.22	[-1.28, 0.85]
Capacity utilization	0.85	[0.29, 1.40]	-2.24	[-2.92, -1.55]	-0.12	[-0.82, 0.60]	1.04	[0.32, 1.76]
2021								
GDP	-1.33	[-2.61, 0.01]	-1.80	[-3.37, 0.13]	-1.43	[-3.44, 0.72]	-0.66	[-3.70, 1.75]
Core inflation	3.20	[-0.03, 6.45]	-1.69	[-4.92, 1.56]	1.18	[-2.06, 4.42]	3.39	[0.14, 6.65]
Consumption	-1.93	[-3.75, -0.07]	-1.71	[-3.56, 0.16]	0.98	[-0.91, 2.83]	2.13	[0.15, 4.07]
Capital formation	0.00	[-0.04, 0.04]	0.00	[-0.04, 0.04]	0.00	[-0.05, 0.04]	0.00	[-0.03, 0.04]
Exports	2.08	[-1.07, 5.25]	0.43	[-2.69, 3.63]	-1.38	[-4.59, 1.82]	-0.27	[-3.49, 2.93]
Imports	2.02	[-0.64, 4.84]	-0.46	[-3.11, 2.23]	-1.13	[-3.88, 1.54]	0.47	[-2.13, 3.13]
Unemployment rate	0.04	[-0.11, 0.28]	0.07	[-0.05, 0.36]	0.04	[-0.11, 0.28]	-0.02	[-0.22, 0.14]
Consumer confidence	0.30	[-0.76, 1.36]	0.95	[-0.12, 2.01]	0.87	[-0.20, 1.94]	0.21	[-0.85, 1.27]
Capacity utilization	0.95	[0.20, 1.69]	1.31	[0.54, 2.08]	1.19	[0.39, 2.00]	-0.45	[-1.30, 0.40]
2022								
GDP	-1.78	[-4.79, 0.70]	-2.20	[-5.26, 0.47]	-2.46	[-5.63, 0.57]	-3.71	[-7.17, -0.23]
Core inflation	5.25	[1.99, 8.50]	3.82	[0.59, 7.07]	0.05	[-3.21, 3.31]	-1.43	[-4.67, 1.81]
Consumption	0.57	[-1.51, 2.62]	1.69	[-0.57, 3.95]	-0.48	[-2.99, 2.07]	-1.46	[-4.37, 1.52]
Capital formation	0.00	[-0.05, 0.04]	0.00	[-0.04, 0.04]	0.00	[-0.04, 0.04]	0.00	[-0.04, 0.04]
Exports	0.34	[-3.05, 3.69]	-0.70	[-4.23, 2.81]	0.92	[-2.83, 4.67]	-0.32	[-4.75, 4.09]
Imports	0.41	[-2.22, 3.16]	1.09	[-1.61, 3.97]	1.16	[-1.67, 4.14]	-1.22	[-4.84, 2.11]
Unemployment rate	-0.06	[-0.32, 0.06]	0.05	[-0.08, 0.28]	-0.13	[-0.49, 0.02]	0.07	[-0.08, 0.41]
Consumer confidence	-1.05	[-2.11, 0.01]	-2.31	[-3.36, -1.24]	-3.96	[-5.02, -2.89]	-4.66	[-5.76, -3.55]
Capacity utilization	-0.15	[-1.02, 0.75]	-0.06	[-0.99, 0.89]	-0.10	[-1.07, 0.88]	0.23	[-0.78, 1.23]

Note: "90% interval" gives the 90 per cent credible interval.